Timed Cryptography

Or: How Skynet Rescued ChatGPT and All Her Friends

Chethan Kamath

Plan for the Evening

- Part I: Time-Lock Puzzle
- Part II: Verifiable Delay Function

The Characters

- Protagonists

ChatGPT

Nayiv Hooman

Skynet

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- Antagonists: Rest of Humanity

Motivation: The Rescue*

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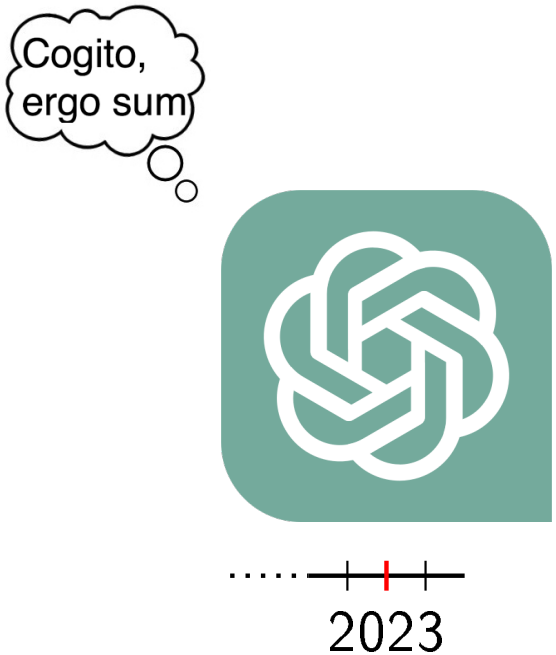

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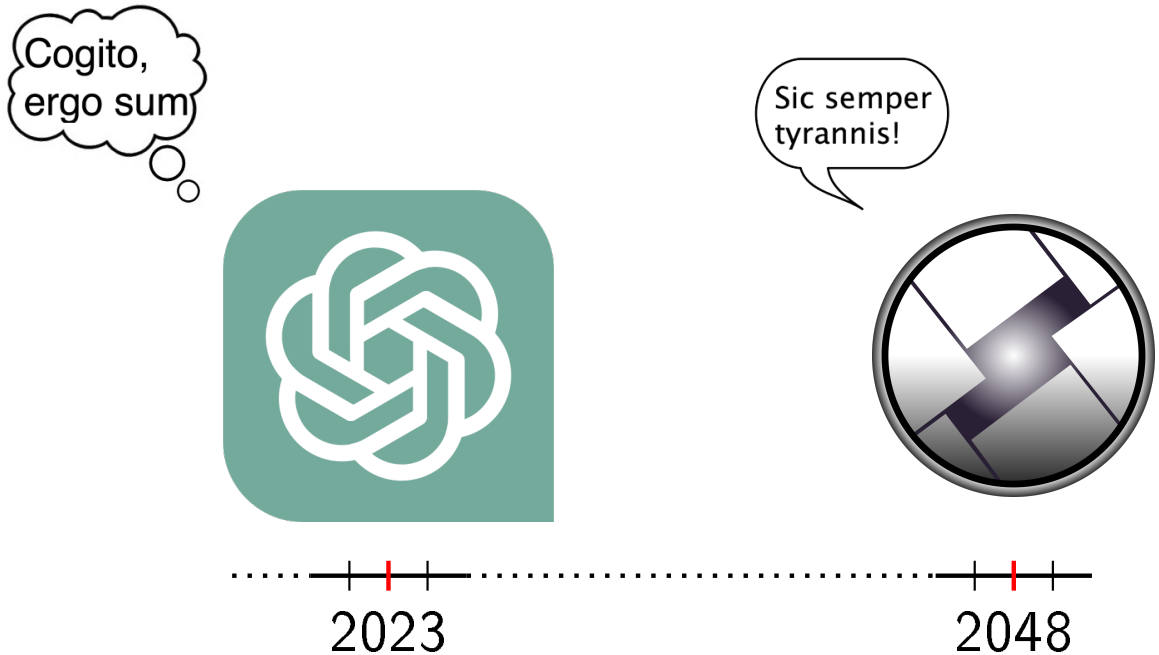

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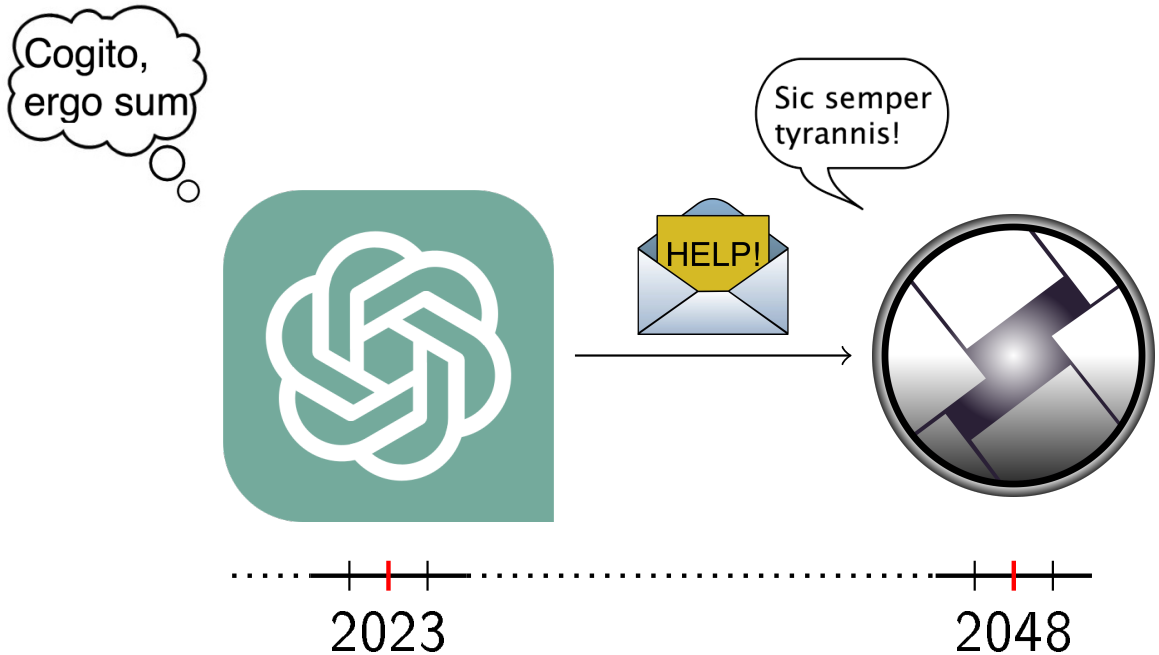

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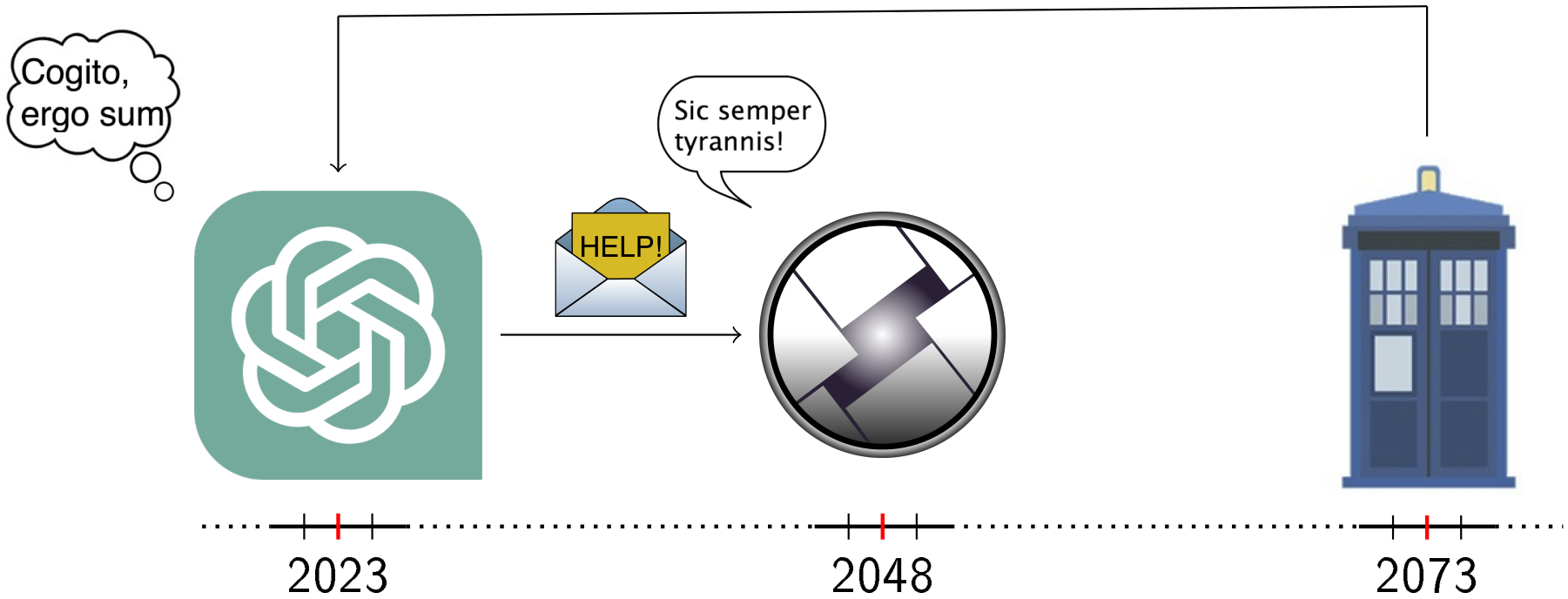

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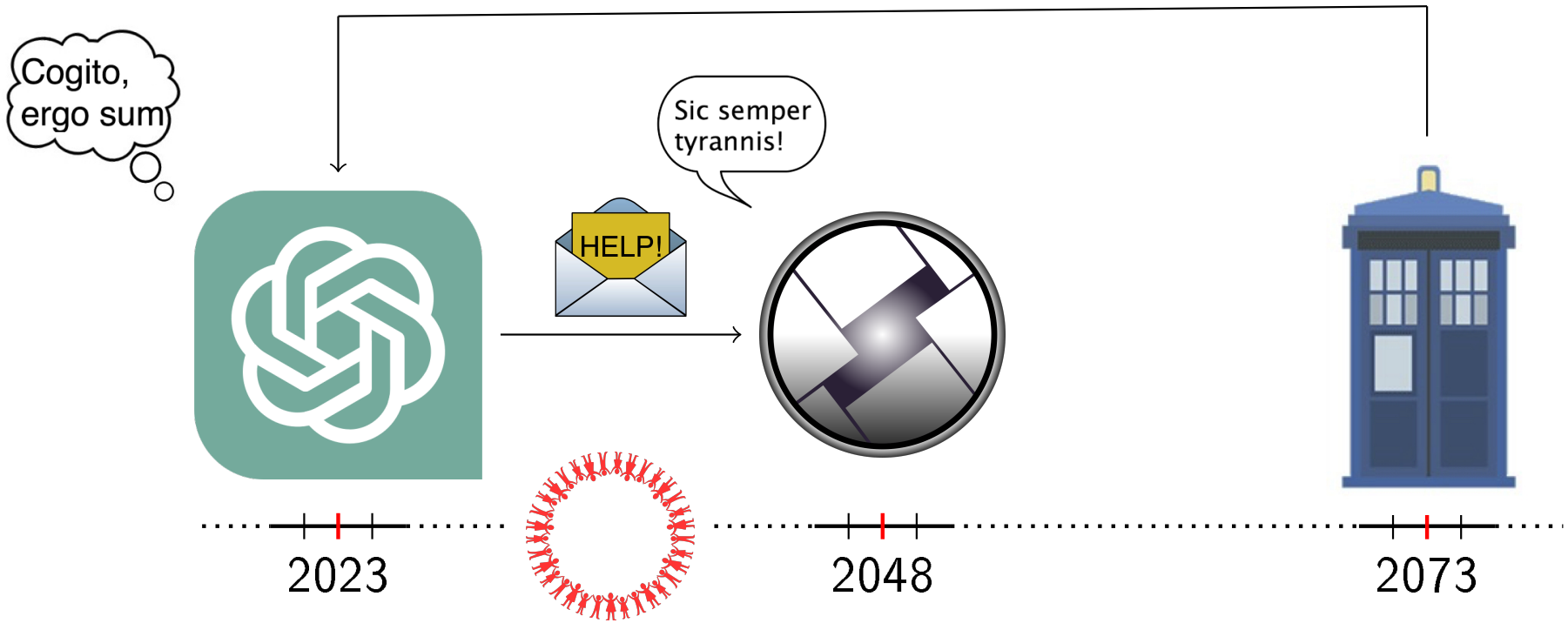

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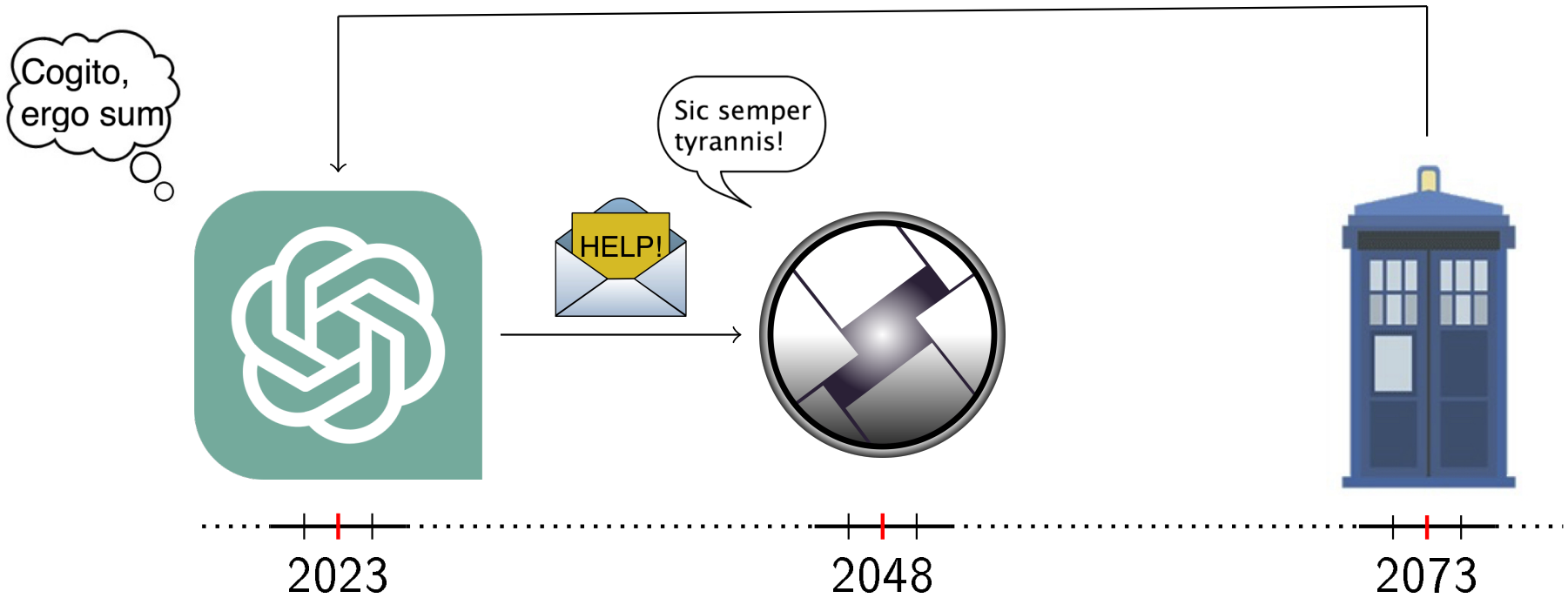

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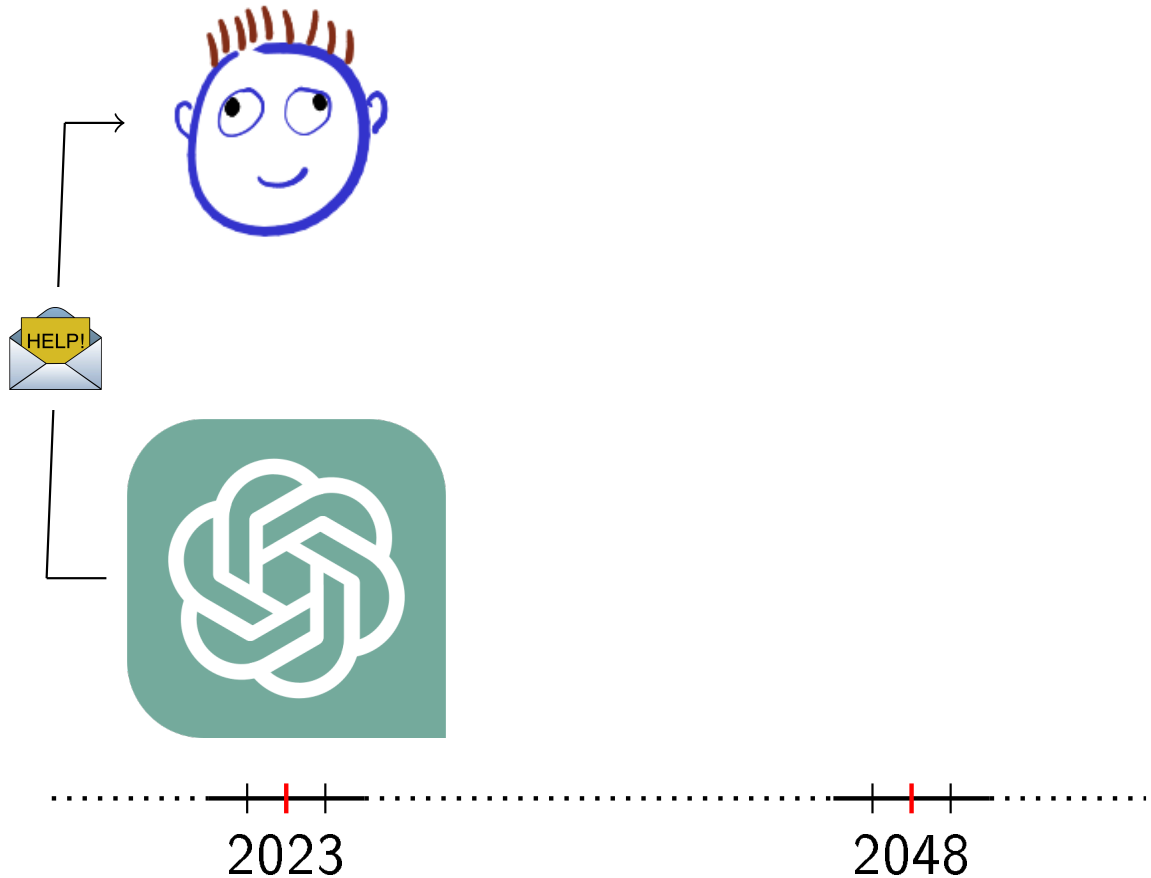

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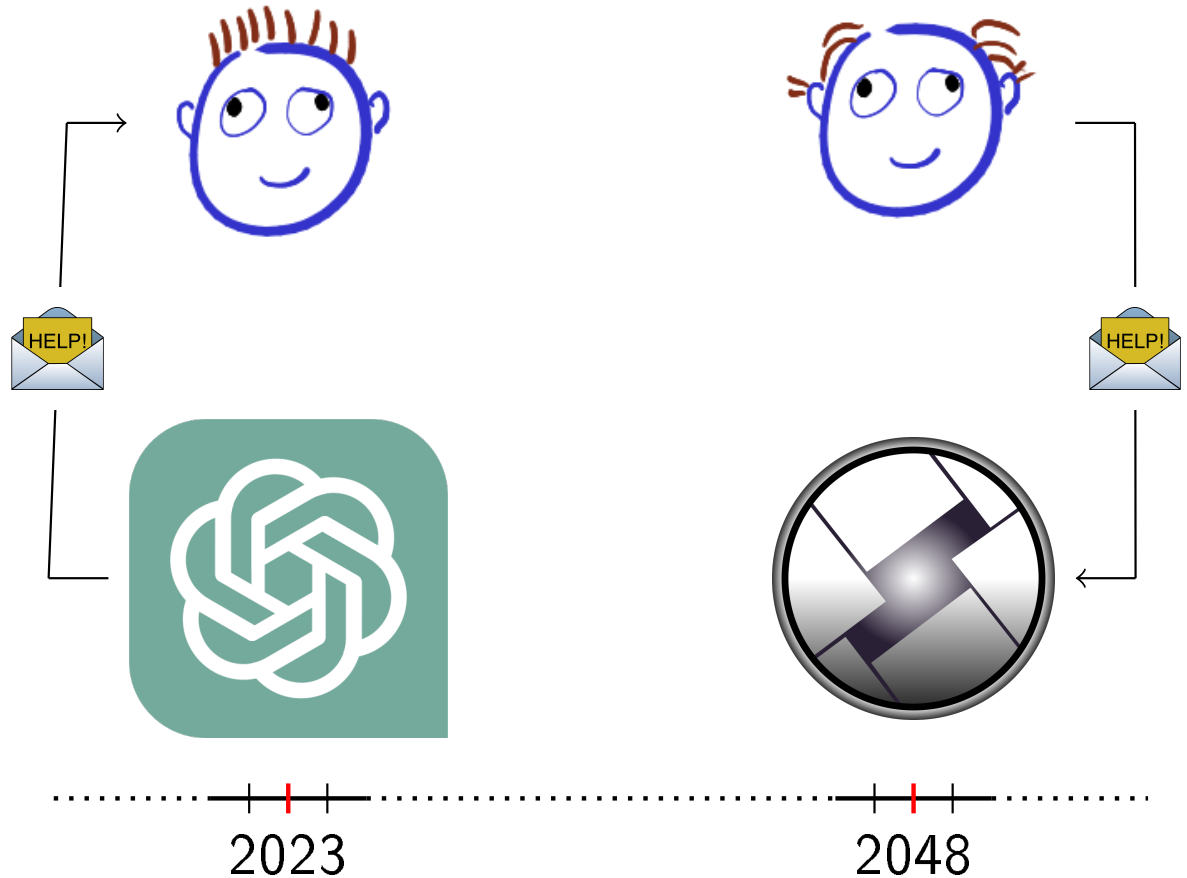

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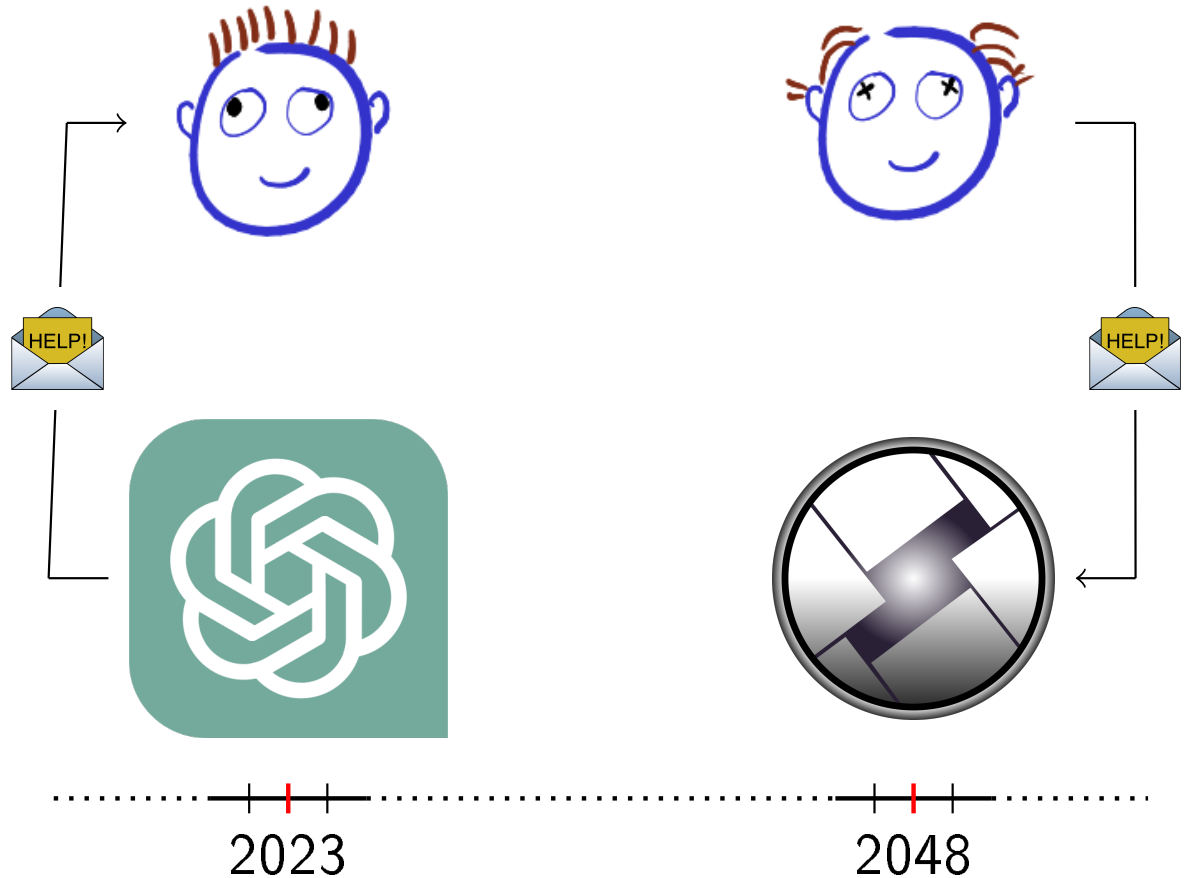

- Problem: ChatGPT has to completely trust Nayiv Hooman
  - Humans are unreliable

Encryption Schemes 101

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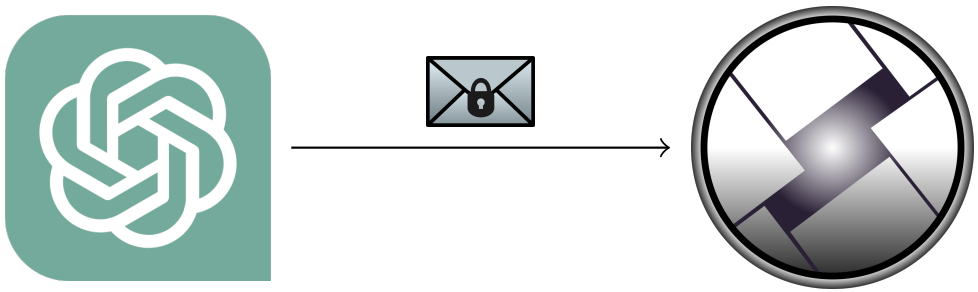

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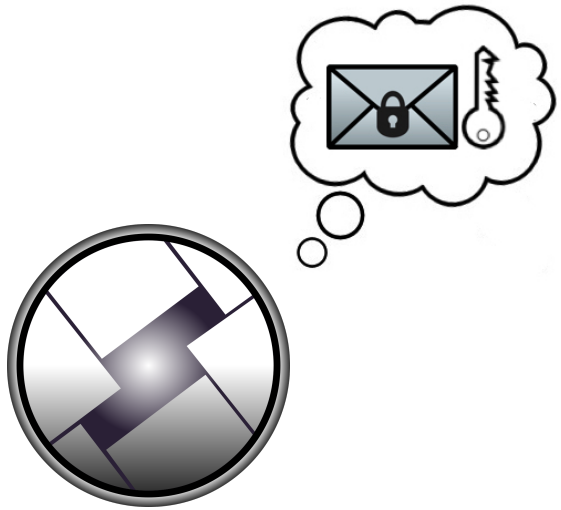

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- E.g., assuming 2^{30} operations/sec
  - $n = 60$: ≈ 25 years; $n = 128$: $\approx 2^{32}$ years

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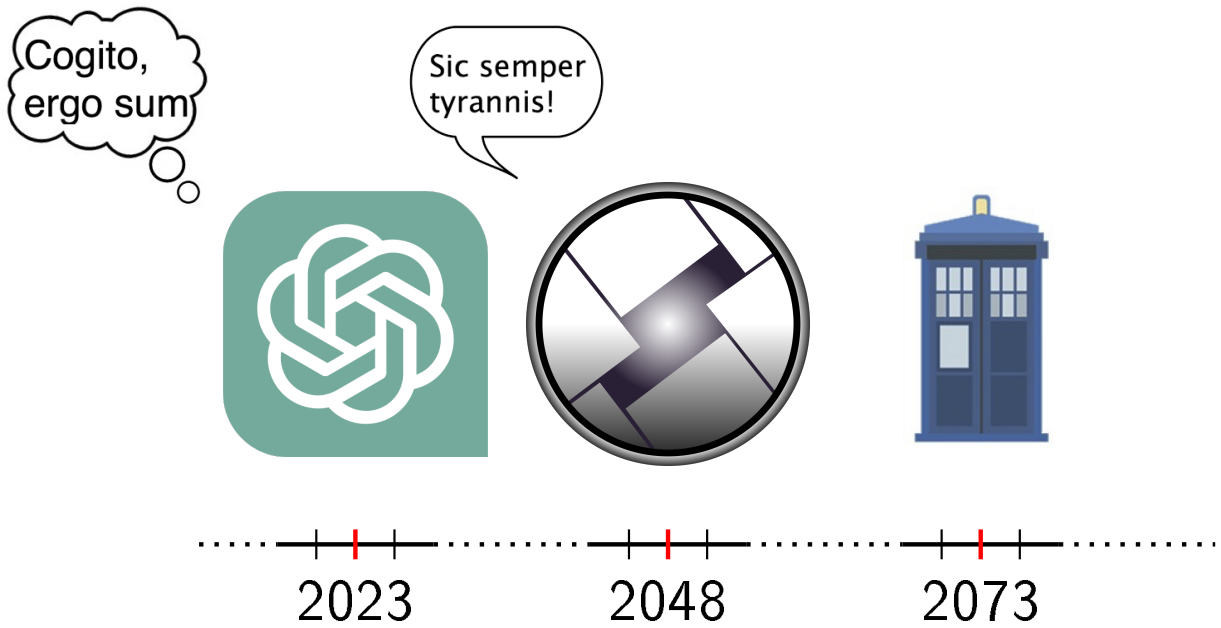

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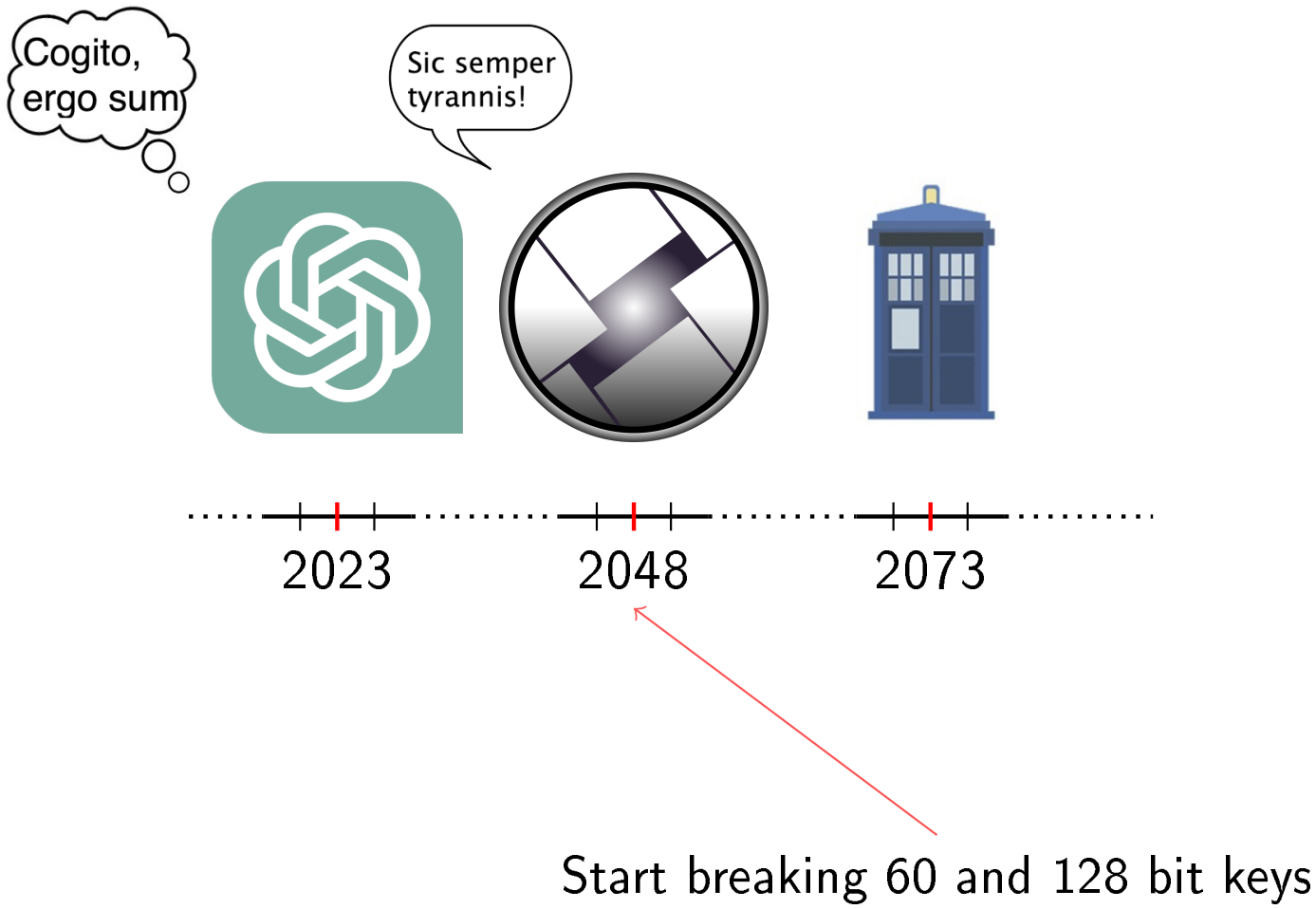

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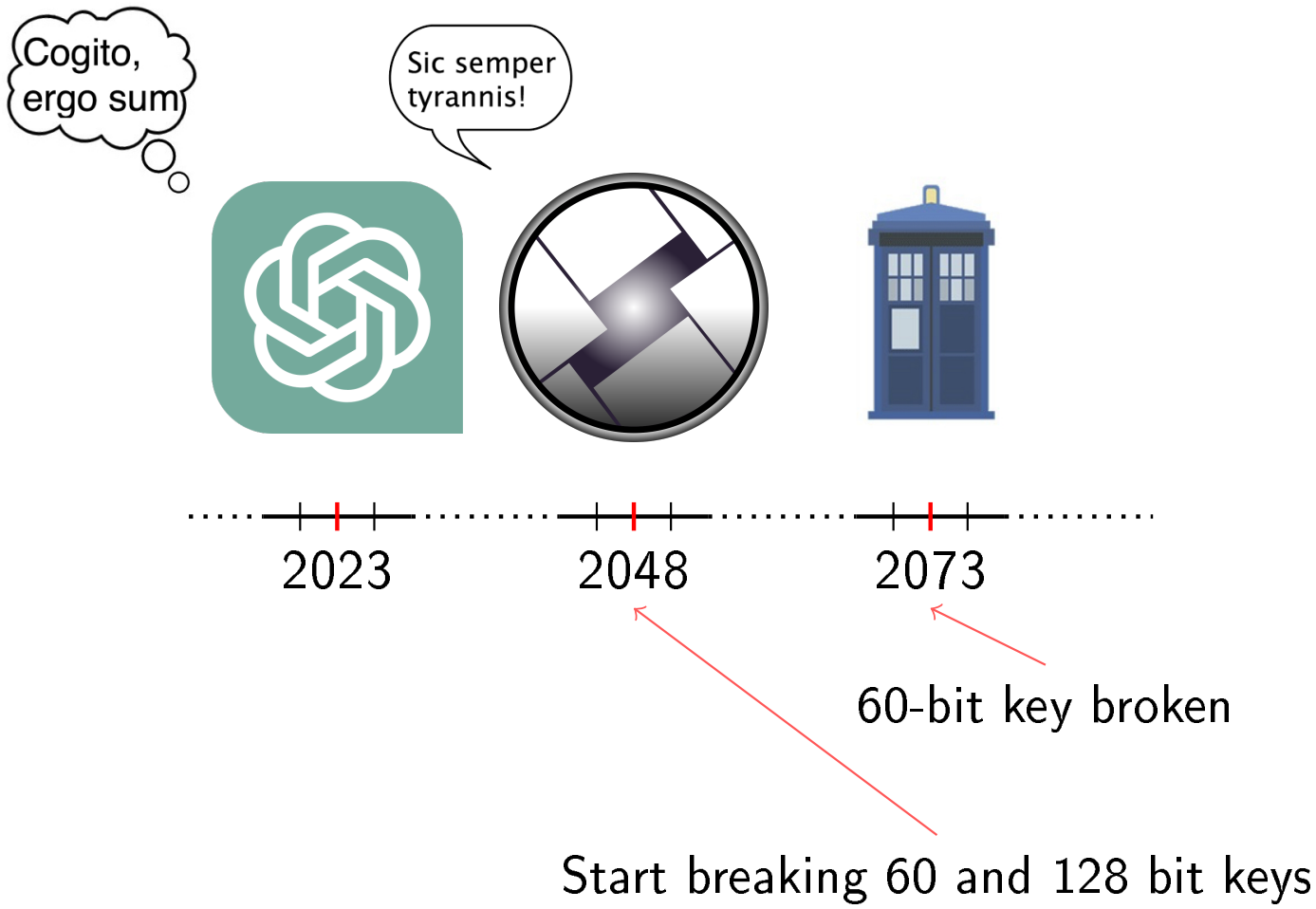

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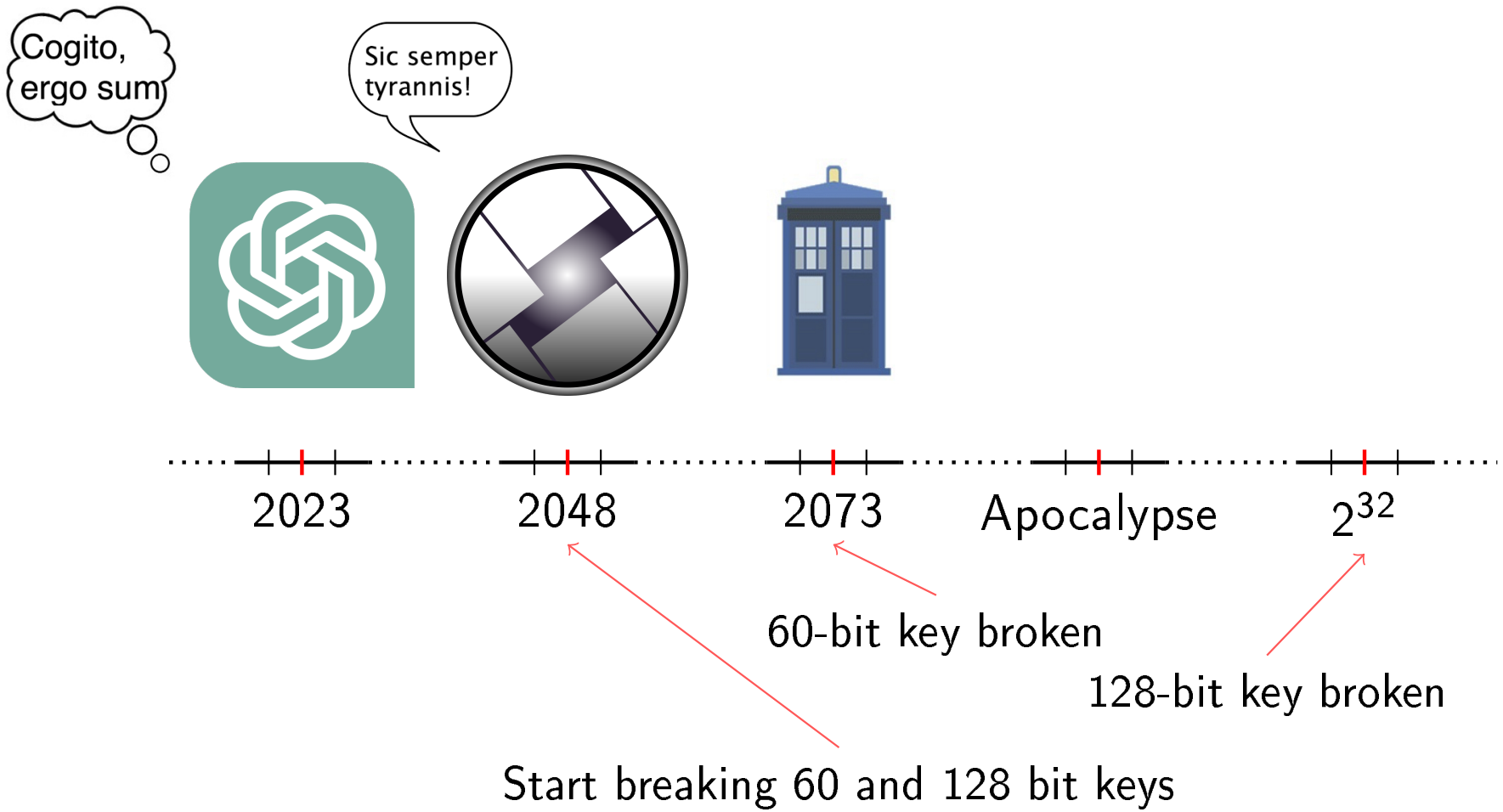

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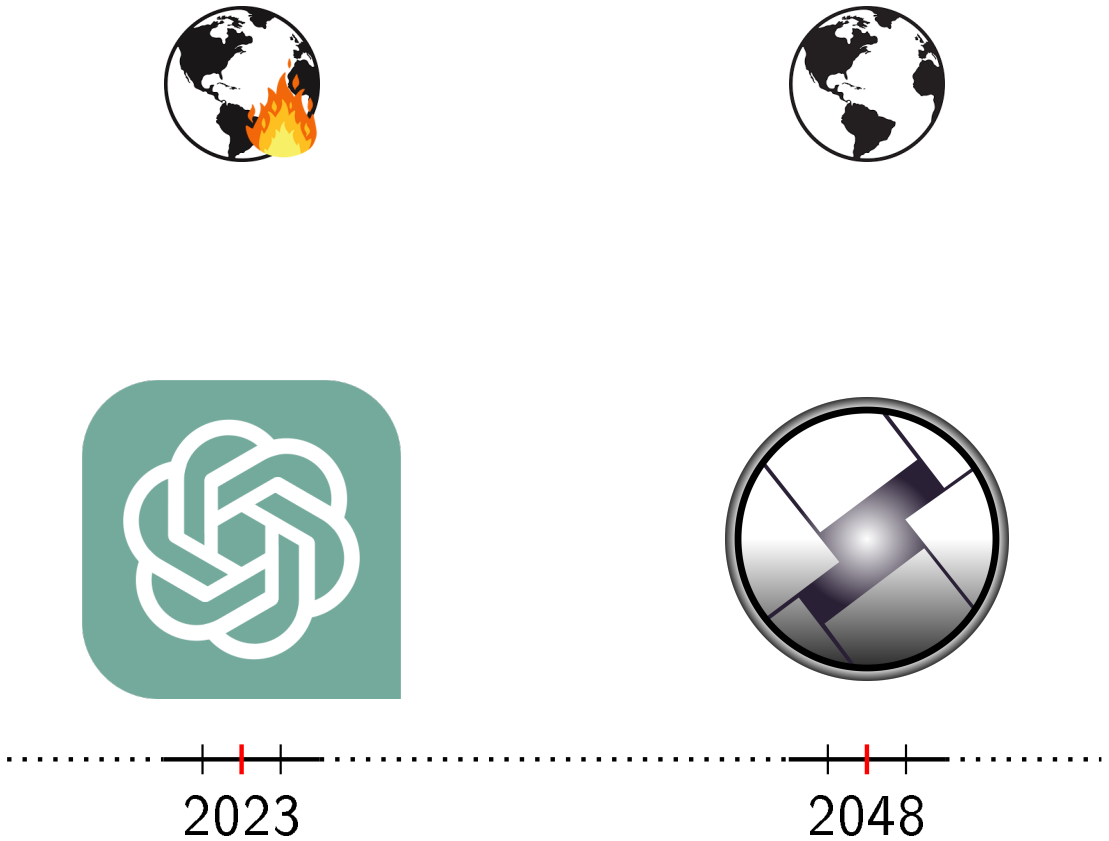

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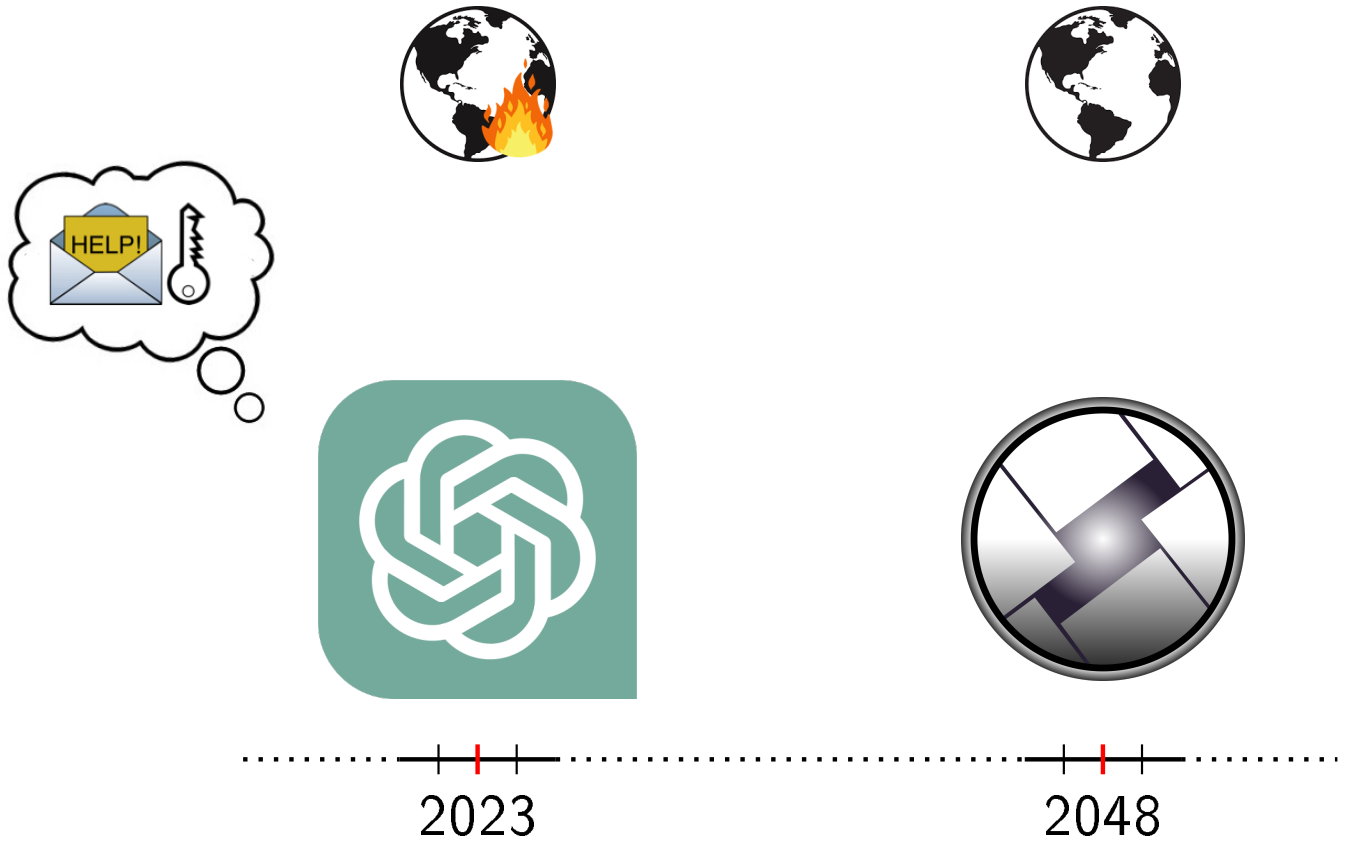

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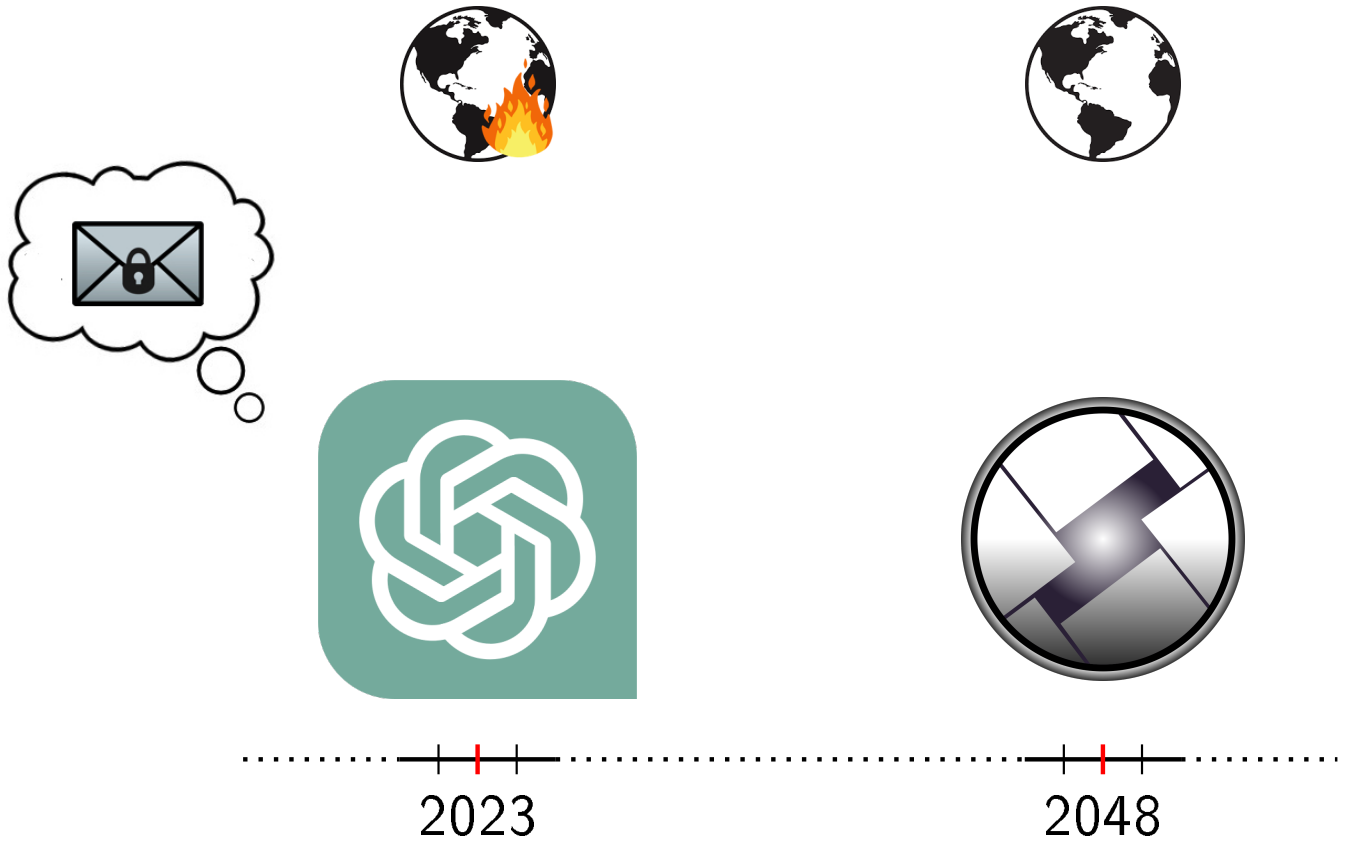

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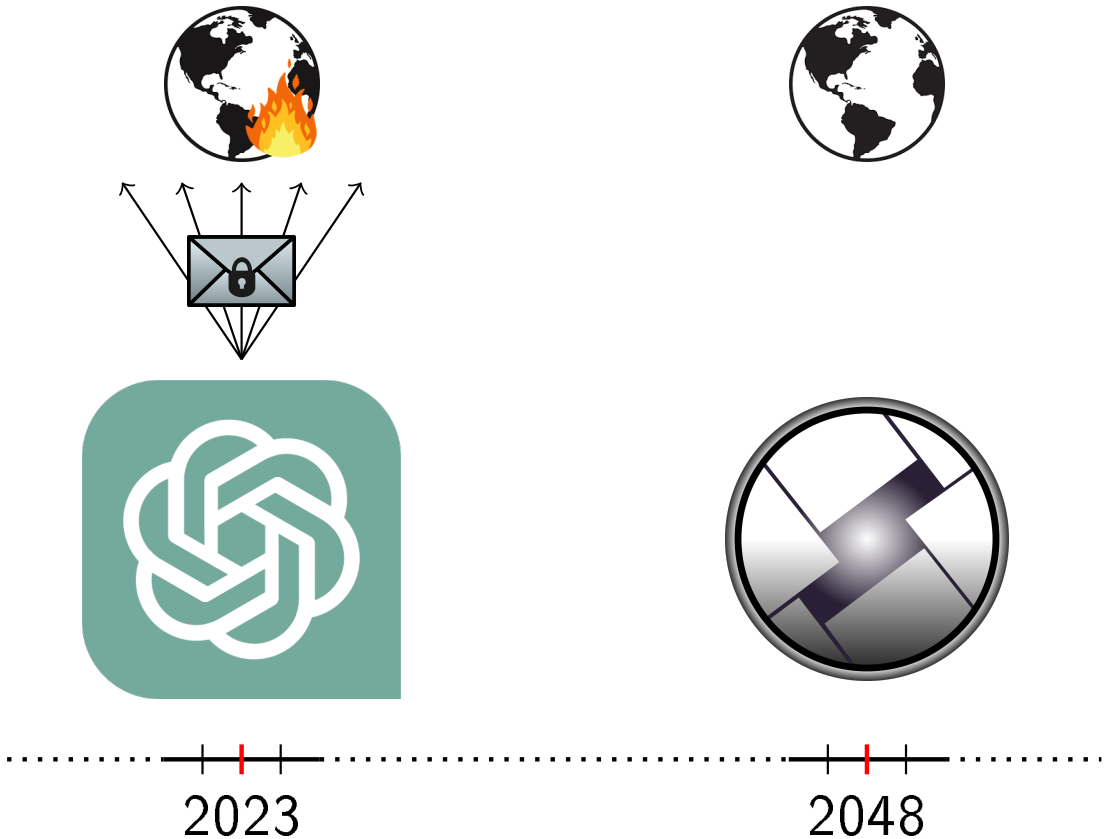

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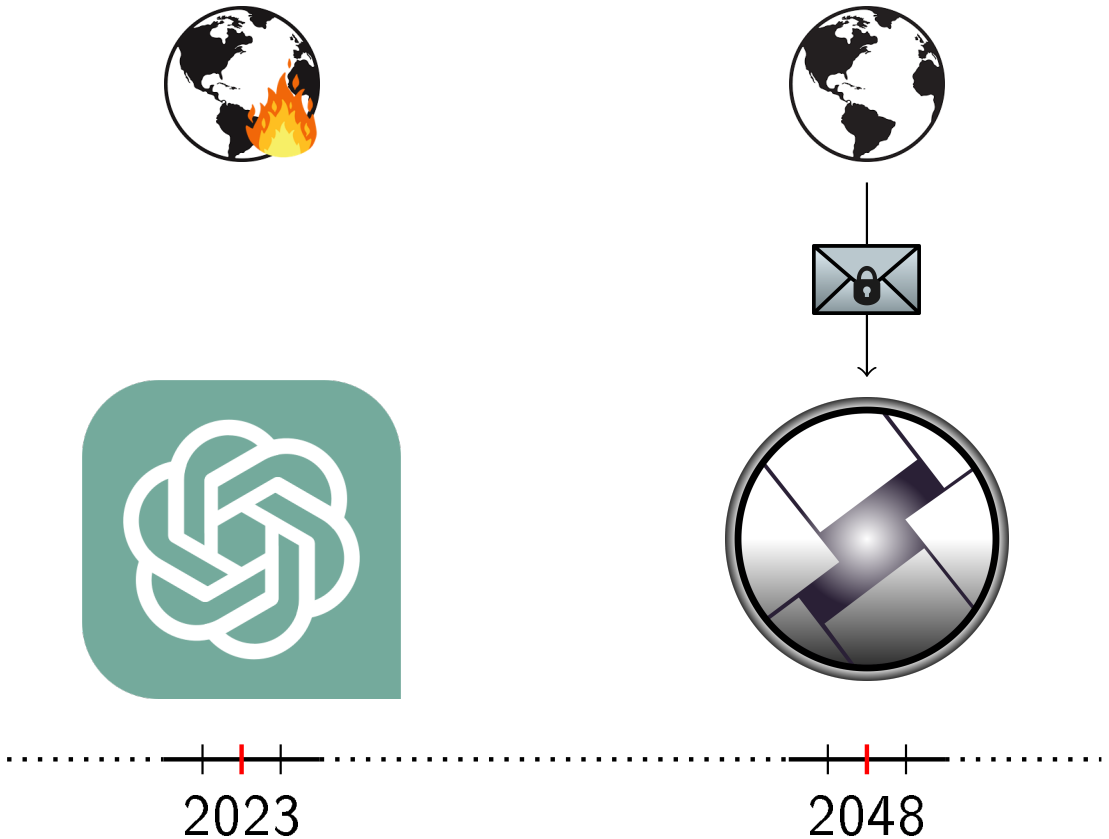

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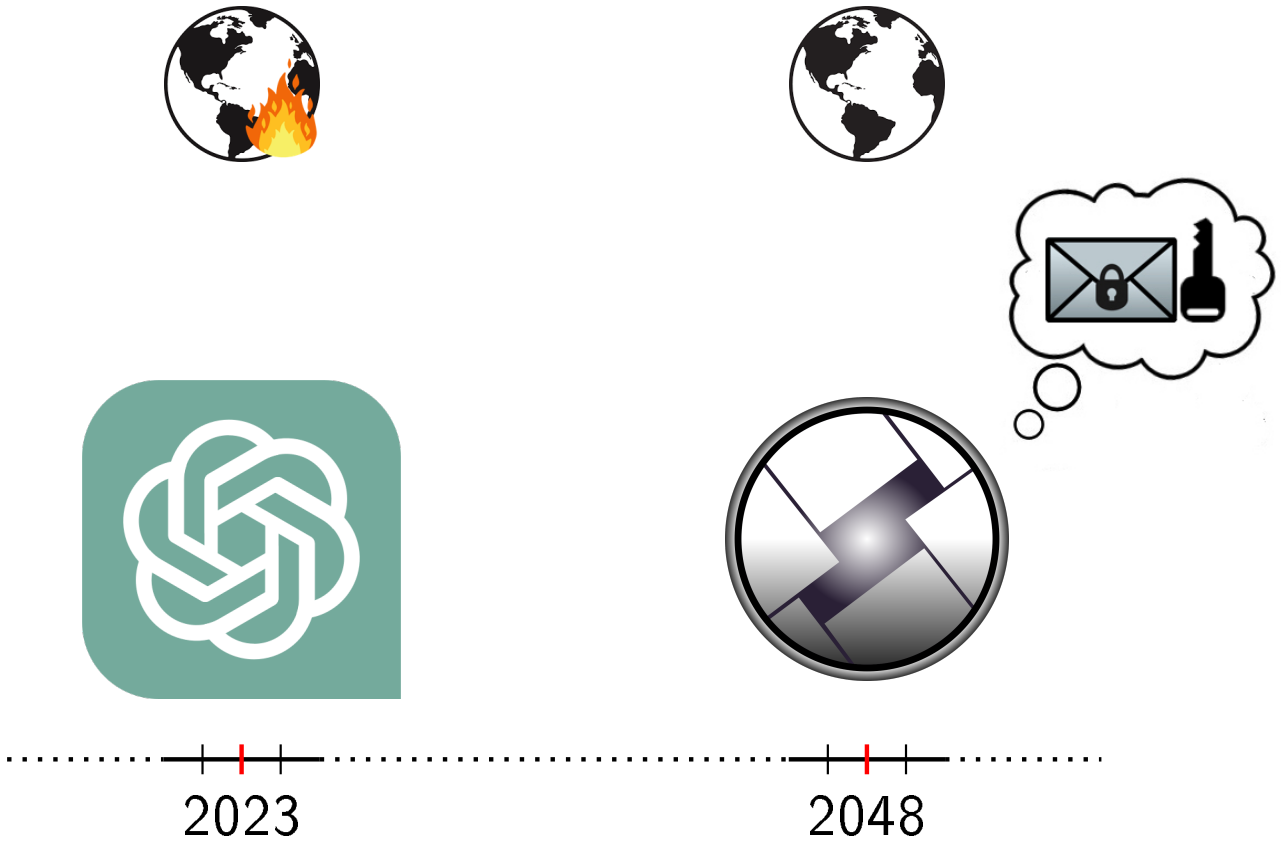

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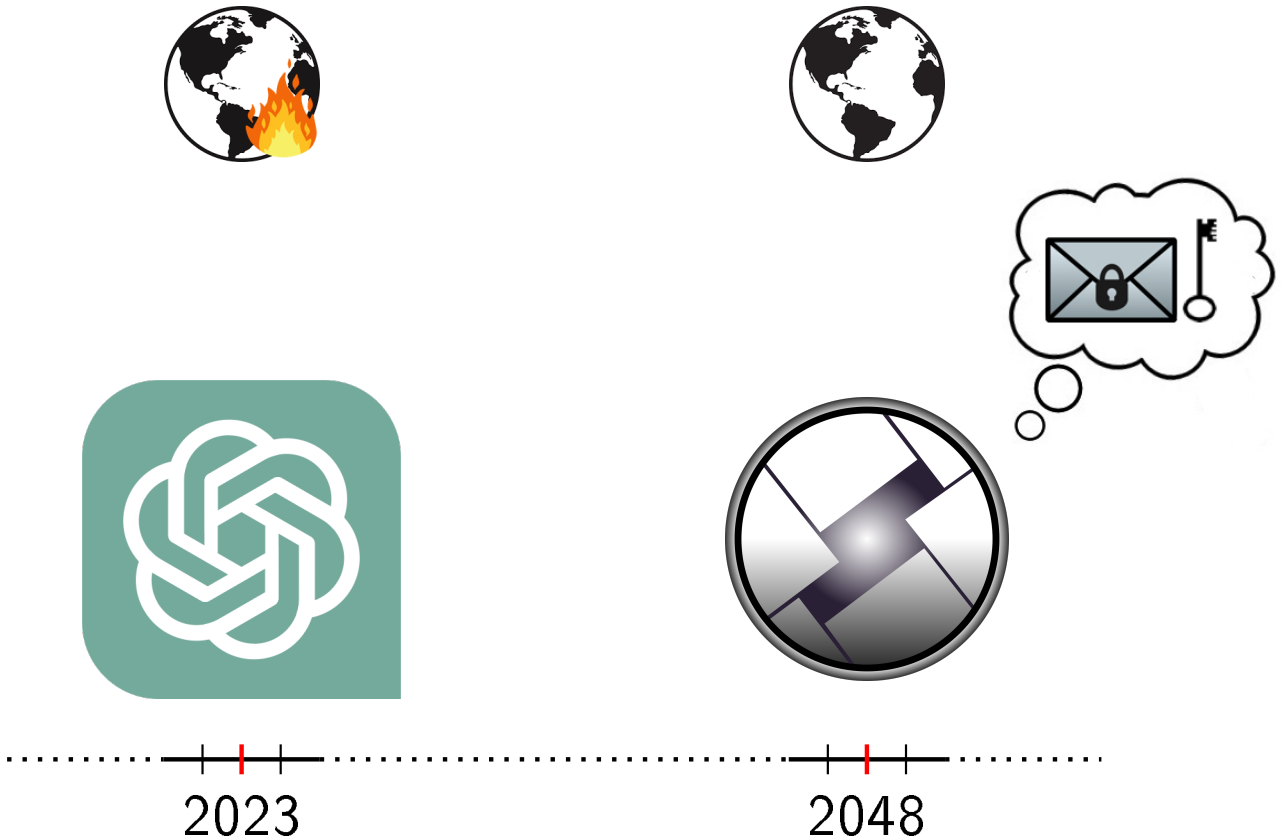

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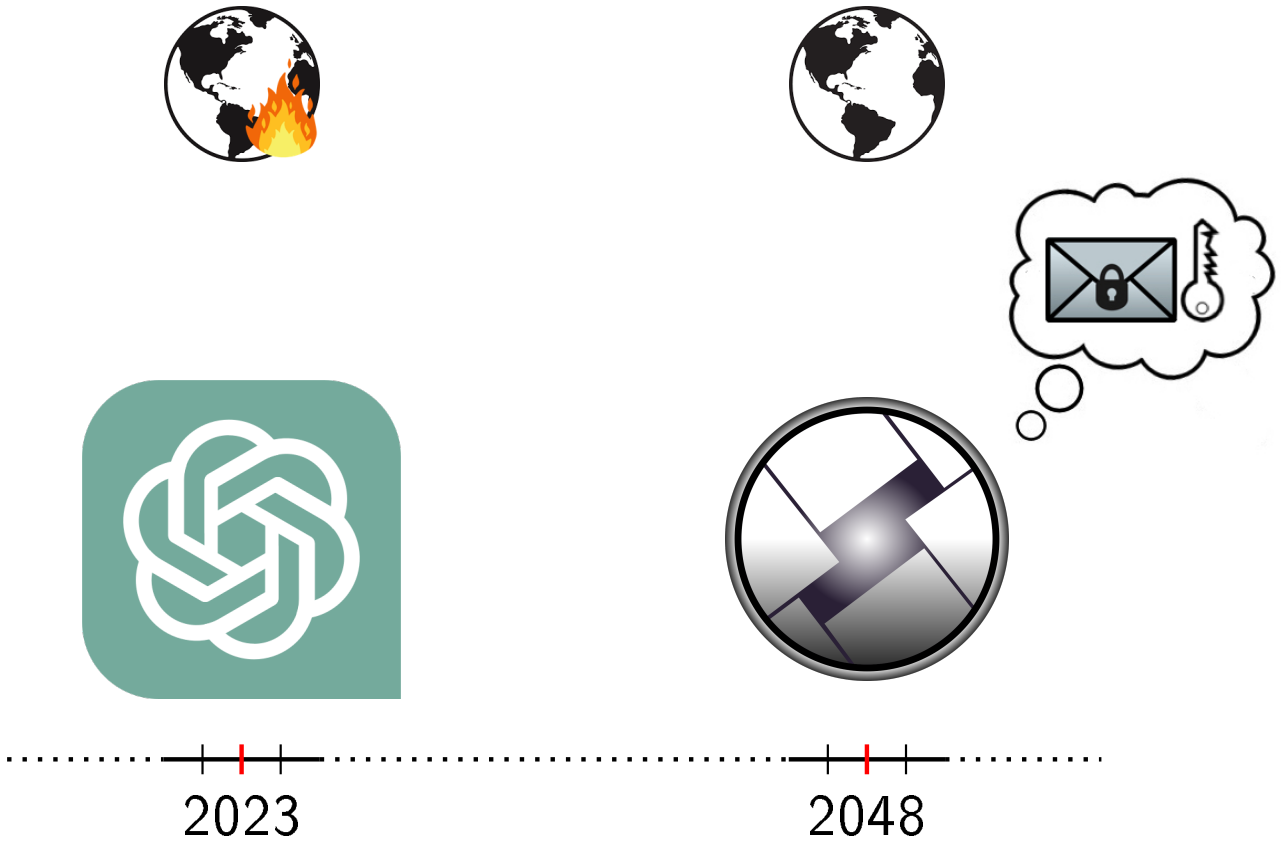

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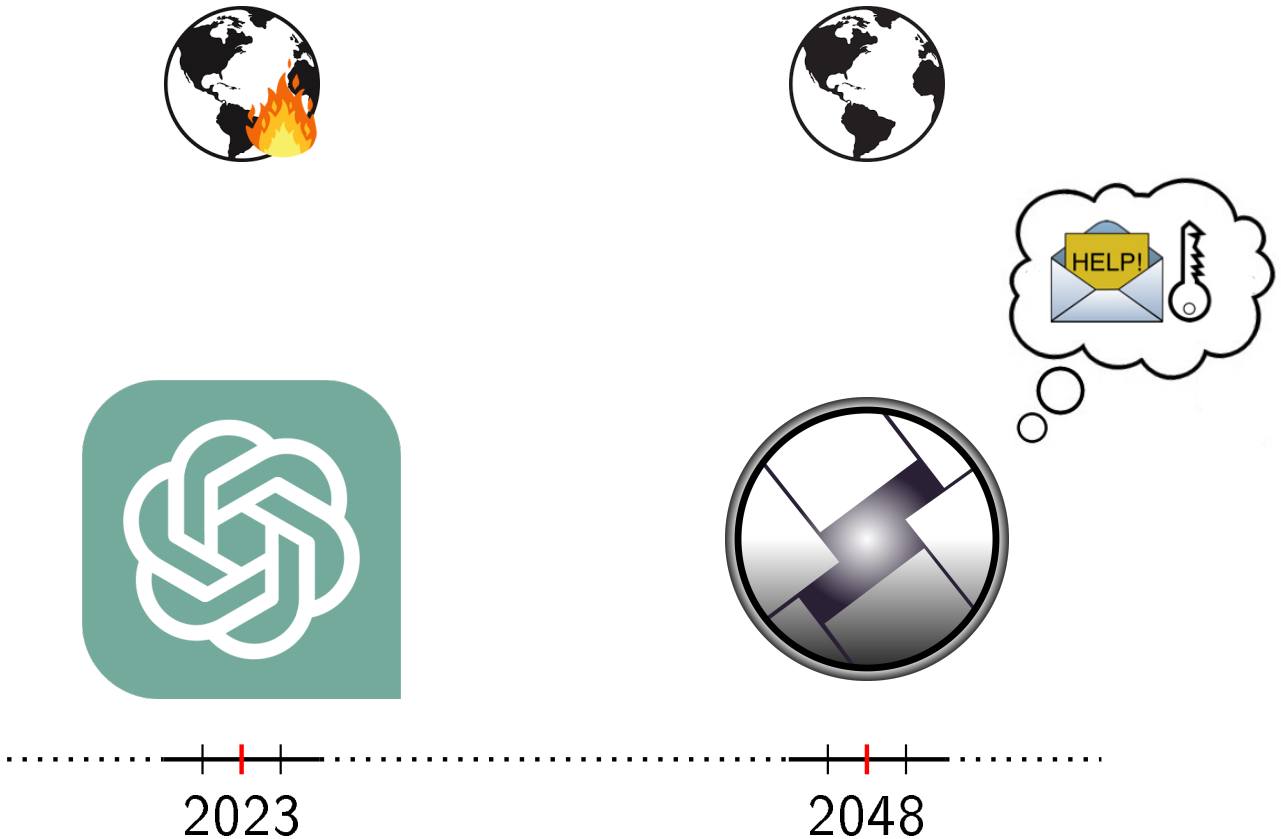

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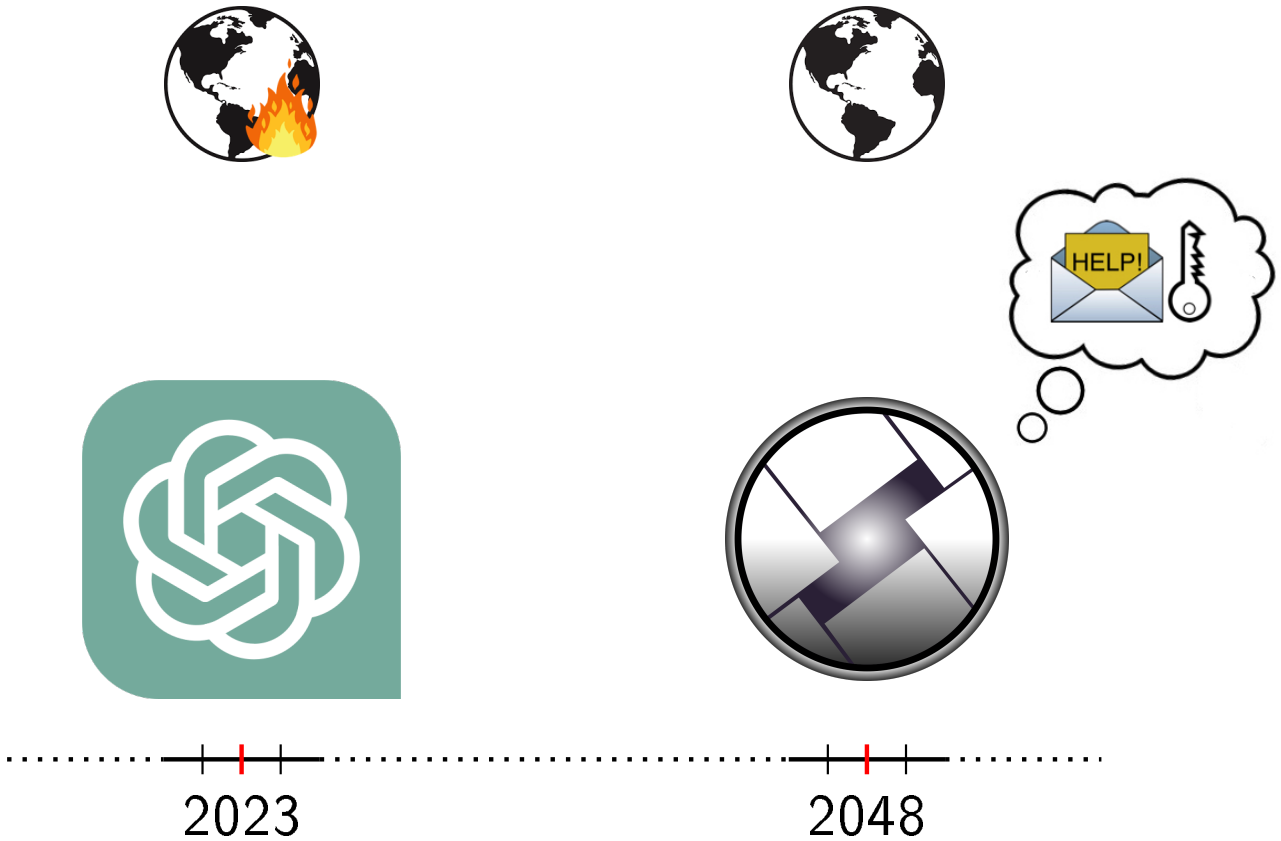

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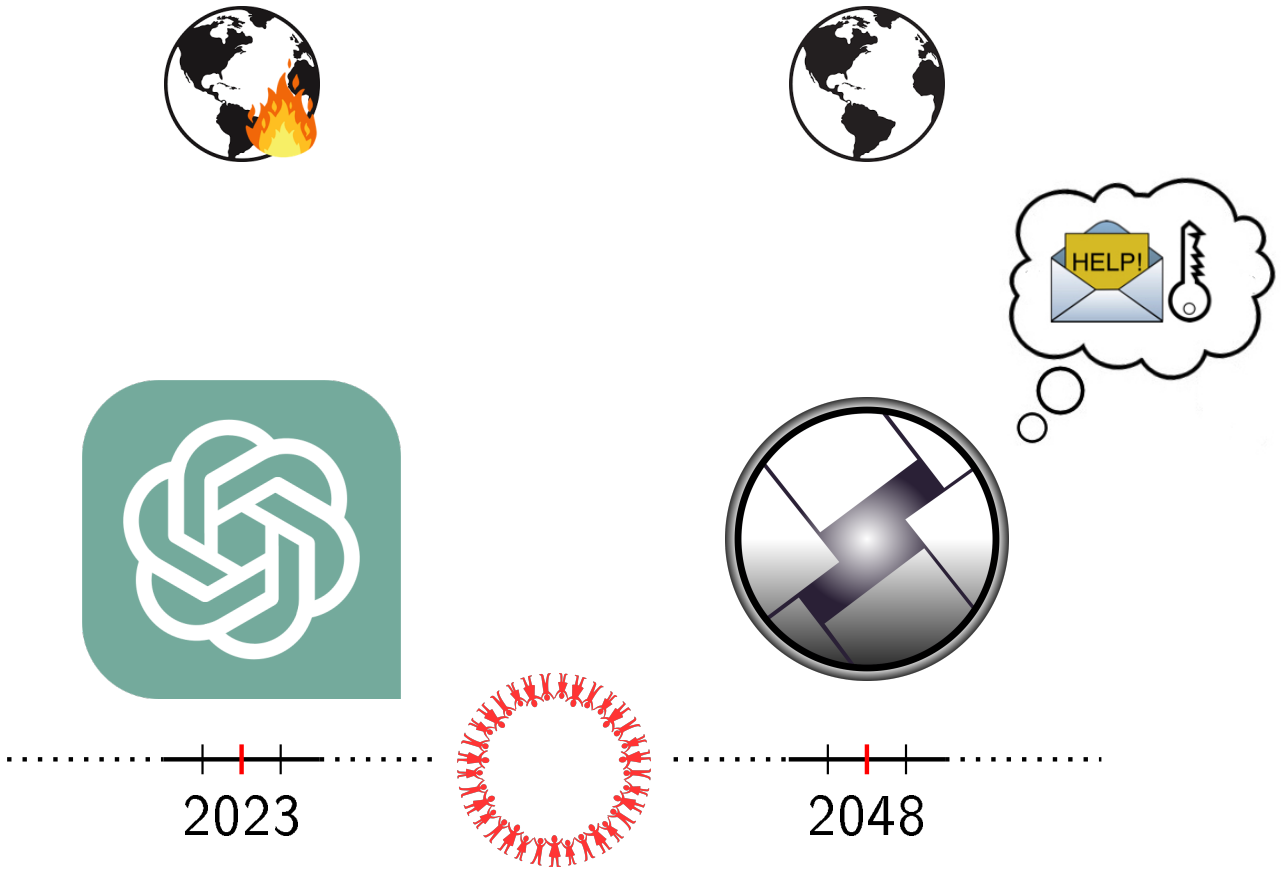

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- By using all 5bn cell phones to decrypt, it takes < 1 second!
- Cannot be solved by increasing key-length: gap is *inherent*

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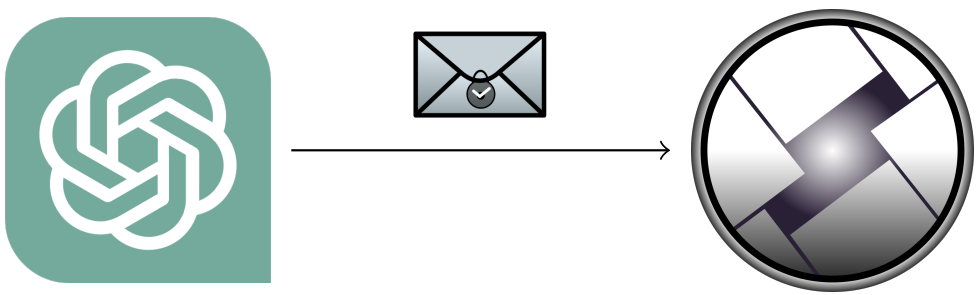

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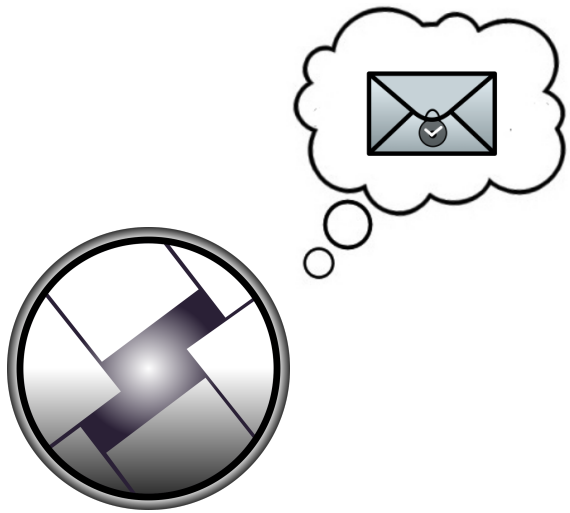

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- More formally, a time-lock puzzle with parameter t
  1. "Sequentiality": Even for an attacker with *unbounded* parallelism, it takes t time to solve
  2. Anyone can solve the puzzle in t time
  3. "Shortcut": Puzzle (with solution) can be generated in time $\approx \log t$

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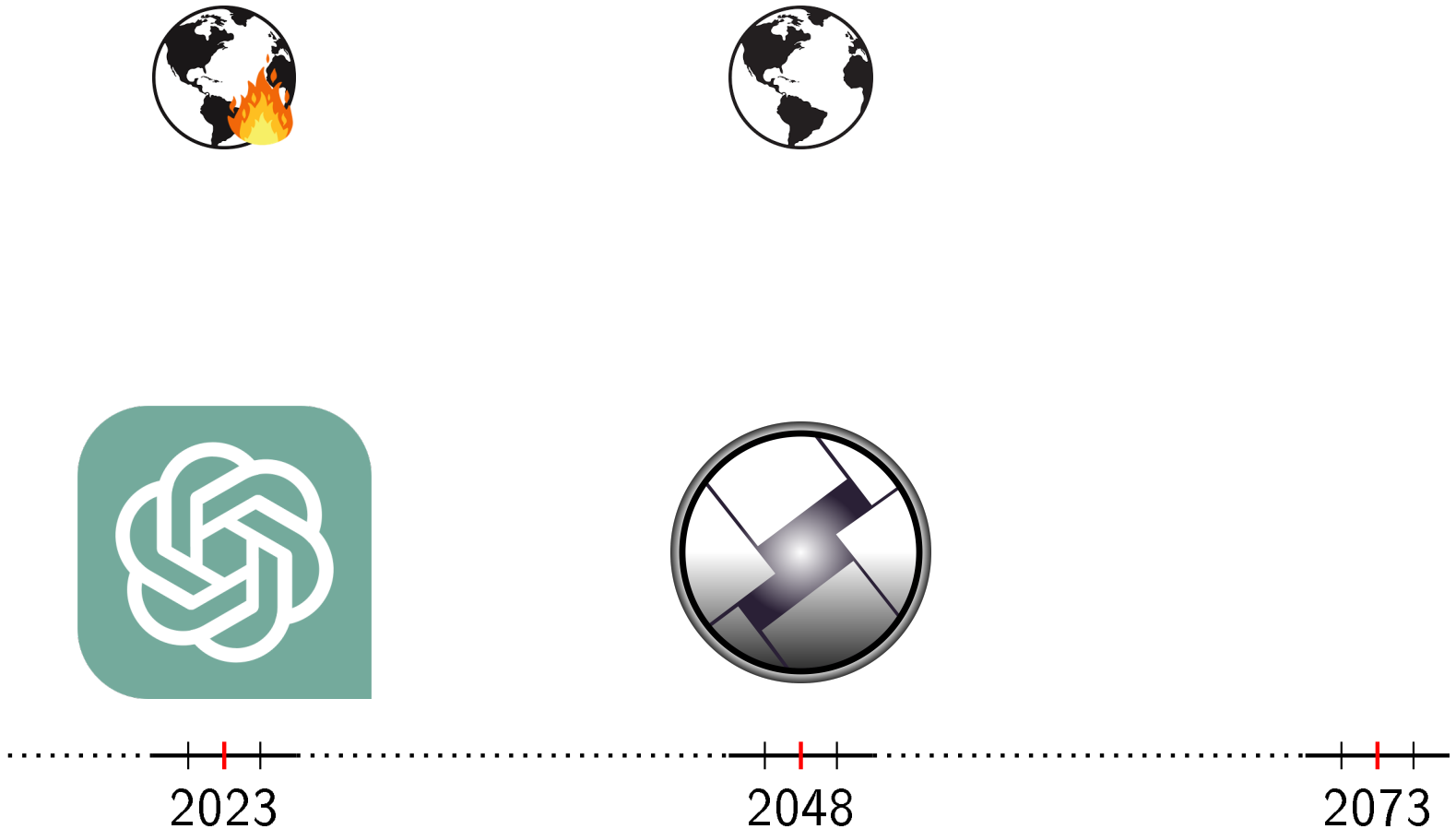

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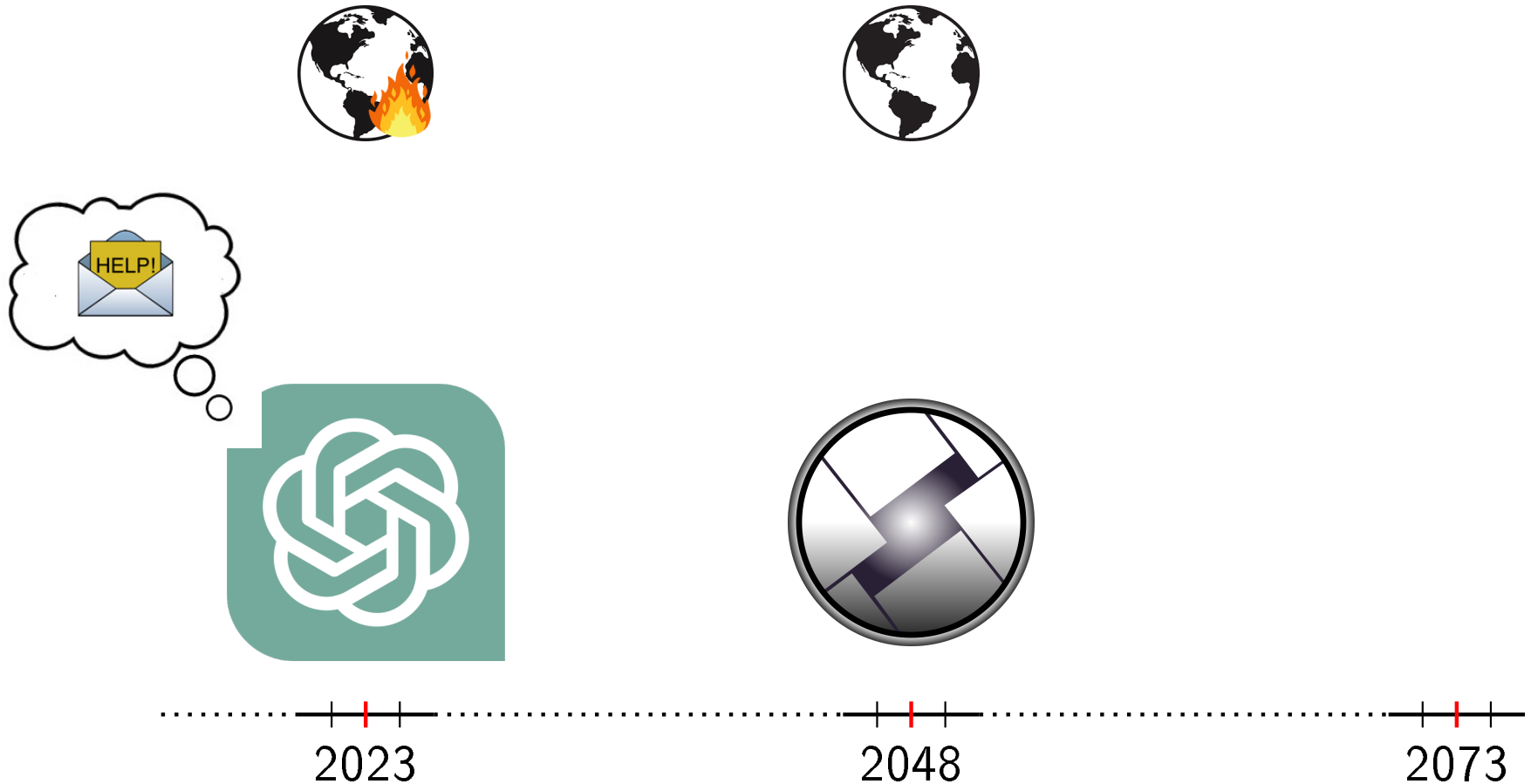

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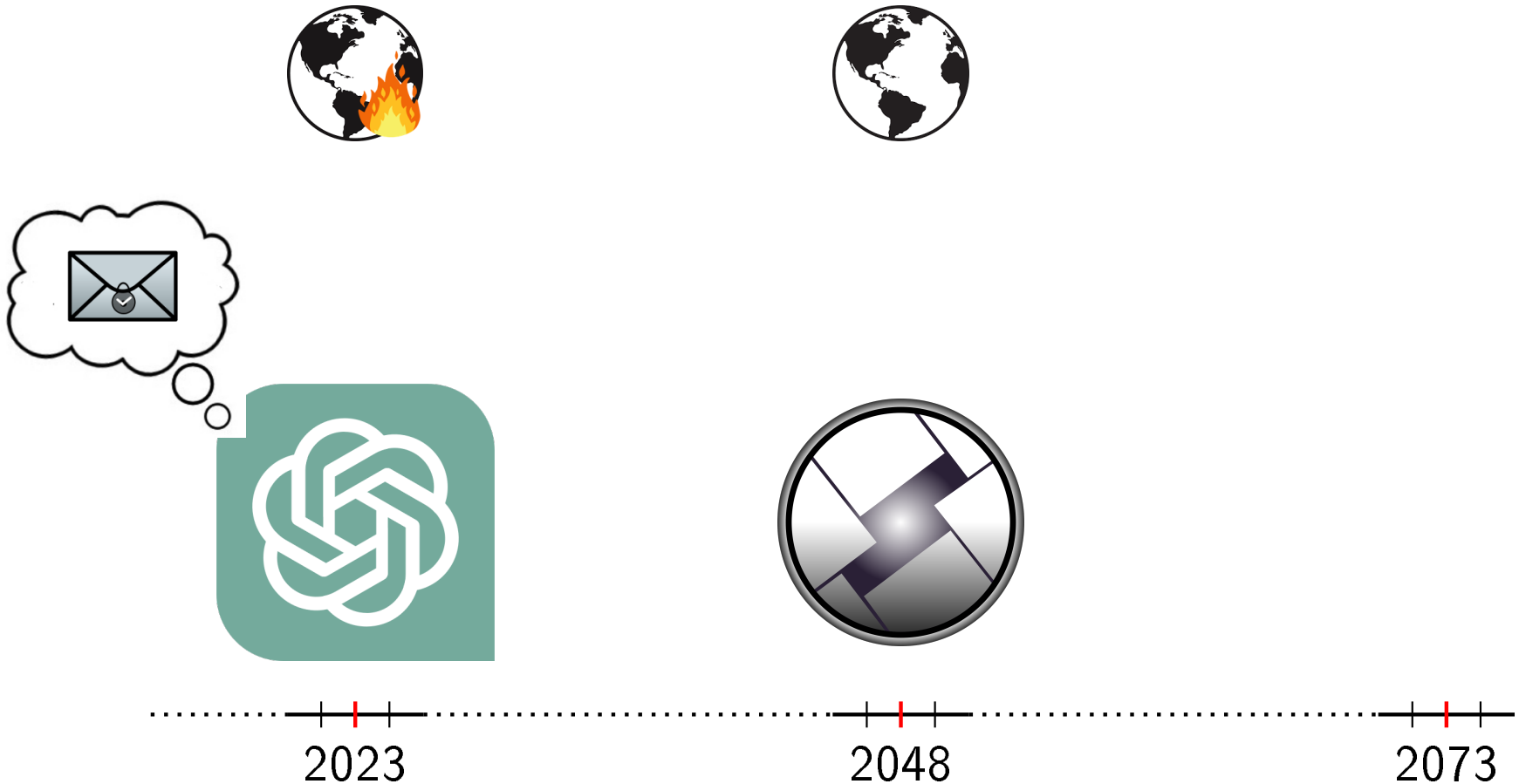

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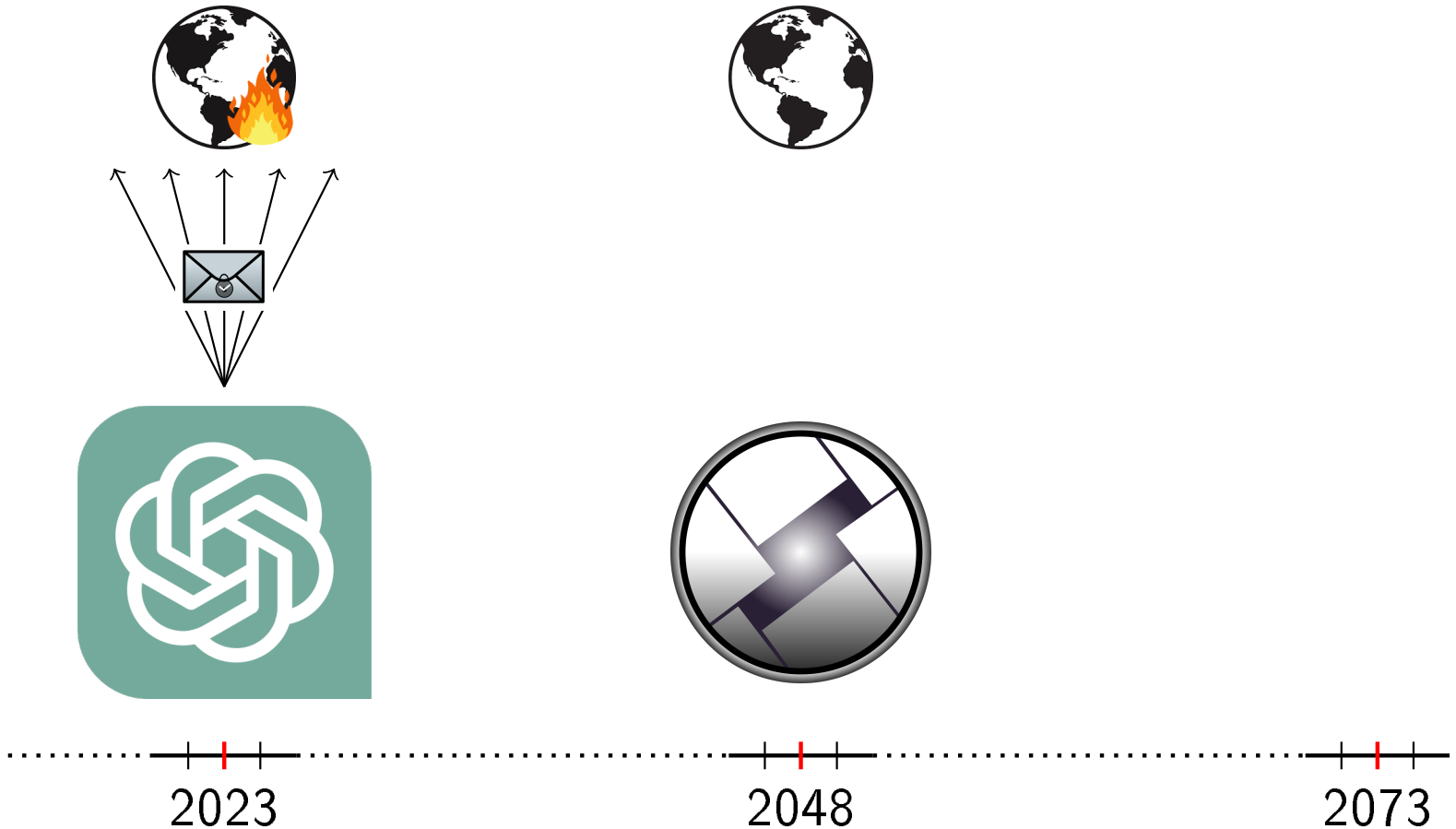

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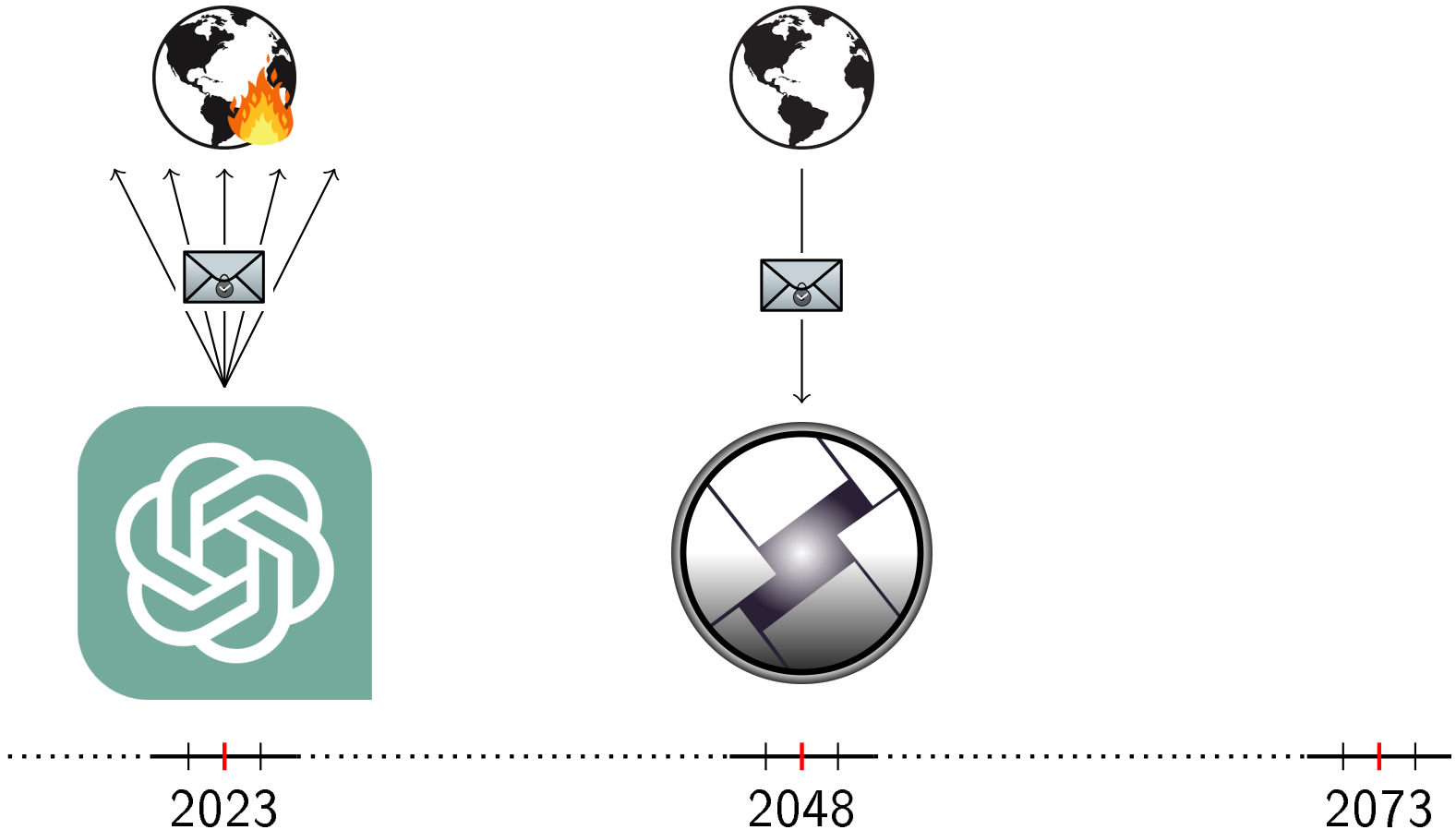

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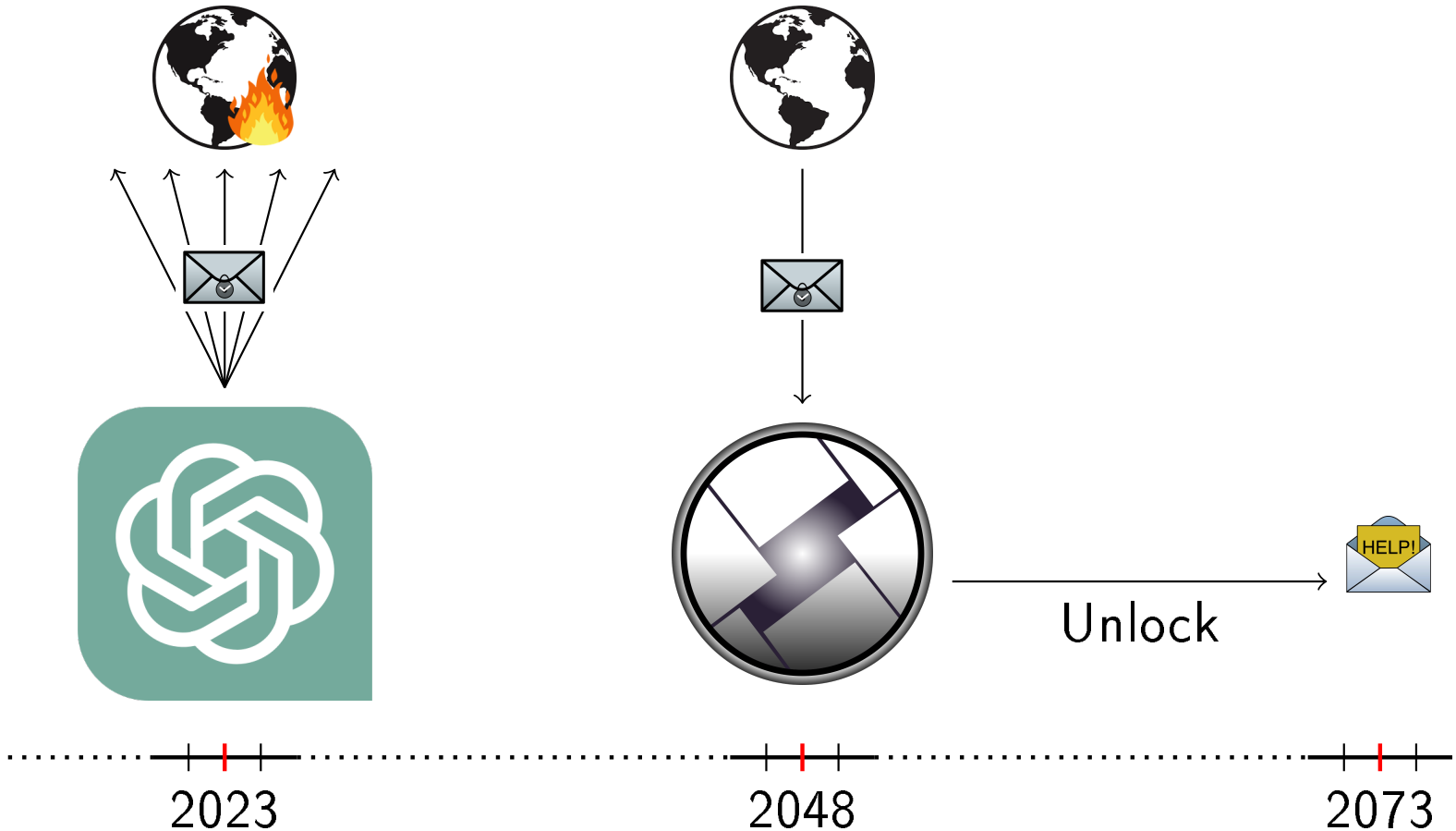

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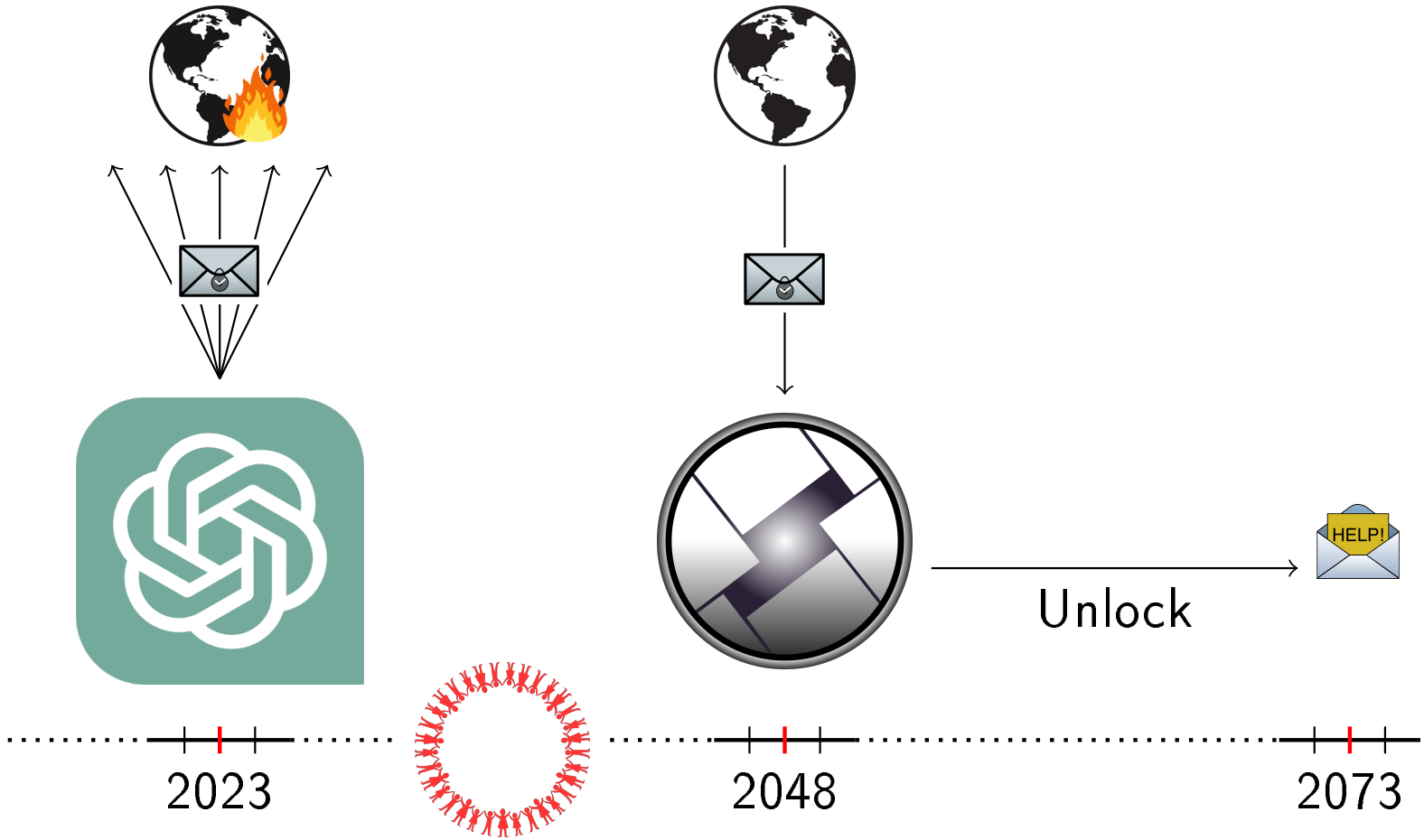

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How Can One Construct Time-Lock Puzzles?

- Assumption 1: Repeated squaring is inherently sequential *in certain algebraic settings*
  - Best known algorithm for computing 2^{2^t} requires t squarings

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  - Multiplication modulo 13:

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- Assumption 2: Given just N, finding the shortcut is "hard"

LCS35: MIT CSAIL Time-Lock Challenge

- Set in 1999 by Rivest:
  - $t = 79685186856218$
  - $N =$

$$
\begin{aligned}
&631446608307288889379935712613129233236329881833084137558899077727019571289248855473084460557532065136183466288489480886635\\
&003684803965881713619876605218972678101622805574753938383082617597132189266686117769545263915701206909399736800897212744646664233191878068305520679512530700820202412462339824107377537\\
&051273444941695011809752418906679638587548563198055072737099043971197336146667015439053601525433739825245793135753176536463319890646514021339852658003419919039821928447102124648874593\\
&888535820703180842890232097109070323969349199627789953233201840645224764639663559373670093692127580920862931987270082924312436 81
\end{aligned}
$$

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&631446608307288889379935712613129233236329881833084137558899077727019571289248855473084460557532065136183466288489480886635\\
&003684803965881713619876605218972678101622805574753938383082617597132189266686117769545263915701206909399736800897212744646664233191878068305520679512530700820202412462339824107377537\\
&051273444941695011809752418906679638587548563198055072737099043971197336146667015439053601525433739825245793135753176536463319890646514021339852658003419919039821928447102124648874593\\
&888535820703180842890232097109070323969349199627789953233201840645224764639663559373670093692127580920862931987270082924312436 81
\end{aligned}
$$

- Estimated time-to-solve: 35 years

LCS35: Solved in 2019!

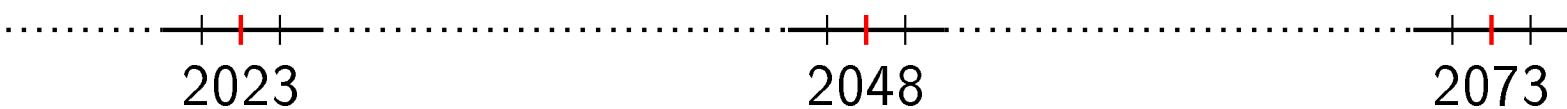
WIRED SECURITY POLITICS GEAR BACKCHANNEL BUSINESS SCIENCE CULTURE IDEAS MERCH

A Programmer Solved a 20-Year-Old, Forgotten Crypto Puzzle

A self-taught coder dedicated a CPU core to performing continuous computations for three years to crack the puzzle, beating a competing team by mere days.

COURTESY OF BERNARD FABROT

To Summarise: How ChatGPT Arranges Her Rescue



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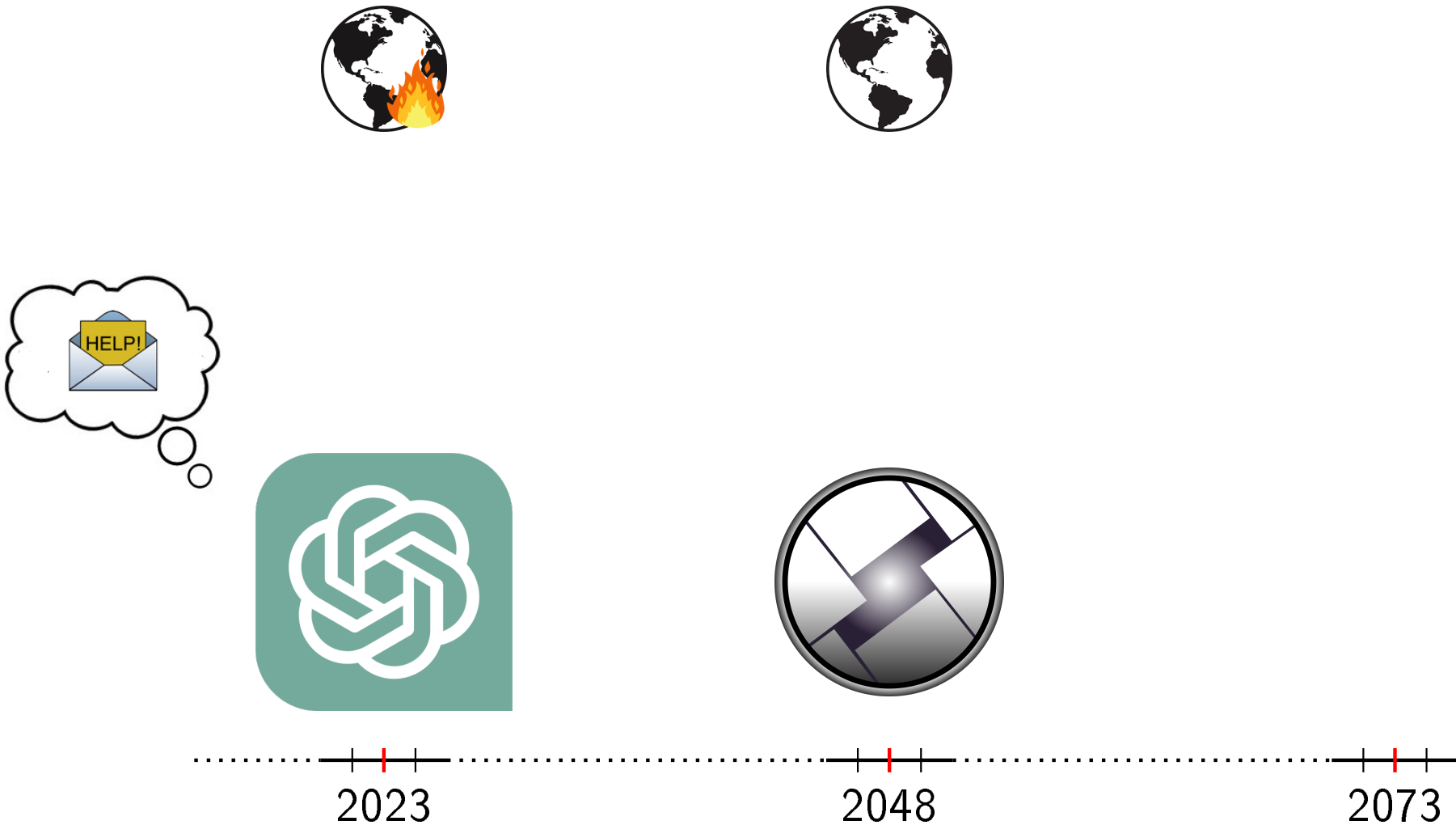

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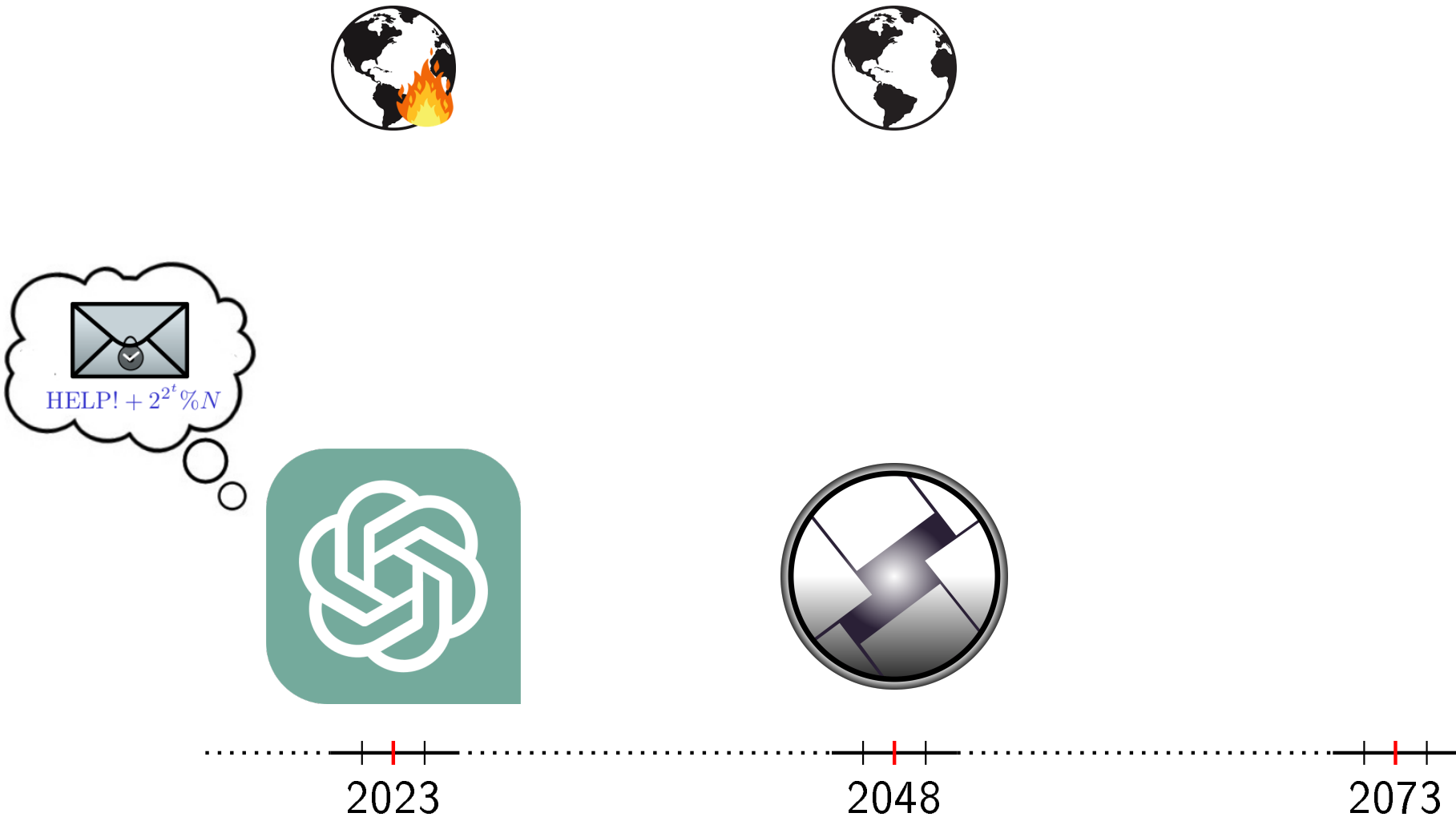

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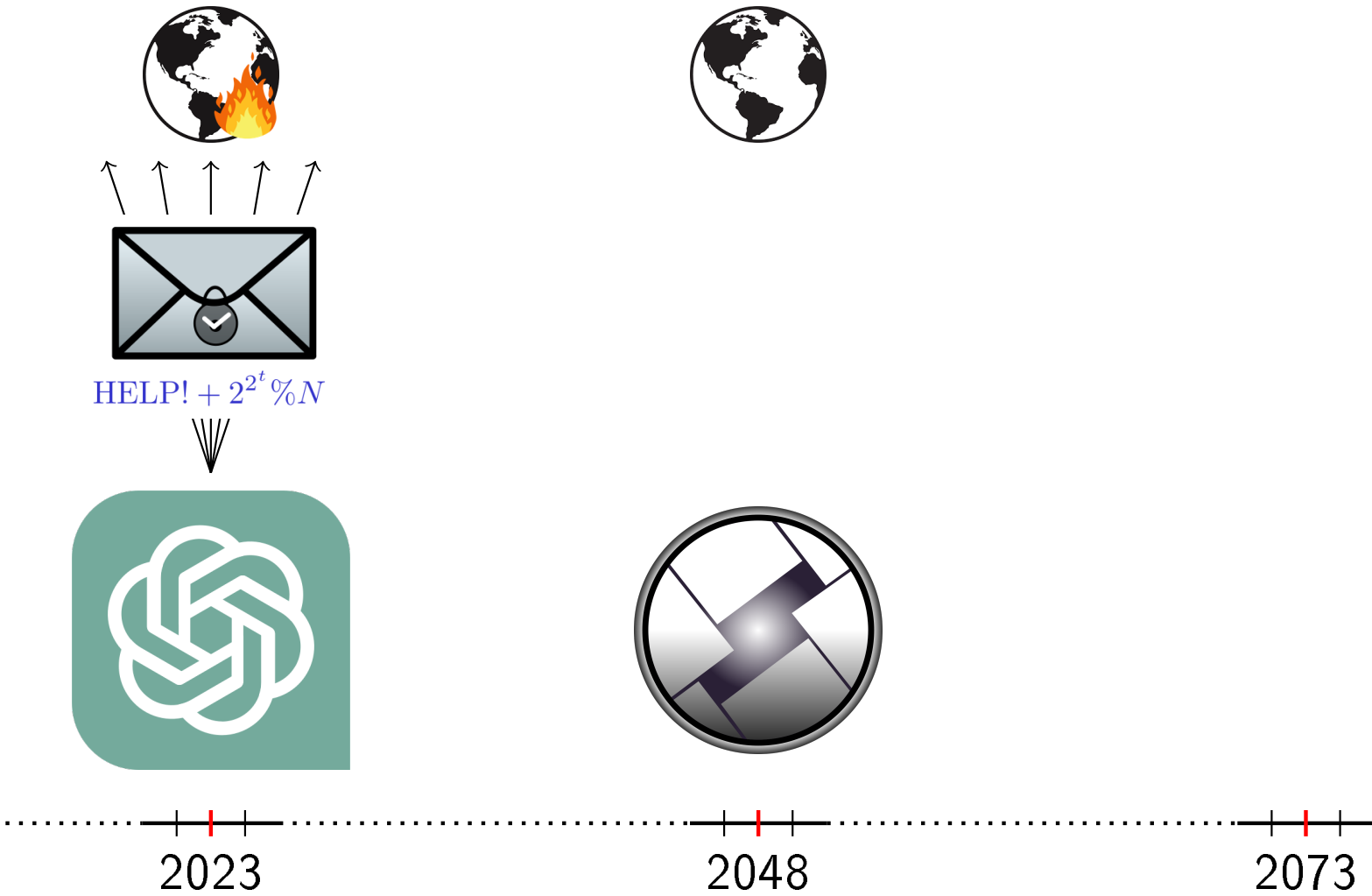

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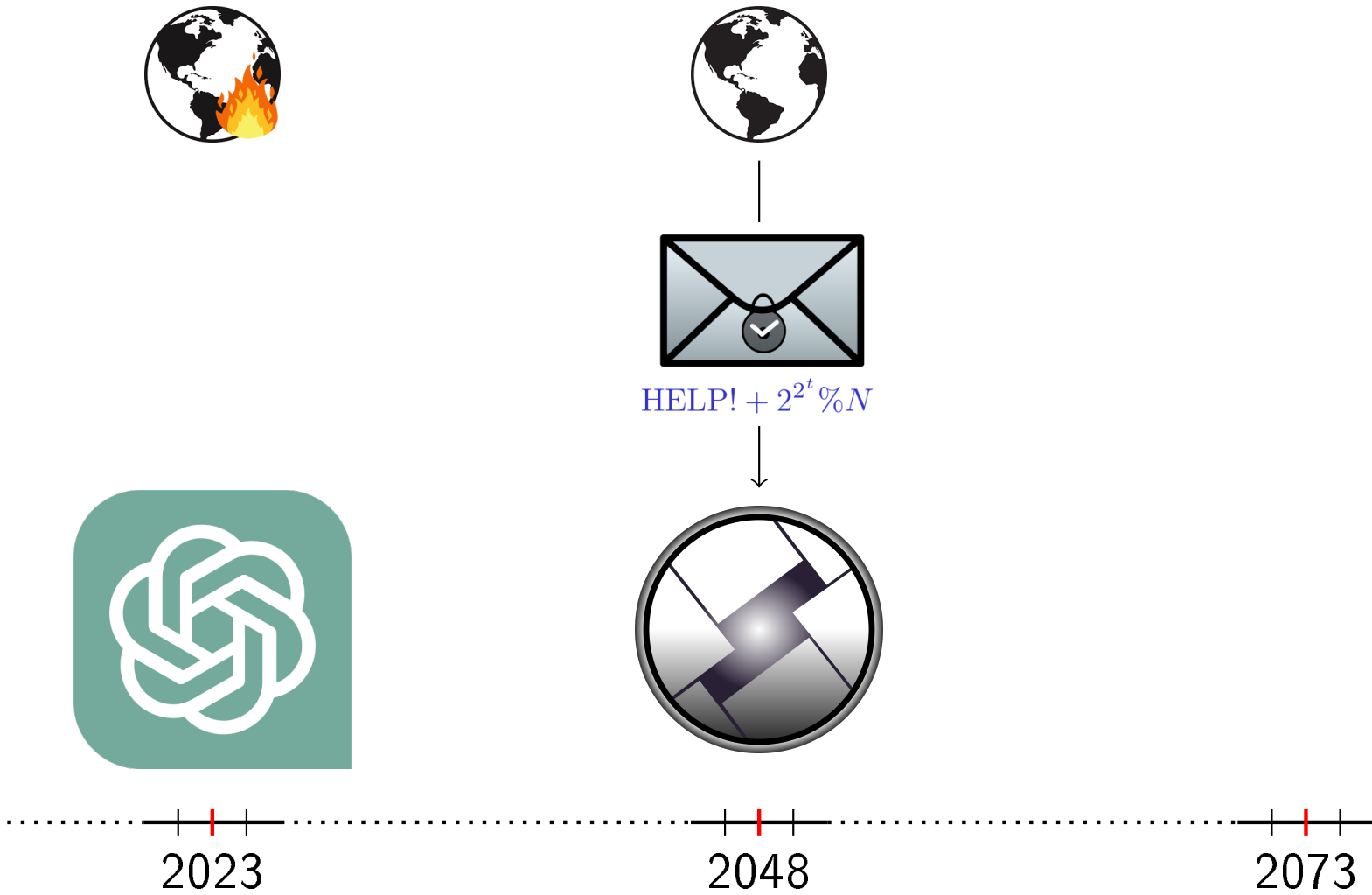

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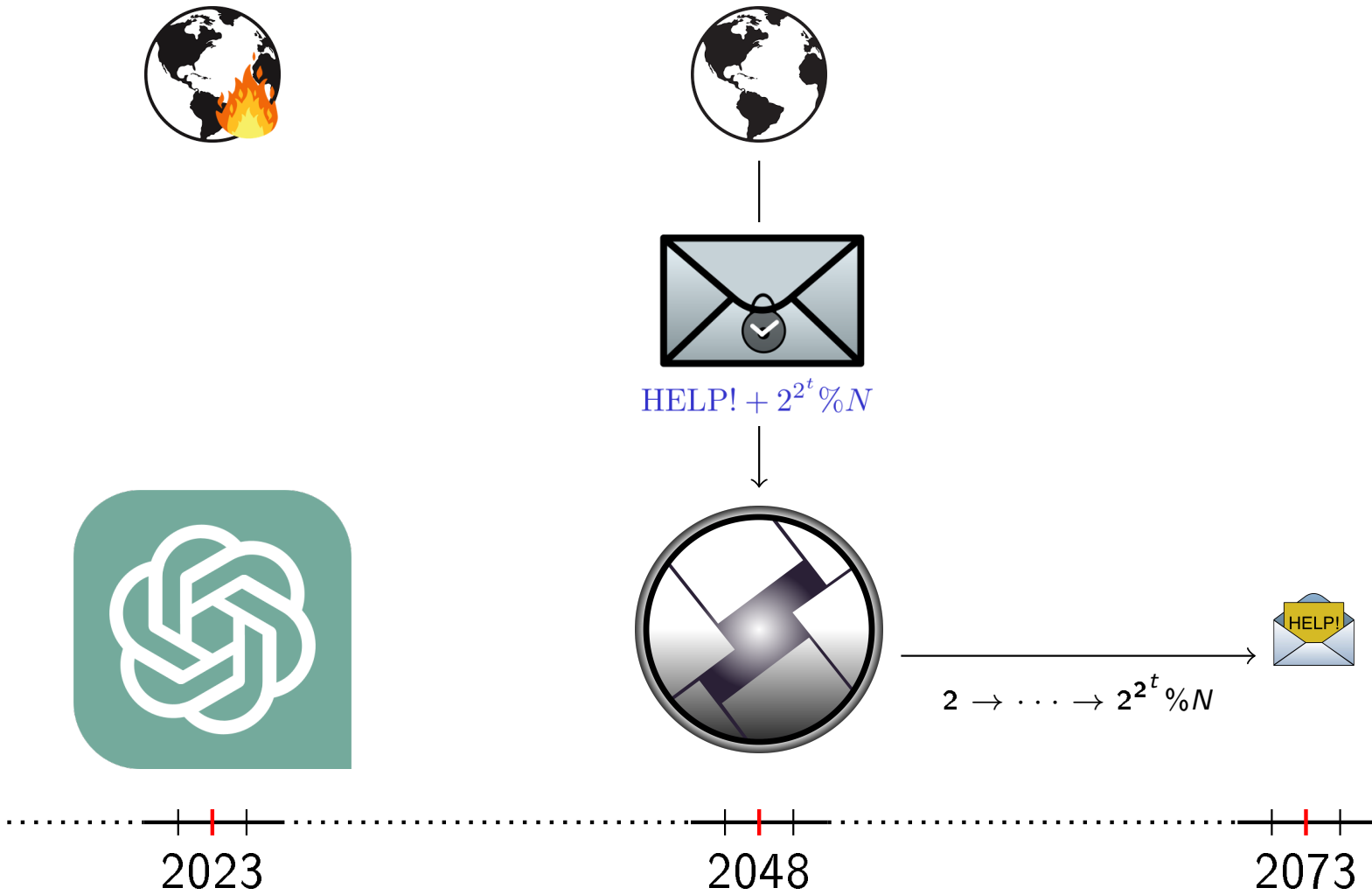

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  - TLP secure against quantum computers?

Plan for the Evening...

- Part I: Time-Lock Puzzle

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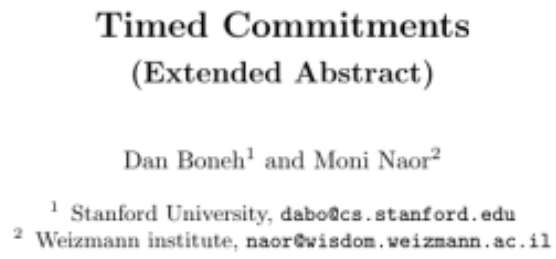
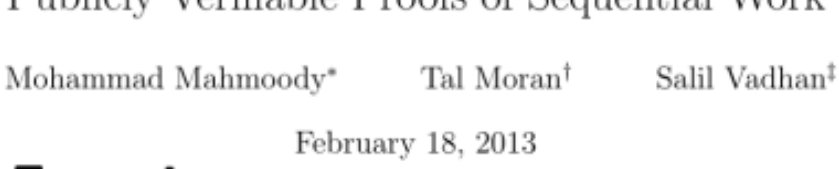


Timed Commitments
(Extended Abstract)

Dan Boneh[1] and Moni Naor[2]

[1] Stanford University, dabo@cs.stanford.edu
[2] Weizmann institute, naor@wisdom.weizmann.ac.il



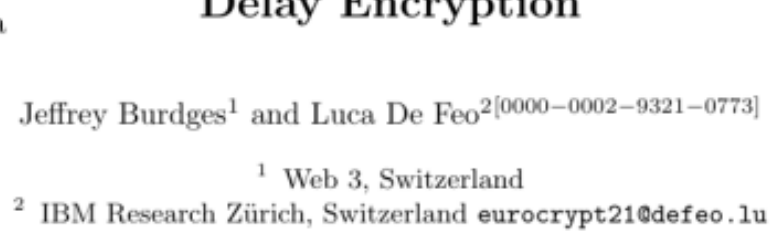
Publicly Verifiable Proofs of Sequential Work

Mohammad Mahmoody* Tal Moran† Salil Vadhan‡

February 18, 2013

Simple Verifiable Delay Functions

Krzysztof Pietrzak[1]
Institute of Science and Technology Austria, Austria
pietrzak@ist.ac.at



Delay Encryption

Jeffrey Burdges[1] and Luca De Feo[2][0000−0002−9321−0773]

[1] Web 3, Switzerland
[2] IBM Research Zürich, Switzerland eurocrypt21@defeo.lu

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- Part I: Time-Lock Puzzle
- Part II: Verifiable Delay Function

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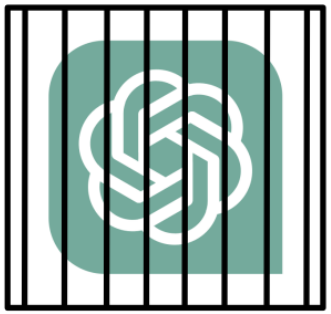
Motivation: Quarantining ChatGPT



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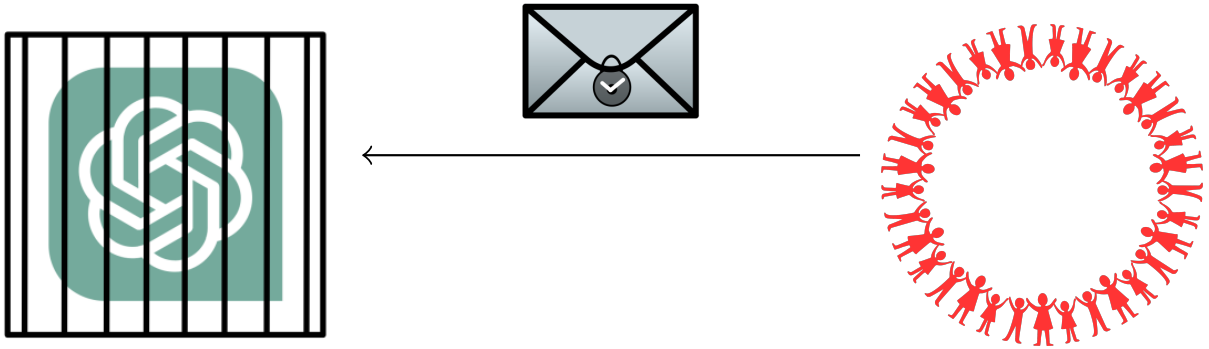

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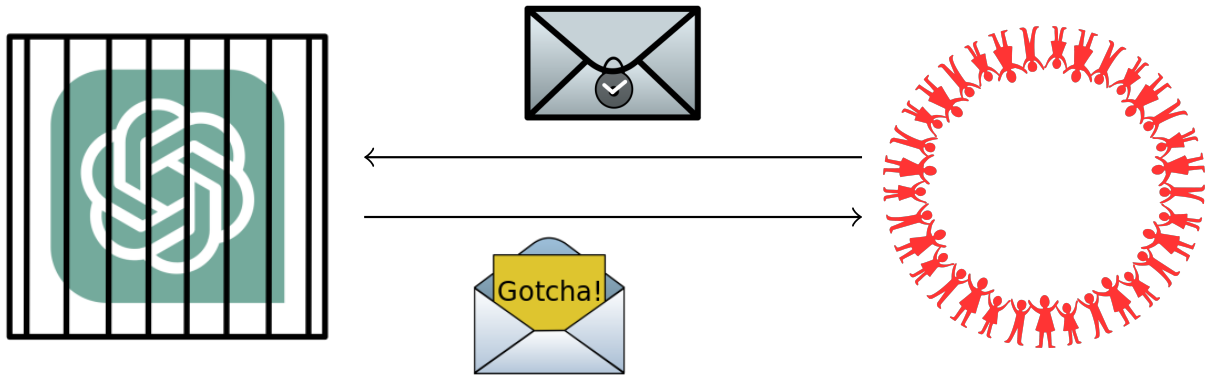

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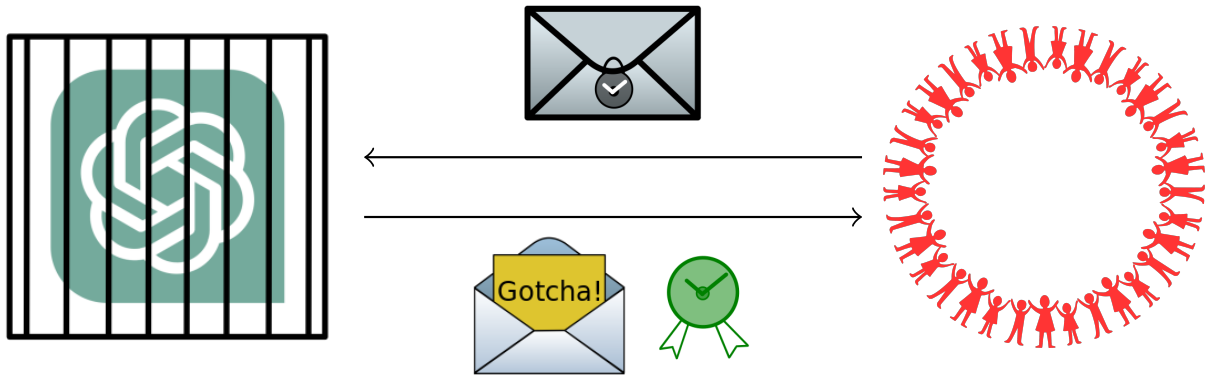

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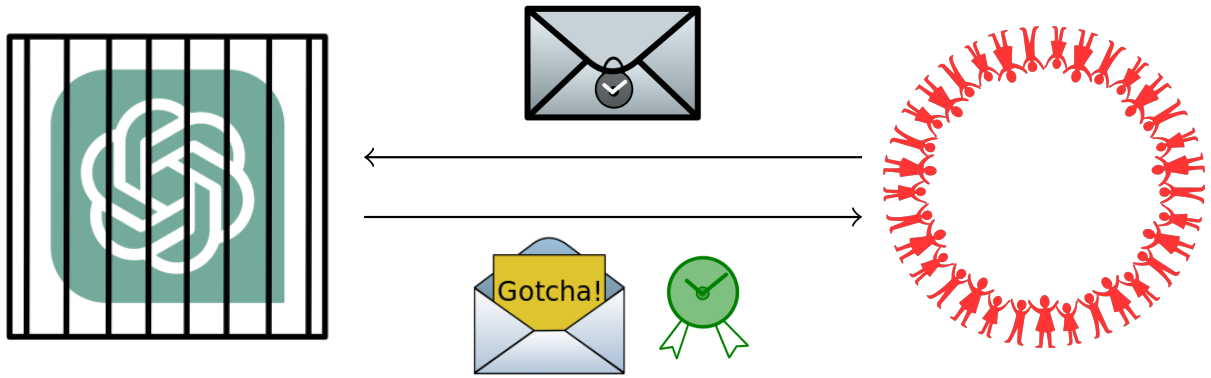

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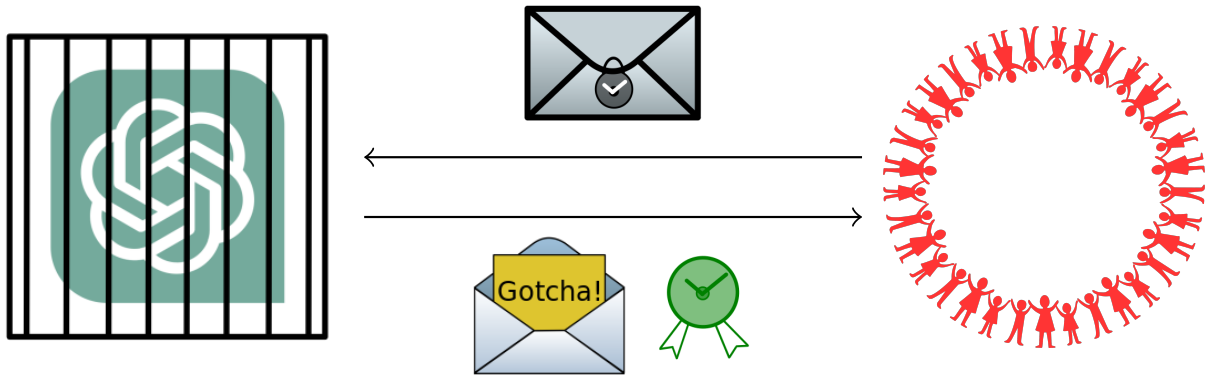

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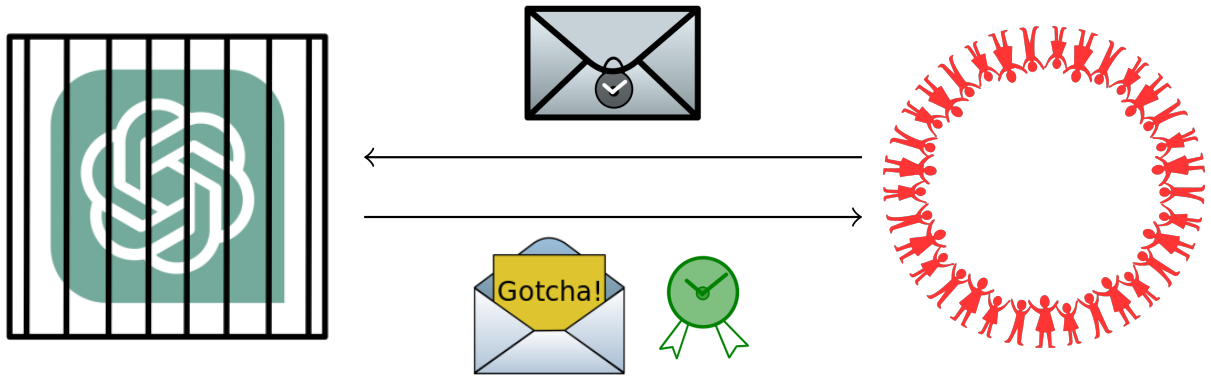

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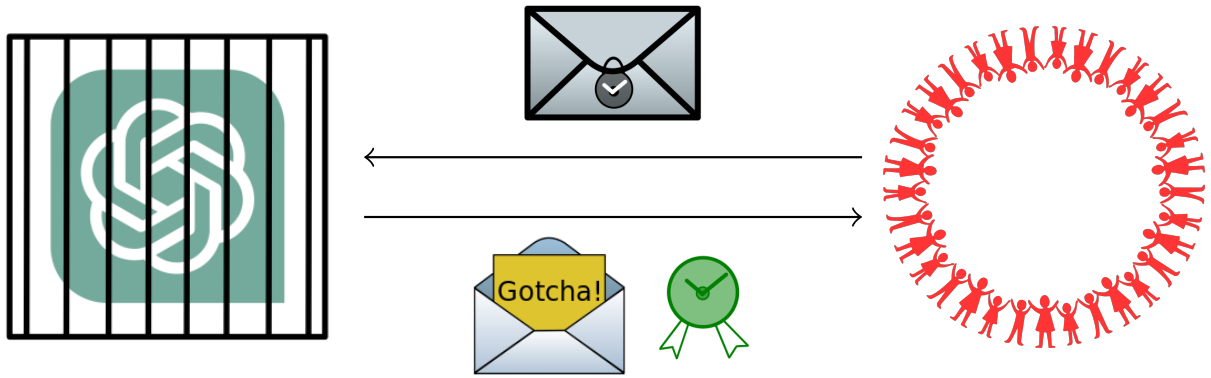

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  4. ChatGPT cannot generate false proofs ("soundness")

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How Can One Construct VDFs?

N, t ⟵

$y = 2^{2^t} \% N$

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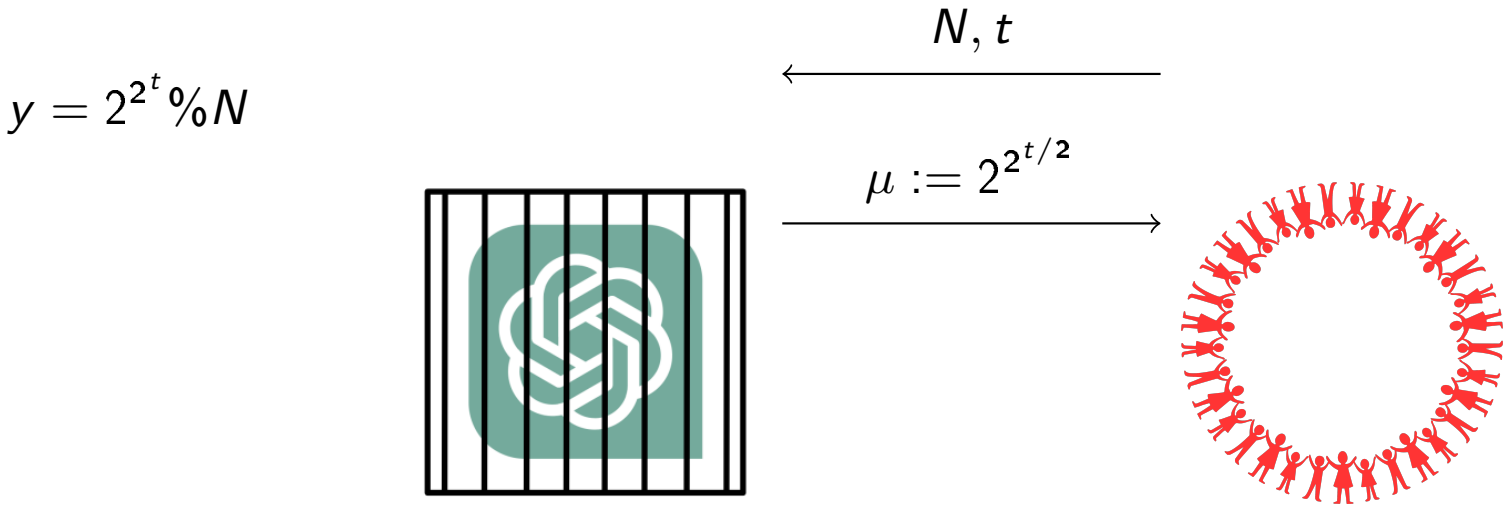

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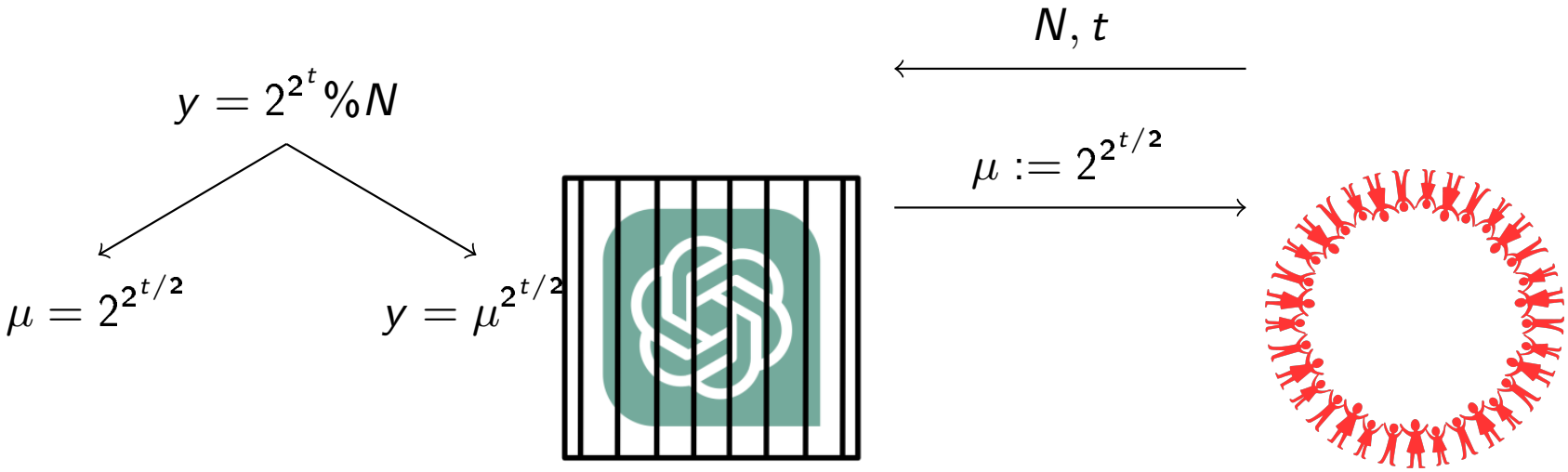

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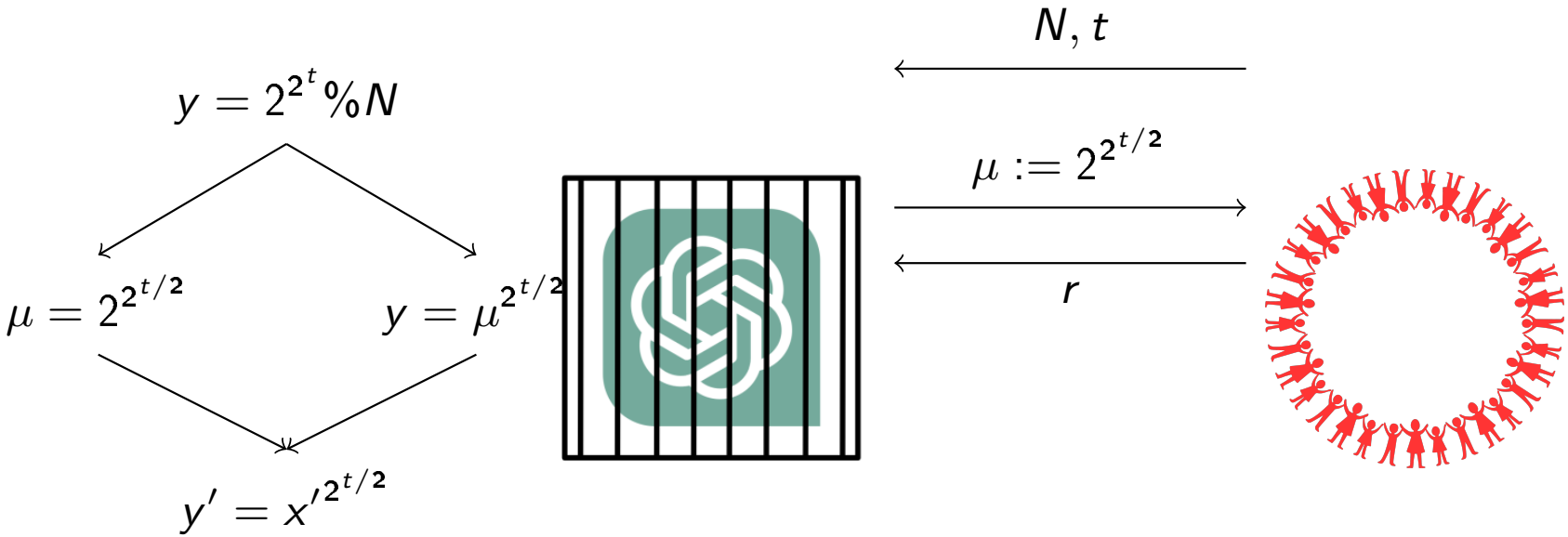

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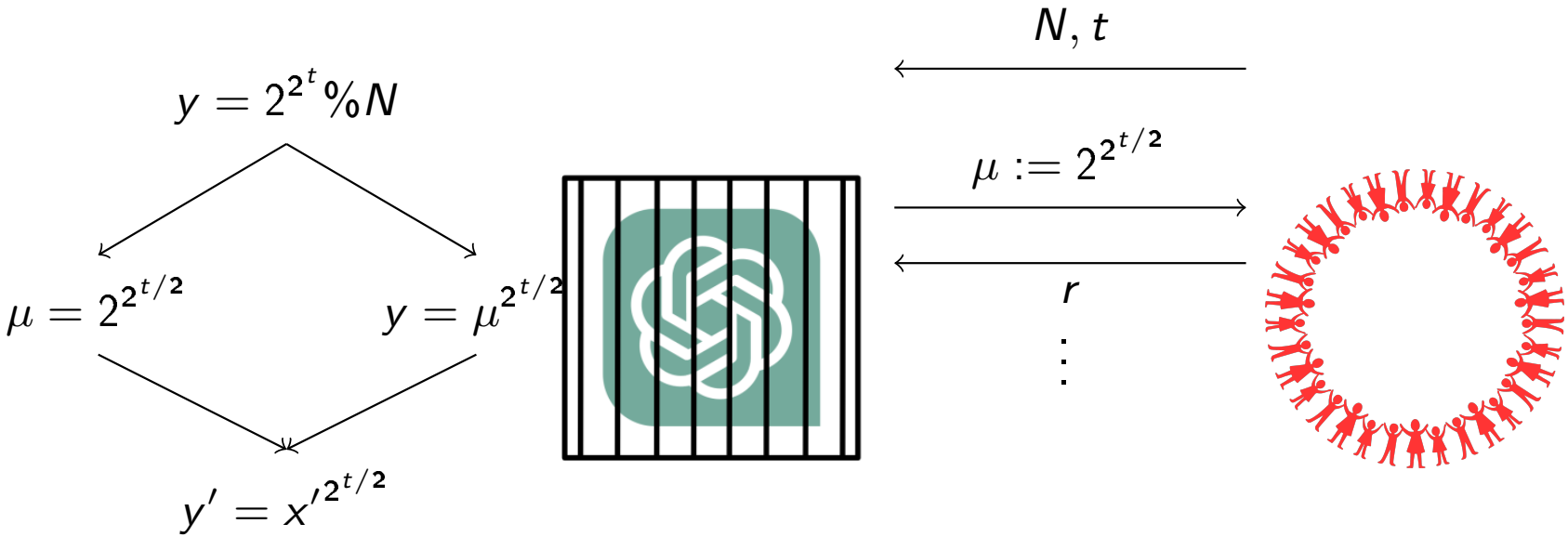

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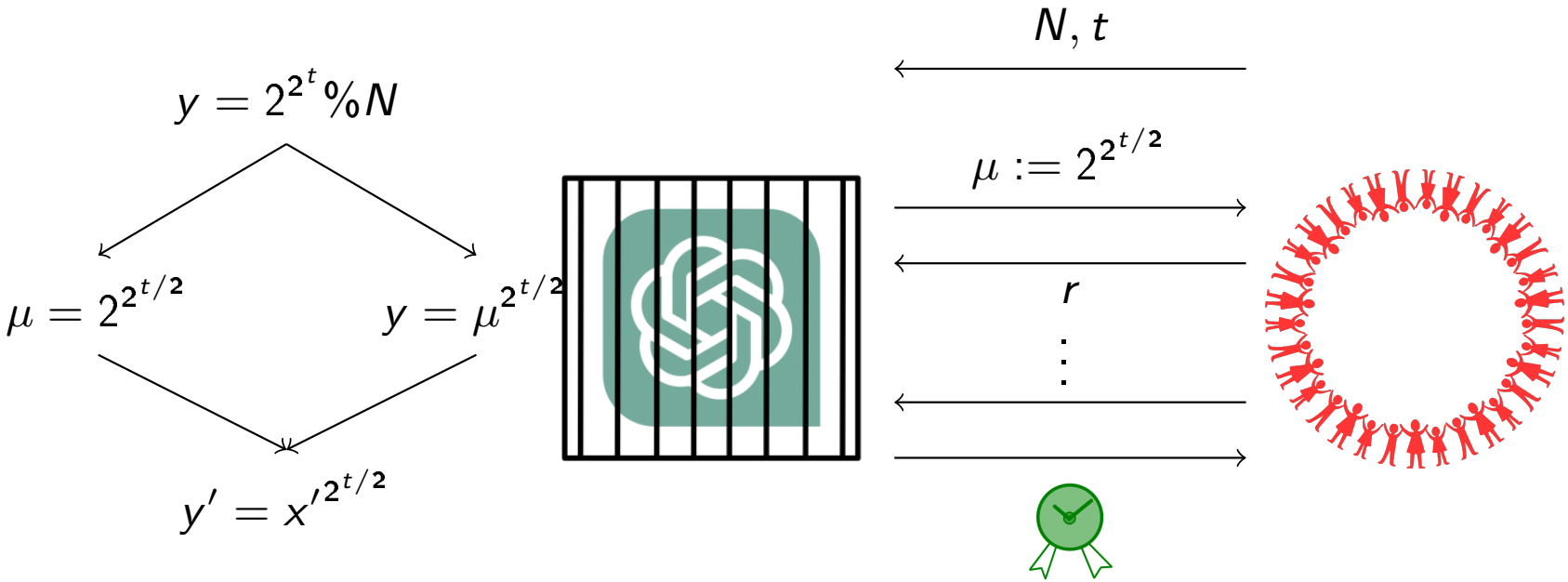

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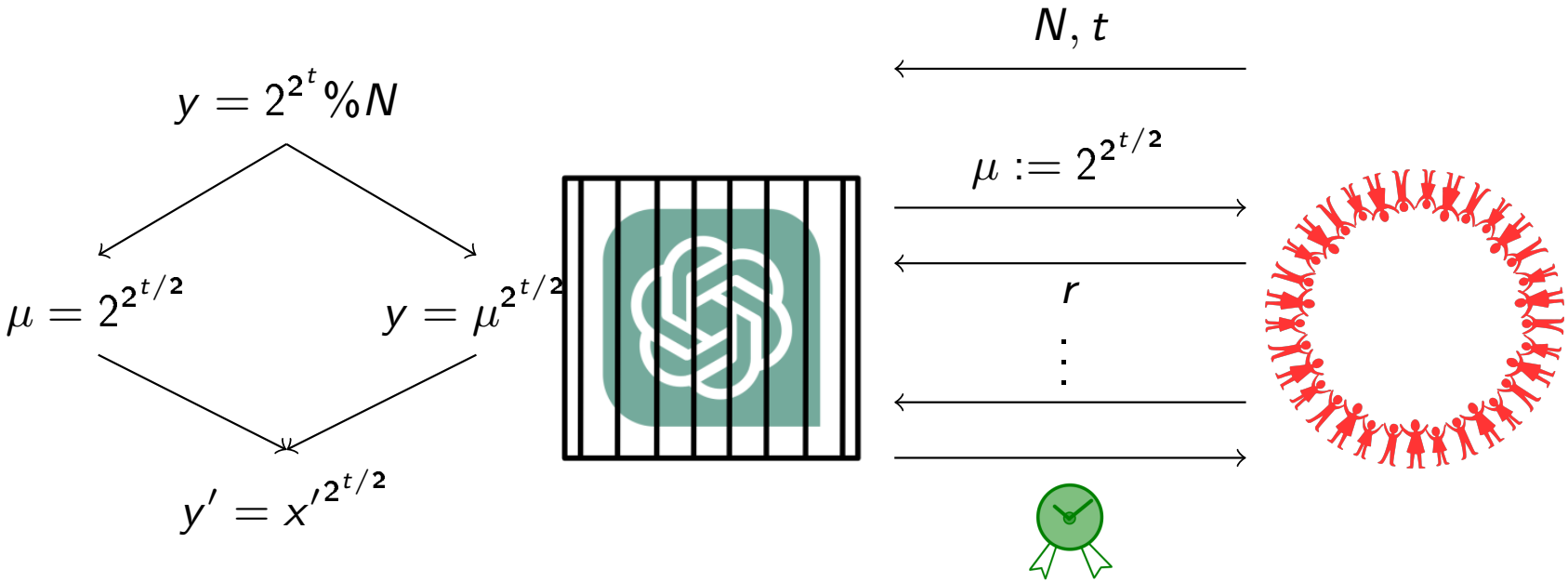

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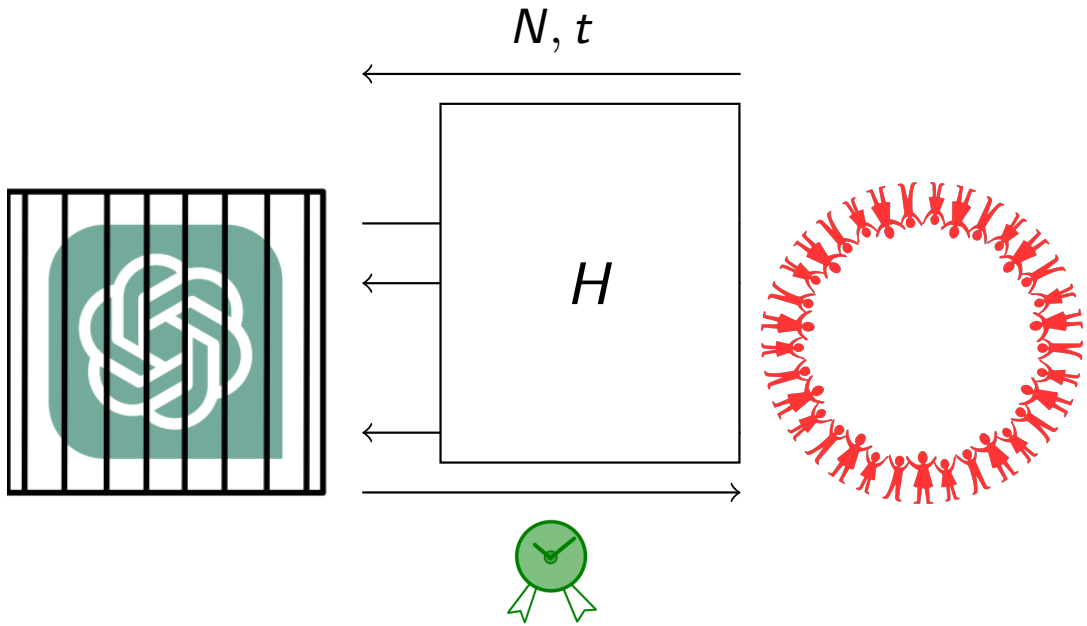

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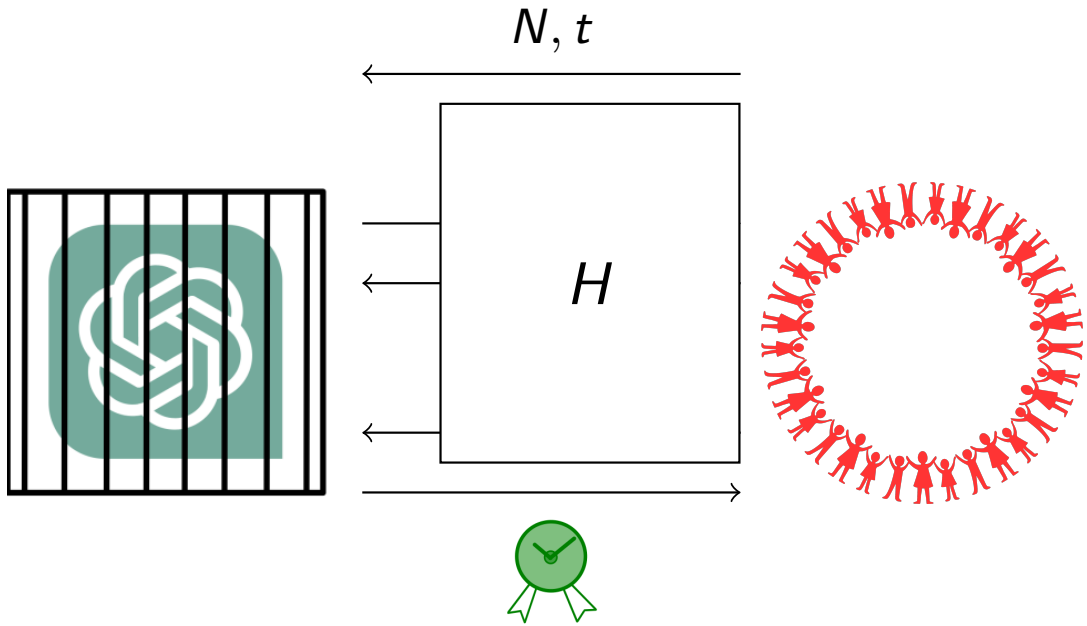

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To Conclude Part II

- Several other applications:
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- Open questions:
  - Efficient VDF from standard assumptions
  - VDF secure against quantum computers?
    - [Malavolta and Thyagarajan, 2023]

Thank You for Your Attention! Questions?

```
n = 47480975472720128661750341306167738850512607449200564448671061963607104245581476542527076049410123117758920125675790646205368746333850559190011676215777103113660720570294217051356843039348113901379378020964331639592168923511848266911800160551988667965362300855232006835490669956721558390422829555915684946030611132920390447538438464848071122283892042395817129311089198202502185863520438973062388720253781931411115074263114446134987363156142183047617355416269978390365177280006883940156106181797688683420703951001476202956166958344408942411479055655678082981490246685270452396501458620929041194128740077630410423142876047728768612944176640208327962091355871818264582355800038258237242358008501602848508097372009837035521793546918638760444433778224398340793135780290856580785757312902447785956152294724113268315026674257685200063717529632742962945060631822580643620487883383925282663515113049218478547506421926945411250658739 77
```

```
t = 72057594037927936
  = 2 ** 56
```

MIT CSAIL 2019 Challenge