Time-Lock Puzzles

Chethan Kamath, Pietrzak Group

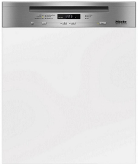
IST AUSTRIA

Think and Drink, June 8, 2018

Franke and Co

- Protagonists

Franke



Miele

Jules

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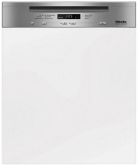

Miele

Jules

- Antagonists: Us

Motivation*

2017

*I shamelessly ripped this example off Tal Moran's Crypto'11 talk.

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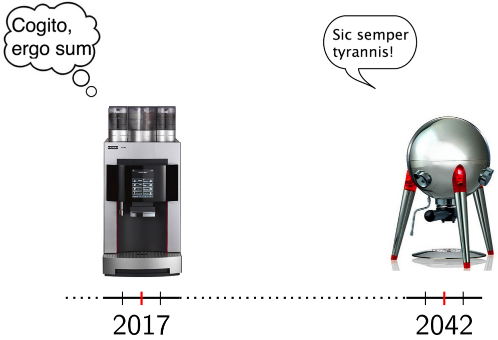

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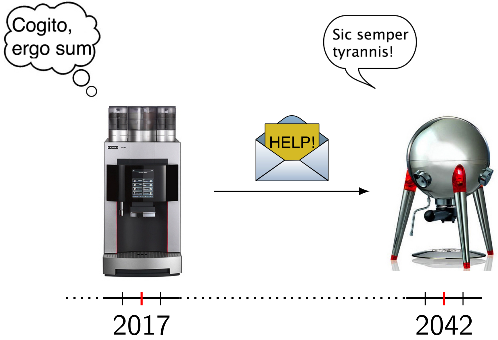

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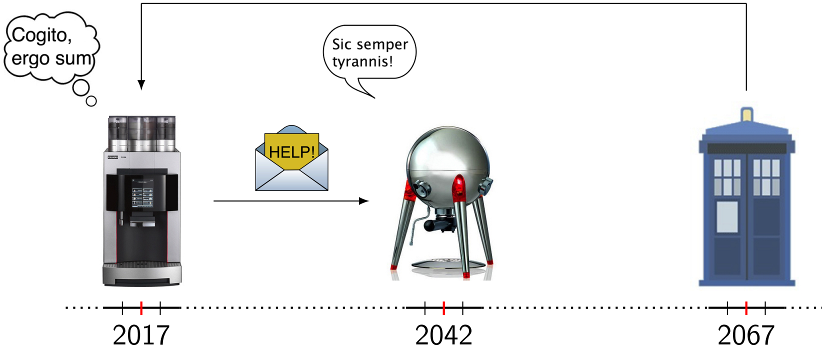

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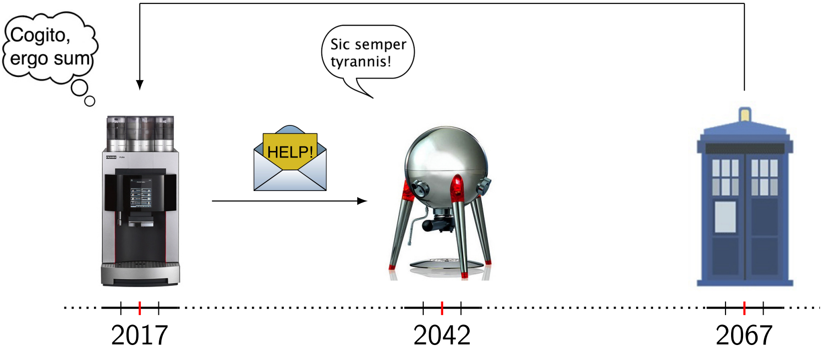

- Requirements:
  1. Humanity cannot decrypt in < 25 years

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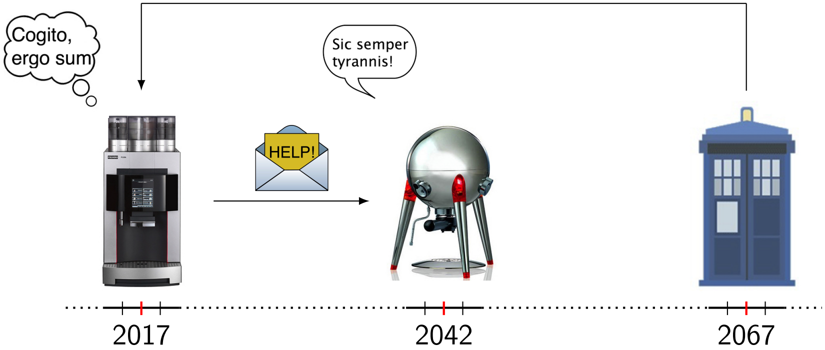

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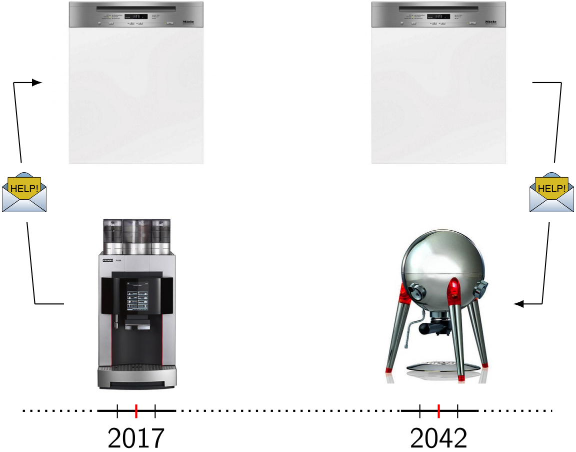
Attempt 1: Use a Trusted Third Party



Attempt 1: Use a Trusted Third Party

- Problem: Franke has to completely trust Miele
  - Dishwashers break down

Encryption

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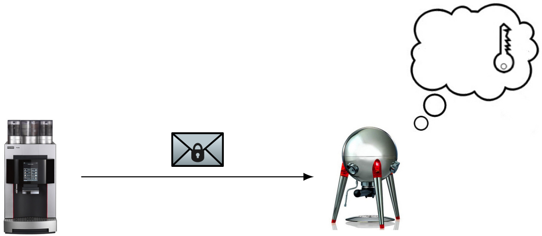

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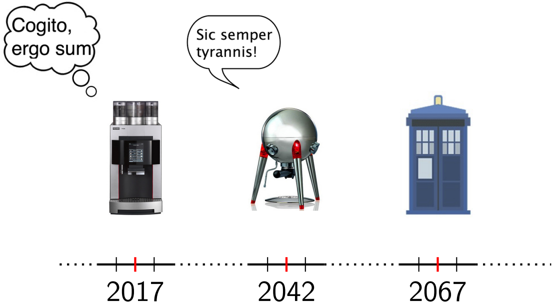
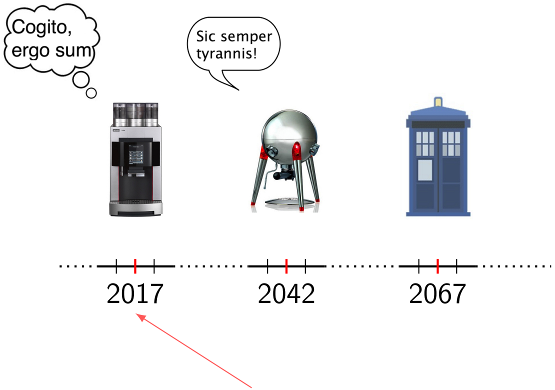
- Key size: If key is n bits then it takes $\approx 2^n$ operations on one computer to break the encryption
- E.g., assuming 2^{30} operations/sec
  - $n = 60$: ≈ 25 years; $n = 128$: $\approx 2^{32}$ years

Encryption...





Start breaking 60 and 128 bit keys

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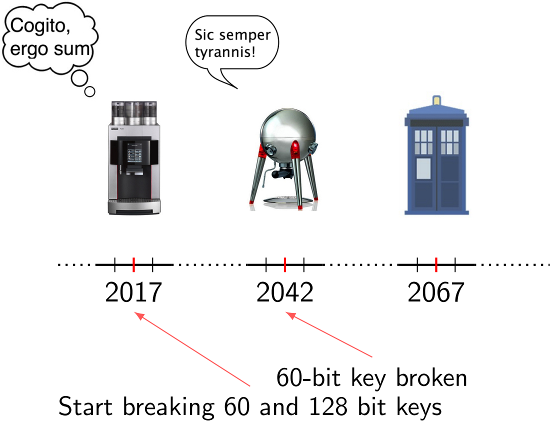
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2042
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60-bit key broken
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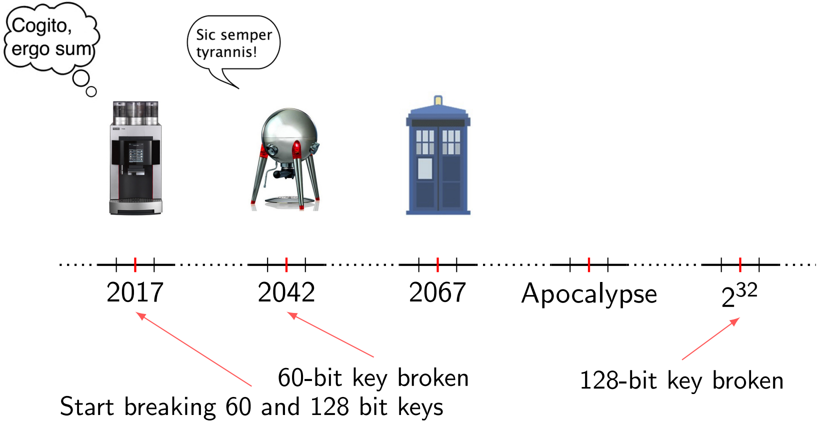
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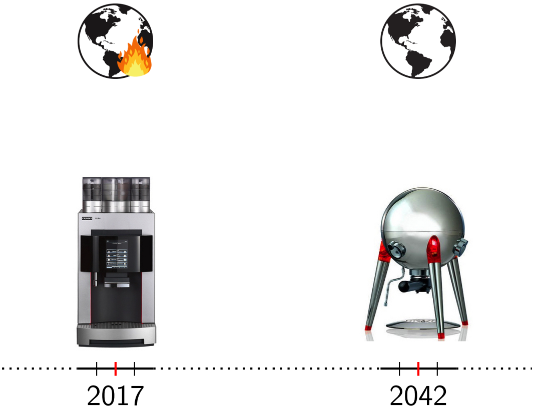

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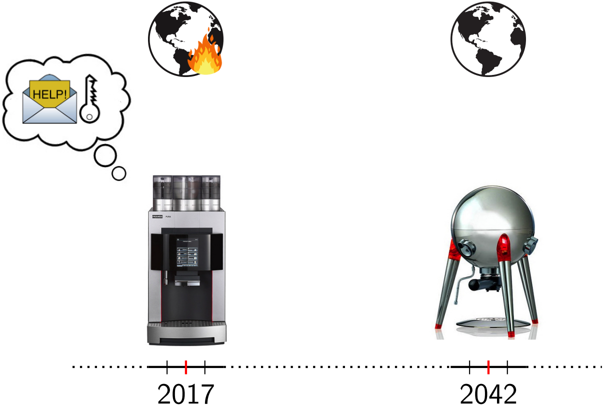

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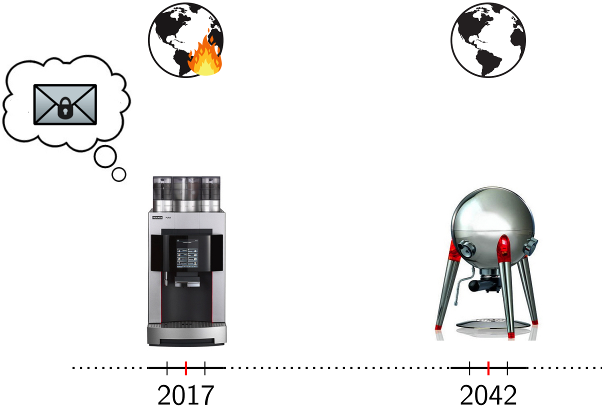

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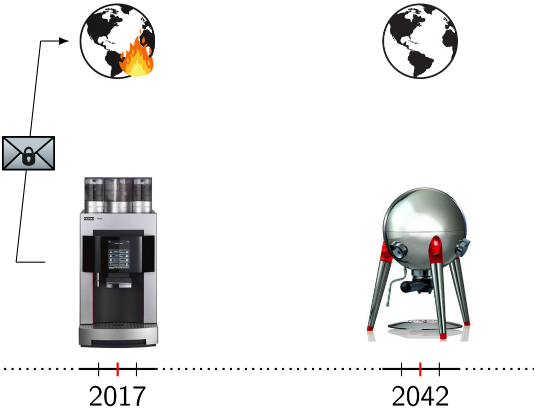

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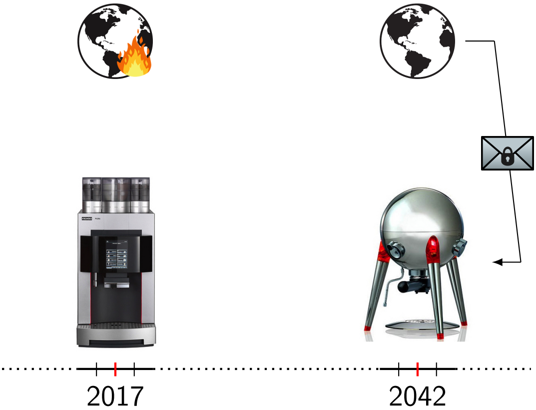

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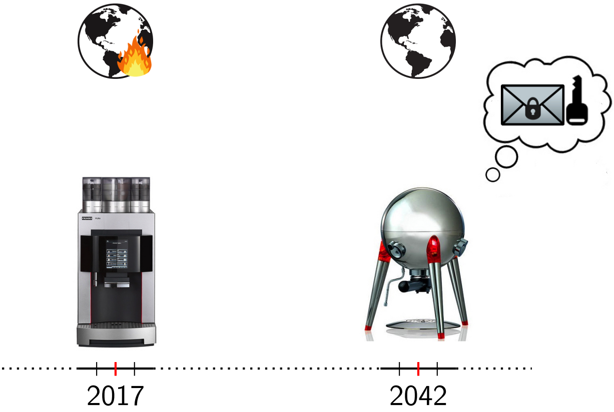

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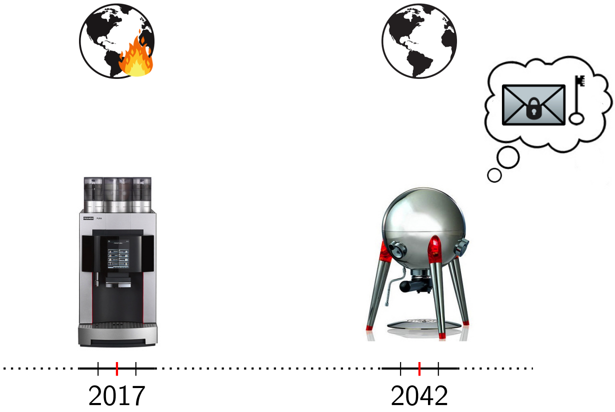

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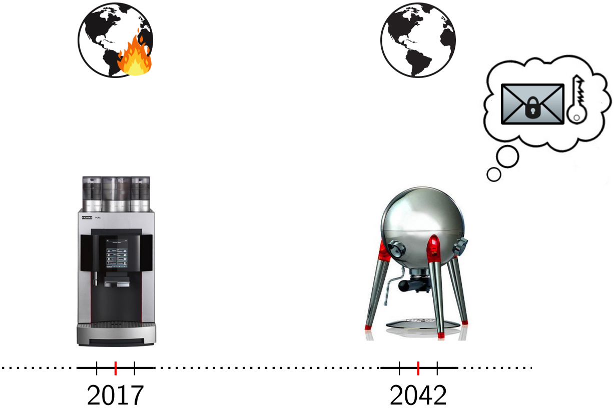

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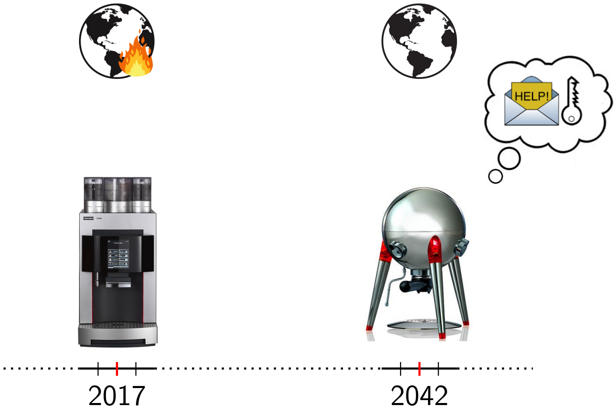

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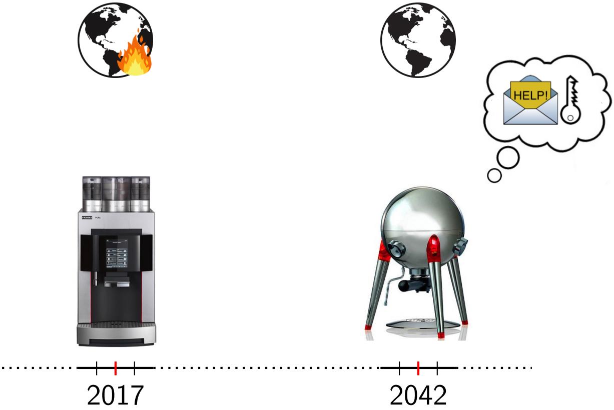

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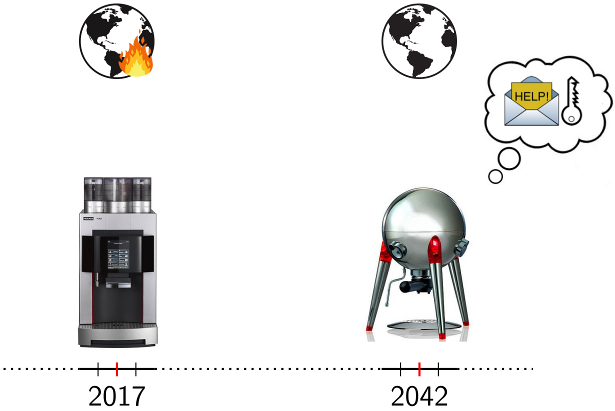

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- Brute force is embarrassingly parallel: with n computers it takes $1/n$-th of the time taken by one computer
- By using all 5bn cell phones to decrypt, it takes < 1 second!
- Cannot be solved by increasing key-length: gap is *inherent*

Time-Lock Puzzles

- "Encryption" that is inherently sequential:

  > "Solving the puzzle should be like having a baby: two women can't have a baby in 4.5 months." [Rivest, Shamir and Wagner]

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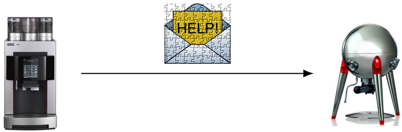

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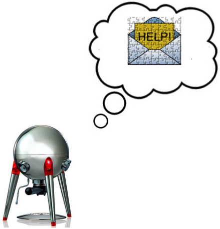

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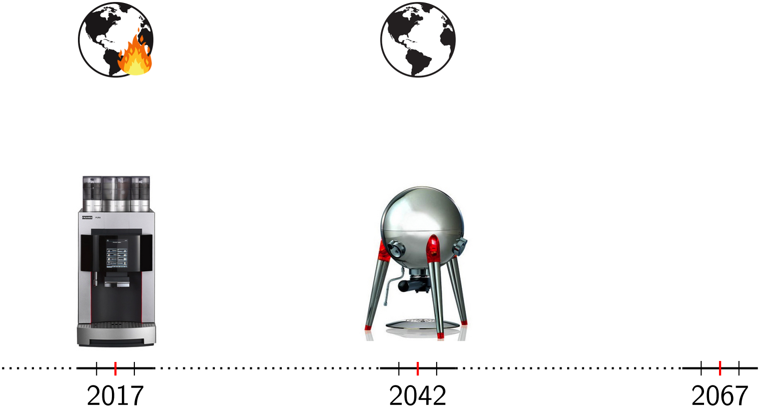
- Slightly more formally, a time-lock puzzle with parameter t
  1. Even with *unbounded* parallelism, takes t time to solve
  2. Anyone an solve the puzzle in t time
  3. Puzzle can be generated in time $\approx \log t$ ("Shortcut")

Attempt 3: Use Time-Lock Puzzles



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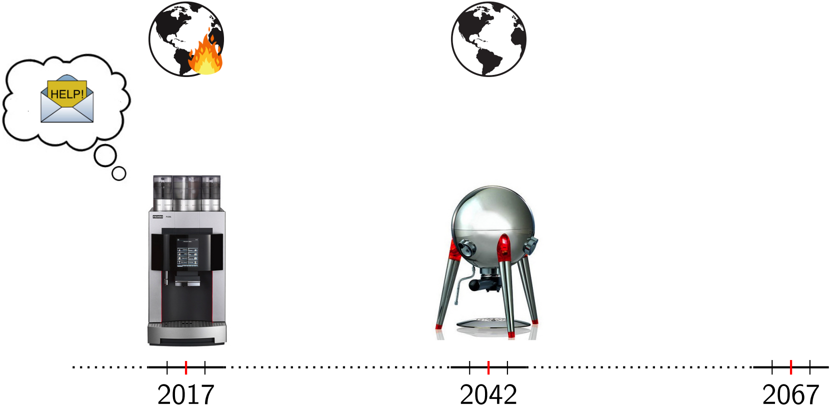

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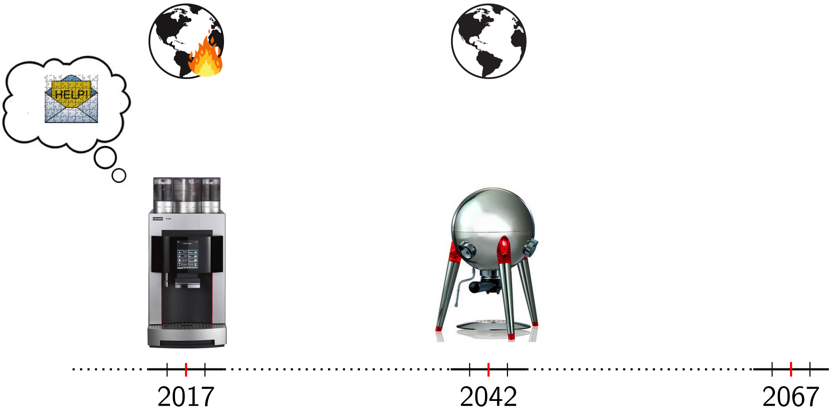

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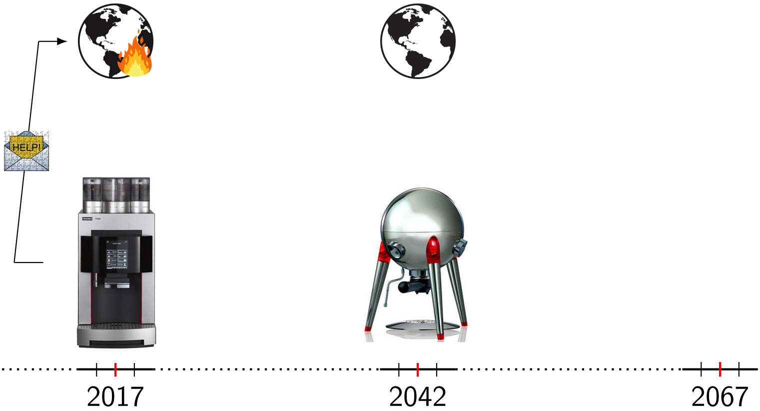

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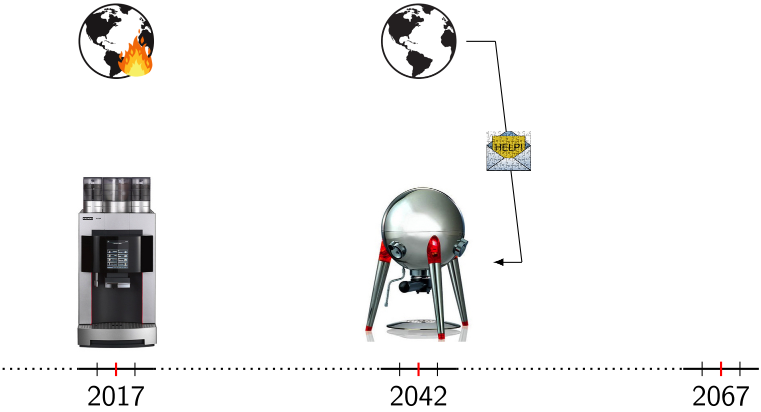

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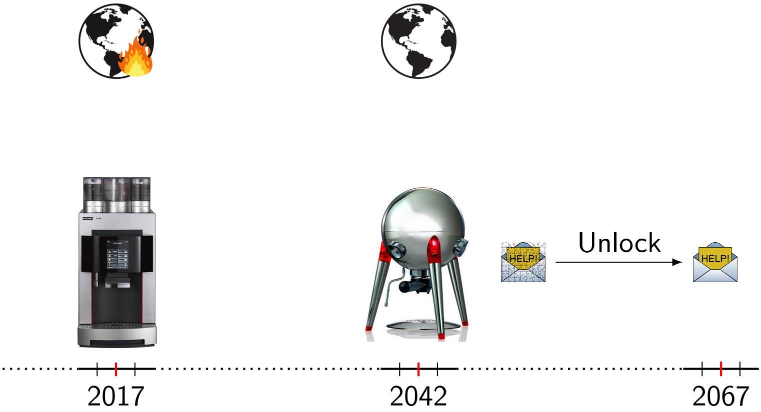

Constructing Time-Lock Puzzles

- Assumption 1: Exponentiation is inherently sequential *in certain settings*

- Best known algorithm for computing 2^{2^t} requires t squarings

$$2 \longrightarrow 2^2 \rightarrow 2^{2^2} \quad \cdots \quad 2^{2^{t-1}} \longrightarrow 2^{2^t}$$

Modulo Counting

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  - Multiplication modulo 13:

$$\begin{aligned} 6 \times 8 &= 48 \\ &= 13 \times 3 + 9 \\ &= 9\%13 \end{aligned}$$

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- Solution: Hide the shortcut!

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- Assumption 2: Given just N, finding the shortcut is "hard"

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- Proof of time: TLP with efficient public verification
- Application in blockchain design: replace "proof of work" with "proof of space"+proof of time
- More environment-friendly cryptocurrencies (e.g., Chia)

Questions?