CS206 Lecture 05

Resolution Theoram Proving

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Plan for Lecture 05

- Introduction
- Resolution Principle
- Producing the Clause
- Answer Extraction

Resolution Principle

- Resolution proves a theoram by negating the statement to be proved and adding the negated goal to the set of axioms that are known to be true.

- Use the resolution rule of inference to show that this leads to a contradiction.

- Once we are able to show that the negated goal is incosistent with the given set of axioms it follows that the original goal must be consistent.

Resolution Principle

Steps for resolution refutation proofs

- Put the premises or axioms into clause form (CNF).
- Add the negation of what is to be proved in the clause form to the axioms.
- Resolve these clauses together producing new clauses that follow logically from them.
- Produce a contradiction by generating the empty clause.
- The substitutions used to give empty clause opposite of negated goal is true.

Resolution Principle

Prove that "Tommy will die" from

the satements:-

- **Tommy is a dog.**
- **All dogs are animals.**
- **All animals will die**

Changing premises to predicates

- dog(X) -> animal(X)
  dog(Tommy)
  animal(Tommy)
- animal(Y) ->die(Y)
  die(Tommy)

Resolution Principle

Equivalent reasoning by resolution

- Convert predicates to clause form

  Predicate form Clause form

  1. Dog(X) -> Animal(X) ~Dog(X) V Animal(X)
  2. Dog(Tommy) Dog(Tommy)
  3. Animal(Y) -> Die(Y) ~Animal(Y) V Die(Y)

Resolution Principle

Negate the conclusion

- Predicate form Clause form

4. ~die(Tommy) ~die(Tommy)

Binary Resolution Proof Procedure

- Binary Resolution Step

  For any two clauses C1 and C2,if there is a literal L1 in C1 that is complementary to a literal L2 in C2, then delete L1 and L2 from C1 and C2 respectively,and construct the disjunction of clauses.The constructed clause is a resolvent of C1 and C2.

Resolution Principle

Examples of Resolution Step

- C1=a V ~b, C2=b V c
  Complemetary Literals : ~b,b
  Resolvent : a V c

- C1=~a V b V c, C2= ~b V d
  Complementary literals: b,~b
  Resolvent:~a V c V d

Binary Resolution Proof Procedure

Justification of Resolution Step

- Theoram Given two clauses C1 and C2,a resolvent C of C1 and C2 is a logical consequence of C1 and C2.

- Proof Let C1=L V C1', C2= ~L V C2', and C=C1' V C2', where C1' and C2' are disjunction of literals.
  Suppose C1 and C2 are true in an interpretation I.

Binary Resolution Proof Procedure

- We want to prove that the resolvent C of C1 and C2 is also true in I.

  -Case 1:L is true in I

  » Then since C2=~L V C2' is true in I

  C2' must be true in I, and thus

  C=C1' V C2' is true in I.

  Case 2:L is false in I

  » Then since C1=L V C1' is true in I

  C1' must be true in I, and thus

  C= C1' V C2' is true in I.

Binary Resolution Proof Procedure

- Resolution in Propositional Logic

1. a <- b Λc $aV\tilde{}bV\tilde{}c$
2. b b
3. $c <- d\Lambda e$ $cV\tilde{}dV\tilde{}e$
4. eVf eVf
5. $d\Lambda\tilde{}f$ d

$\tilde{}f$

Resolution Principle