System Validation

Lecture 5: Computation Tree Logic

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January 24, 2003

Overview of lecture

$\Rightarrow$ *Introduction*

- Computation tree logic
  - Syntax and semantics
  - Some formulas express the same
- Model-checking CTL
- Fairness
- The difference between PLTL and CTL
- Practical use of CTL

Linear and branching temporal logic

- *Linear* temporal logic:

  "statements about (all) paths starting in a state"

  – $s \models \mathbf{G}\,(x \leqslant 20)$ iff for all possible paths starting in s always $x \leqslant 20$

- *Branching* temporal logic:

  "statements about all or some paths starting in a state"

  – $s \models \mathbf{AG}\,(x \leqslant 20)$ iff for **all** paths starting in s always $x \leqslant 20$
  – $s \models \mathbf{EG}\,(x \leqslant 20)$ iff for **some** path starting in s always $x \leqslant 20$

Why branching temporal logic?

- Expressiveness of linear and most branching temporal logics is incomparable:
  - there are properties that can be expressed in linear, but not in most branching TL
  - there are properties that can be expressed in most branching, but not in linear TL

- The model-checking algorithms are different, and so are their time and space complexities

model checking was originally developed for a branching temporal logic
[Emerson & Clarke 1981]

Branching temporal logics

There are various branching temporal logics:

- Hennessy-Milner logic
- Computation Tree Logic (CTL)
- Extended Computation Tree Logic (CTL^*)
  - combines PLTL and CTL into a single framework
- Alternation-free modal μ-calculus
- Modal μ-calculus
- Propositional dynamic logic

Overview of lecture

- Introduction

⇒ *Computation tree logic*

  - *Syntax and semantics*
  - *Some formulas express the same*

- Model-checking CTL
- Fairness
- The difference between PLTL and CTL
- Practical use of CTL

Propositional linear temporal logic

Is the smallest set of formulas generated by the rules:

1. each atomic proposition p is a formula
2. if Φ and Ψ are formulas, then $\neg\Phi$ and $\Phi \vee \Psi$ are formulas
3. if Φ and Ψ are formulas, then $\mathbf{X}\,\Phi$ (“next”) and $\Phi\,\mathbf{U}\,\Psi$ (“until”) are formulas.

derived operators $\mathbf{G}$ (always) and $\mathbf{F}$ (eventually)

how to specify that for every computation it is always possible to return to the initial state? $\mathbf{G}\,\mathbf{F}$ *start?*

Propositional branching temporal logic

- Extend PLTL with *path quantifiers*:
  - $\mathbf{A}$, where $\mathbf{A}\,\varphi$ denotes that φ holds over all paths
  - $\mathbf{E}$, where $\mathbf{E}\,\varphi$ denotes that there exists some path satisfying φ
- $\mathbf{A}\,\varphi$ and $\mathbf{E}\,\varphi$ are called *state*-formulas
- PLTL-formula φ is called a *path*-formula

how to specify that for every computation it is
always possible to return to the initial state? $\mathbf{AG}\,\mathbf{EF}$ *start!*

Computation tree logic

CTL is the smallest set of formulas generated by the rules:

1. State-formulas:
   (a) each atomic proposition p is a state-formula
   (b) if Φ and Ψ are state-formulas, then $\neg\Phi$ and $\Phi \vee \Psi$ are state-formulas
   (c) if φ is a path-formula, then $\mathbf{E}\,\varphi$ and $\mathbf{A}\,\varphi$ are state-formulas

2. Path-formulas:
   (a) if Φ and Ψ are state-formulas, then $\mathbf{X}\,\Phi$ and $\Phi\,\mathbf{U}\,\Psi$ are path-formulas.

$\mathbf{X}$ and $\mathbf{U}$ are always directly preceded by $\mathbf{E}$ or $\mathbf{A}$

Derived operators

$$\mathbf{F}\,\Phi \equiv \text{true}\,\mathbf{U}\,\Phi$$

$$\mathbf{G}\,\Phi \equiv \neg\,\mathbf{F}\,\neg\,\Phi$$

$$\mathbf{EF}\,\Phi \equiv \mathbf{E}\,(\text{true}\,\mathbf{U}\,\Phi)$$ "potentially Φ"

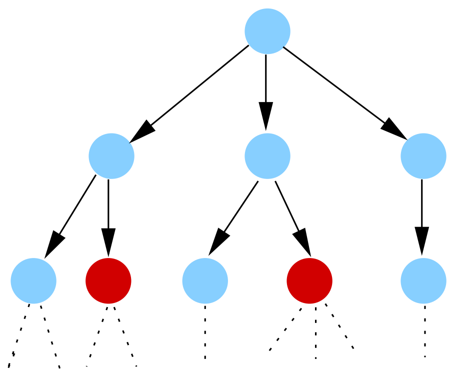
$$\mathbf{AG}\,\Phi \equiv \neg\,\mathbf{EF}\,\neg\,\Phi$$ "invariantly Φ"

$$\mathbf{AF}\,\Phi \equiv \mathbf{A}\,(\text{true}\,\mathbf{U}\,\Phi)$$ "inevitably Φ"

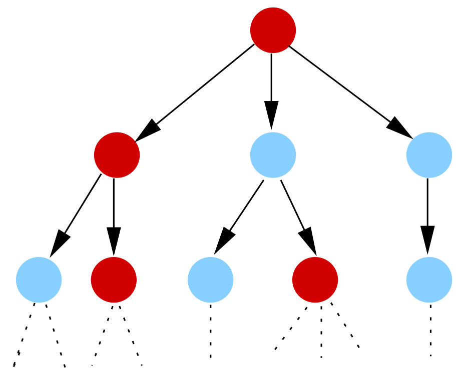
$$\mathbf{EG}\,\Phi \equiv \neg\,\mathbf{AF}\,\neg\,\Phi$$ "potentially always Φ"

the boolean connectives are derived as usual

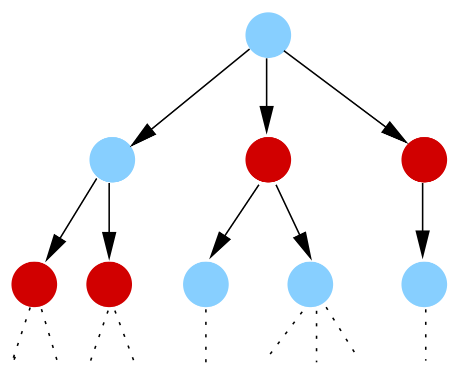
Derived operators



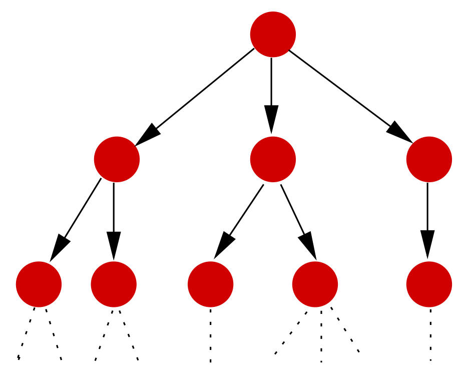
EF *red*



EG *red*



AF *red*



AG *red*

Some example CTL-formulas

let AP be the set of atomic propositions over variable x, boolean operators $<$, $\geqslant$ and $=$, and function $x + c$ for constant c

- the following formulas are *legal* CTL-formulas over AP:
  - $\neg\,(x + 7 < 21)\ \vee\ (x = 64)$
  - $\mathbf{AF}\,(x + 12 \geqslant 10)$
  - $\mathbf{EG}\,(x \geqslant 0\ \wedge\ x < 200)$
  - $x = 10 \Rightarrow \mathbf{AX}\,\mathbf{E}\,(x \geqslant 10\,\mathbf{U}\,x = 0)$
- the following formulas are *illegal* CTL-formulas over AP:
  - $\neg\,(x + x < 21)\ \vee\ (x^3 = 64)$
  - $\mathbf{E}\,(\mathbf{F}\,(x \geqslant 10)\ \wedge\ \mathbf{G}\,(x \geqslant 0))$
  - $\mathbf{E}\,(x \geqslant 20\ \wedge\ \mathbf{X}\,x = 20)$

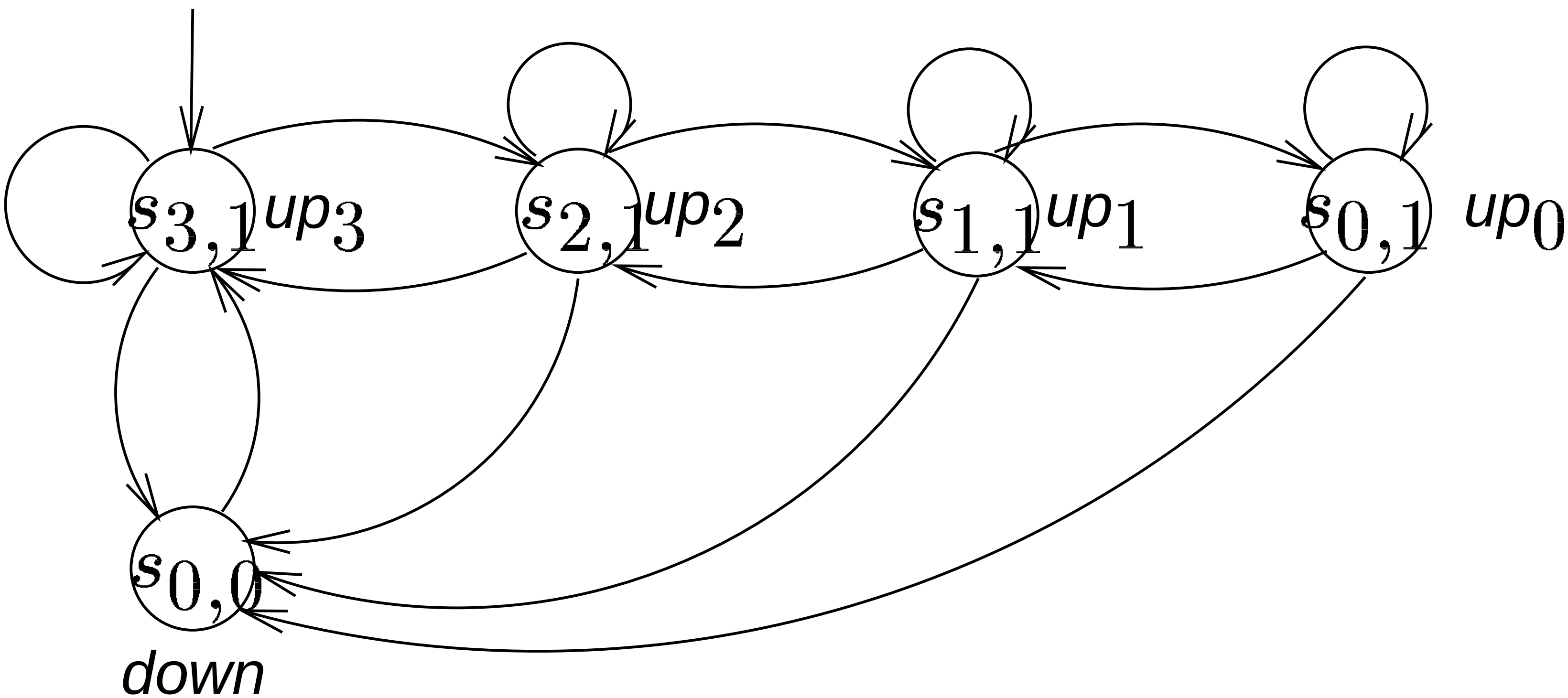
Interpretation of CTL

Formal interpretation of CTL-formulas is defined in terms of a Kripke structure $\mathcal{M} = (S, I, R, Label)$ where

- S is a countable set of states,
- $I \subseteq S$ is a set of initial states,
- $R \subseteq S \times S$ is a transition relation with $\forall s \in S. \, (\exists s' \in S. \, (s, s') \in R)$
- $Label : S \longrightarrow 2^{AP}$ is an interpretation function on S.

$Label(s)$ is the set of the atomic propositions that are valid in s

Example Kripke structure



Semantics of CTL: state-formulas

Defined by a relation $\models$ such that

$\mathcal{M}, s \models \Phi$ if and only if formula Φ holds in state s of structure $\mathcal{M}$

$s \models p$	iff	$p \in Label(s)$
$s \models \neg\,\Phi$	iff	$\neg\,(s \models \Phi)$
$s \models \Phi \;\vee\; \Psi$	iff	$(s \models \Phi) \;\vee\; (s \models \Psi)$
$s \models \mathbf{E}\,\varphi$	iff	$\sigma \models \varphi$ for *some* path σ that starts in s
$s \models \mathbf{A}\,\varphi$	iff	$\sigma \models \varphi$ for *all* paths σ that start in s

Semantics of CTL: path-formulas

A *path* in $\mathcal{M}$ is an infinite sequence of states $s_0\, s_1\, s_2 \ldots$ such that $s_0 \in I$ and $(s_i, s_{i+1}) \in R$ for all $i \geqslant 0$

Define a relation $\models$ such that

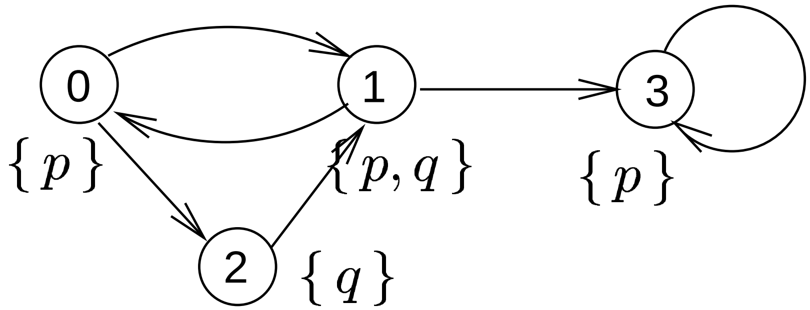
> $\mathcal{M}, \sigma \models \varphi$ if and only if path σ in model $\mathcal{M}$ satisfies formula φ

$\sigma \models \mathbf{X}\,\Phi$ iff $\sigma[1] \models \Phi$

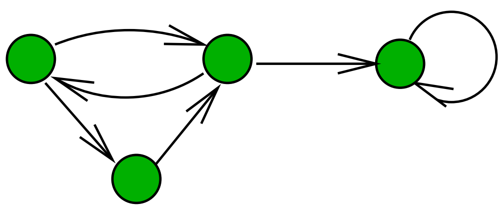
$\sigma \models \Phi\,\mathbf{U}\,\Psi$ iff $(\exists\, j \geqslant 0.\, \sigma[j] \models \Psi \;\wedge\; (\forall\, 0 \leqslant k < j.\, \sigma[k] \models \Phi))$

where $\sigma[i]$ denotes the $(i{+}1)$-th state in the path σ

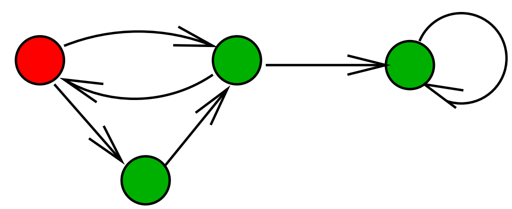
Example of semantics of CTL



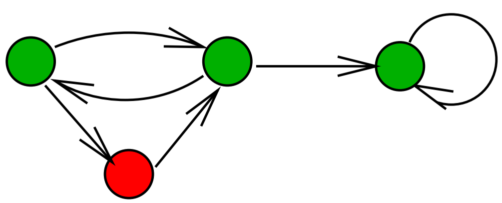
$\mathbf{EX}\, p$



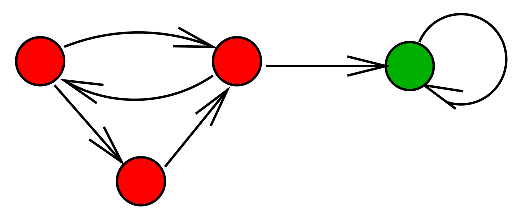
$\mathbf{AX}\, p$



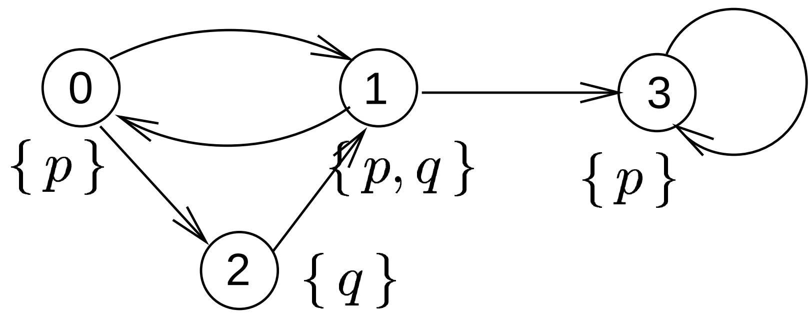
$\mathbf{EG}\, p$



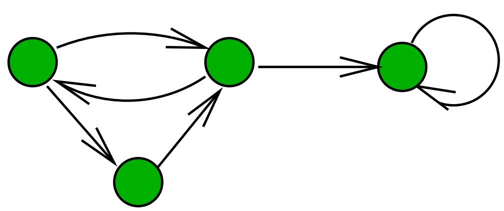
$\mathbf{AG}\, p$



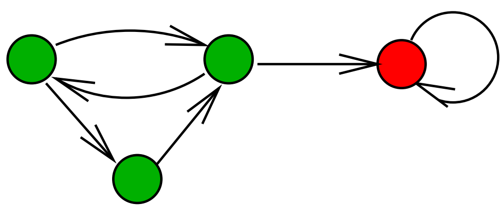
Example of semantics of CTL (cont'd)



$\mathbf{EF}\,\mathbf{EG}\,p$



$\mathbf{A}\,(p\,\mathbf{U}\,q)$



Some important validities for CTL

PLTL expansion rules:
(last lecture)

$$\Phi\,\mathbf{U}\,\Psi \equiv \Psi \vee (\Phi \wedge \mathbf{X}\,(\Phi\,\mathbf{U}\,\Psi))$$

$$\mathbf{F}\,\Phi \equiv \Phi \vee \mathbf{X}\,\mathbf{F}\,\Phi$$

$$\mathbf{G}\,\Phi \equiv \Phi \wedge \mathbf{X}\,\mathbf{G}\,\Phi$$

CTL expansion rules:

$$\mathbf{E}\,(\Phi\,\mathbf{U}\,\Psi) \equiv \Psi \vee (\Phi \wedge \mathbf{EX}\,\mathbf{E}\,(\Phi\,\mathbf{U}\,\Psi))$$

$$\mathbf{A}\,(\Phi\,\mathbf{U}\,\Psi) \equiv \Psi \vee (\Phi \wedge \mathbf{AX}\,\mathbf{A}\,(\Phi\,\mathbf{U}\,\Psi))$$

$$\mathbf{EF}\,\Phi \equiv \Phi \vee \mathbf{EX}\,\mathbf{EF}\,\Phi$$

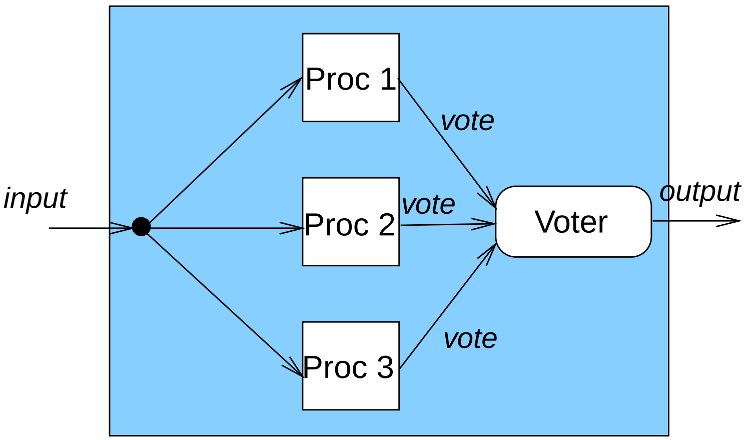
$$\mathbf{AF}\,\Phi \equiv \Phi \vee \mathbf{AX}\,\mathbf{AF}\,\Phi$$

$$\mathbf{EG}\,\Phi \equiv \Phi \wedge \mathbf{EX}\,\mathbf{EG}\,\Phi$$

$$\mathbf{AG}\,\Phi \equiv \Phi \wedge \mathbf{AX}\,\mathbf{AG}\,\Phi$$

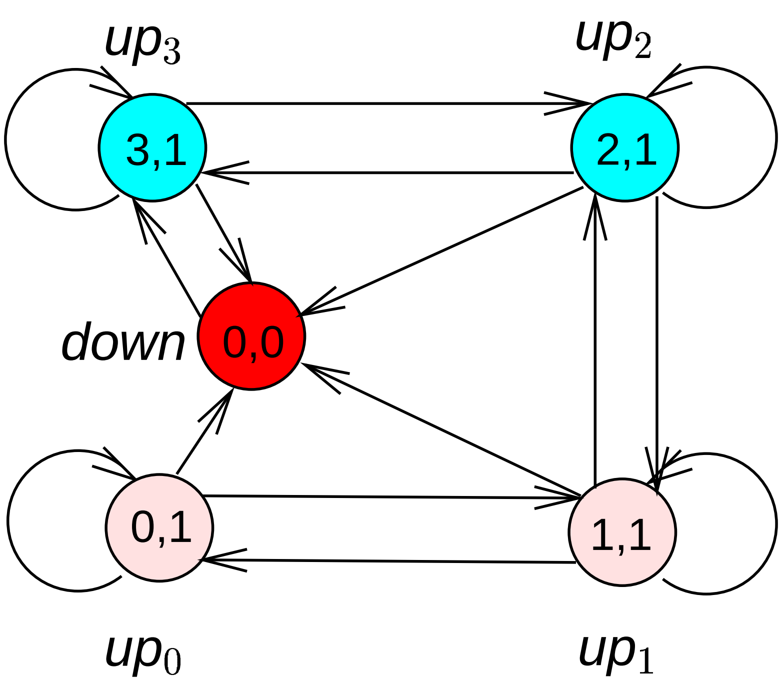
Specifying properties in CTL

- Triple Modular Redundant system: 3 processors and a single voter
  - processors run same program; voter takes a majority vote
  - each component (processor and voter) is failure-prone
  - there is a single repairman for repairing processors and voter



- Modelling assumptions:
  - if voter failes, whole system goes down
  - after repair of voter, system starts "as new"
  - state $= (\#\text{processors}, \#\text{voters})$

Specifying properties in CTL



- Possibly, the system never goes down: $\mathbf{EG} \neg\, down$
- Inevitably, the system never goes down: $\mathbf{AG} \neg\, down$
- It is always possible to start as new: $\mathbf{AG}\,\mathbf{EF}\, up_3$ (not $\mathbf{AF}\, up_3$)
- The system only goes down while being operational:

$$\mathbf{A}\,((up_3 \vee up_2)\,\mathbf{U}\, down)$$

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$\Rightarrow$ *Model-checking CTL*

- Fairness
- The difference between PLTL and CTL
- Practical use of CTL

Model checking CTL

- how to check whether state s satisfies Φ?
  - compute *recursively* the set $Sat(\Phi)$ of states that satisfy Φ
  - check whether state s belongs to $Sat(\Phi)$
- recursive computation:
  - determine the sub-formulas of Φ
  - start to compute $Sat(p)$, for all atomic propositions p in Φ
  - then check the smallest sub-formulas that contain p
  - check the formulas that contain these sub-formulas
  - and so on....... until formula Φ is checked

Model checking CTL

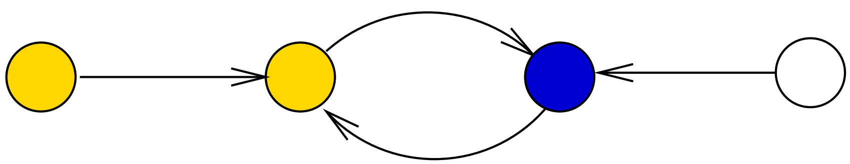
- how to check whether state s satisfies Φ?
  - compute *recursively* the set $Sat(\Phi)$ of states that satisfy Φ
  - check whether state s belongs to $Sat(\Phi)$

- recursive bottom-up computation:
  - consider the parse-tree of Φ
  - start to compute $Sat(p)$, for all leafs in the tree
  - then go one level up in the tree and check the formula of that node
  - then go one level up and check the formula of that node
  - and so on....... until the root of the tree (i.e., Φ) is checked

Model checking CTL: pseudo-algorithm

- $Sat(p)$ is the set of states labelled with atomic proposition p
- $Sat(\Phi \lor \Psi)$ is $Sat(\Phi) \cup Sat(\Psi)$
- $Sat(\neg\Phi)$ equals $S - Sat(\Phi)$
- $Sat(\mathbf{EX}\,\Phi)$ is the set of states that can directly move to $Sat(\Phi)$
- $Sat(\mathbf{AX}\,\Phi)$ is the set of states that can directly only move to $Sat(\Phi)$
- $Sat(\mathbf{E}\,(\Phi\,\mathbf{U}\,\Psi))$ is computed iteratively:
  - $S^0 = Sat(\Psi)$
  - S^1 = $S^0 \cup \Phi$-states that can directly move to S^0
  - S^2 = $S^1 \cup \Phi$-states that can directly move to S^1
  - .... until $S^{k+1} = S^k$

Computing $Sat(\mathbf{E}\,(yellow\,\mathbf{U}\,blue))$



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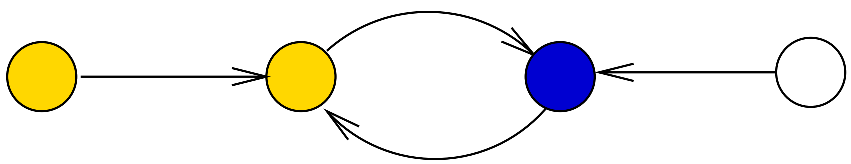
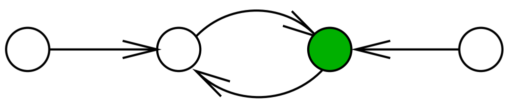


first iteration



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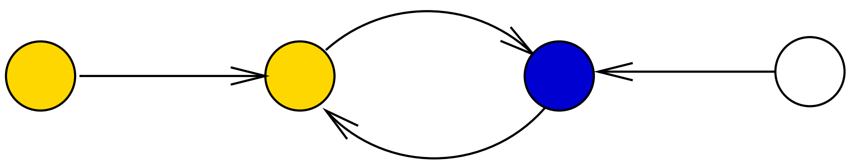
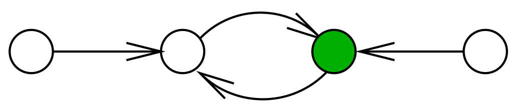
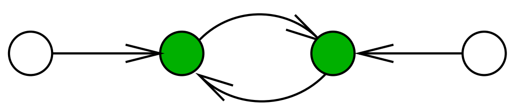


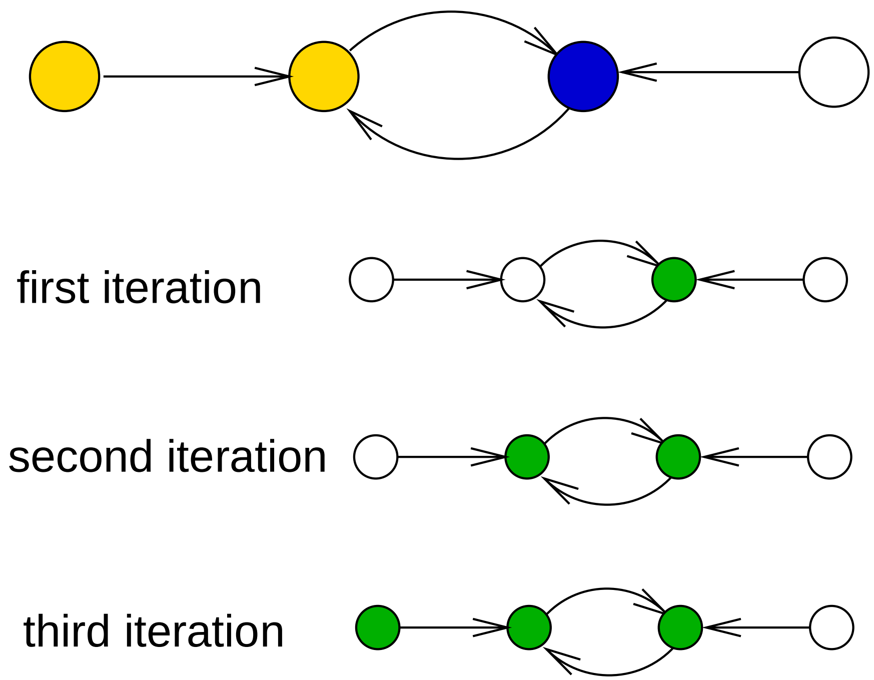
first iteration



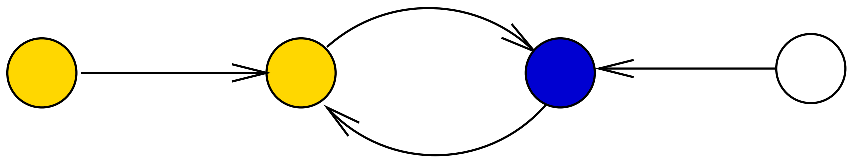
second iteration

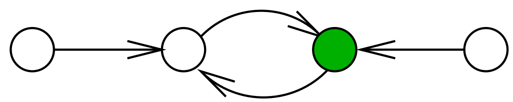


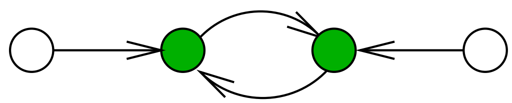
Computing $Sat(\mathbf{E}\,(yellow\,\mathbf{U}\,blue))$

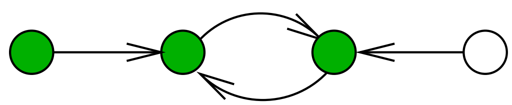


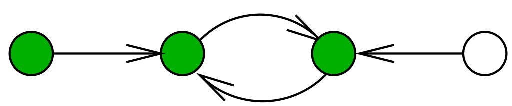
Computing $Sat(\mathbf{E}\,(yellow\,\mathbf{U}\,blue))$



first iteration 

second iteration 

third iteration 

fourth iteration  done!

Overview of model-checking CTL

- Algorithm: bottom-up traversal of the parse tree of the formula
- For until-formulas: a fixed-point computation
- For $\mathbf{EG}$-formulas: a more efficient algorithm using detection of strongly connected components
- Special attention has to be devoted to fairness issues
- Worst case time-complexity is $\mathcal{O}(|\Phi| \cdot N^2)$

  where $|\Phi|$ is the length of Φ and N is the number of states in the system model
- Tools: NuSMV, Cadence SMV, Uppaal, CADP, ...

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$\Rightarrow$ *The difference between PLTL and CTL*

- Fairness
- Practical use of CTL

PLTL versus CTL

- Their expressiveness is incomparable:
  - there is no equivalent PLTL-formula for $\mathbf{AG}\,\mathbf{EF}\,p$
  - there is no equivalent CTL-formula for $\mathbf{A}\,(\mathbf{F}\,(p \;\wedge\; \mathbf{X}\,p))$
    - each path reaches a point at which p holds for two consecutive moments
    - $\mathbf{AF}\,(p \;\wedge\; \mathbf{AX}\,p)$ and $\mathbf{AF}\,(p \;\wedge\; \mathbf{EX}\,p))$ do not express the same
  - but there do exist common formulas like $\mathbf{A}\,(p\,\mathbf{U}\,q)$ and $\mathbf{AG}\,p$
- Complexity of model checking is different:
  - model checking PLTL is PSPACE-complete: $\mathcal{O}(System^2 \cdot 2^{Formula})$
  - model checking CTL is in polynomial time: $\mathcal{O}(System^2 \cdot Formula)$

don't think that CTL model checking is more efficient
as CTL-formulas are sometimes much longer than PLTL-formulas!

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$\Rightarrow$ *Fairness*

- Practical use of CTL

Fairness: modelling concurrency

Consider the parallel execution of two processes: (initially $x = 0$)

process $P =$ **while** $\langle\ (x \geqslant 0)$ **do** $x := x + 1\ \rangle$ **od**

process $Q = x := -1$

- Does this parallel program ever terminate?
- Expected runs: $P\,Q\,P\,Q\,P\,Q \ldots$ or $P\,P\,Q\,P\,Q\,Q\,P\,P \ldots$ or the like
- But not: $P\,P\,P\,P\,P\ \ldots$ (no Q) or $Q\,Q\,Q\ \ldots$ (no P)
- *Fairness* is modeled by fair scheduling assumptions – described as temporal logic-formulas – over the processes

Typical fairness assumptions (in PLTL)

- *Unconditional fairness*: property *running* is true infinitely often:

$$\mathbf{G}\,\mathbf{F}\, running$$

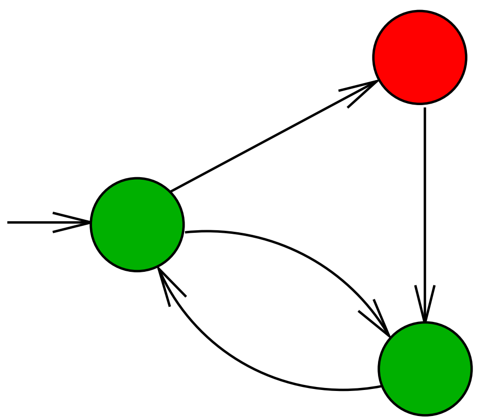
- *Weak fairness*: if *enabled* is eventually continuously true, *running* holds infinitely often:

$$\mathbf{F}\,\mathbf{G}\, enabled \;\Rightarrow\; \mathbf{G}\,\mathbf{F}\, running$$

- *Strong fairness*: if *enabled* holds infinitely often, *running* does so too:

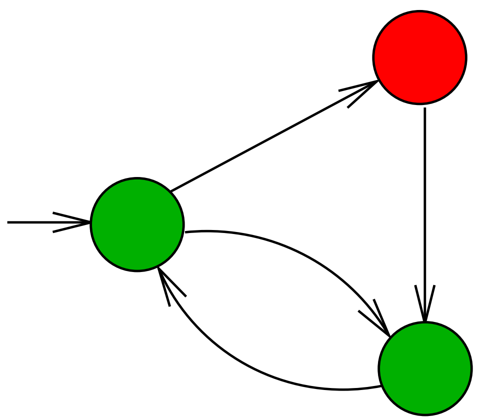
$$\mathbf{G}\,\mathbf{F}\, enabled \;\Rightarrow\; \mathbf{G}\,\mathbf{F}\, running$$

Fair versus unfair computations



do we have $\mathbf{AG}\,(green \;\Rightarrow\; \mathbf{AF}\, red)$?

Fair versus unfair computations



- no, since there exists an entirely *green* path!
- but, is this a "fair" path?
- no, as becoming *red* is possible infinitely often
- how to exclude these *unfair* computations?
- add a fairness assumption, e.g., $\mathbf{AG}\,\mathbf{AF}\,$*red*!
- then $\mathbf{AG}\,(\mathit{green} \Rightarrow \mathbf{AF}\,\mathit{red})$ is valid as the unfair computations are ignored

$\Rightarrow$ fairness assumptions rule out "unrealistic" runs

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$\Rightarrow$ *Practical use of CTL*

Practical properties in CTL

- Reachability
  - simple reachability $\mathbf{EF}\,\Psi$
  - conditional reachability $\mathbf{E}\,(\Phi\,\mathbf{U}\,\Psi)$
  - reachability from any state $\mathbf{AG}\,(\mathbf{EF}\,\Phi)$
- Safety (“something bad never happens”)
  - simple safety $\mathbf{AG}\,\neg\Phi$
  - conditional safety $\mathbf{A}\,(\Phi\,\mathbf{U}\,\Psi)\ \vee\ \mathbf{AF}\,\Phi$
- Liveness $\mathbf{AG}\,(\Phi\ \Rightarrow\ \mathbf{AF}\,\Psi)$
- Fairness $\mathbf{AG}\,(\mathbf{AF}\,\Phi)$

How to use CTL in practice?

Capture commonly-used types of formulas in specification patterns

- *Specification pattern*: generalized description of a commonly occurring requirement on the permissable paths in a model
  - parameterizable: only state-formulas to be instantiated
  - high-level: no detailed knowledge of TL is required
  - formalism-independent: by mappings onto TL at hand

- *Scope* of a pattern: the extent of the computation over which the pattern must hold, such as
  - global: the entire computation
  - after: the computation after a given state
  - between: any part of the computation from one state to another

Most commonly used specification patterns for CTL

Investigation of 555 requirement specifications reveals that the following patterns are most widely used for state-formulas P, Q and R: (Dwyer et al, 1998)

pattern	*scope*	*PLTL-formula*	*frequency*
response	global	$\mathbf{AG}\,(P \Rightarrow \mathbf{AF}\,Q)$	43.4 %
universality	global	$\mathbf{AG}\,P$	19.8 %
absence	global	$\mathbf{AG}\,\neg P$	7.4 %
precedence	global	$\mathbf{AG}\,\neg P \vee \mathbf{A}\,(\neg P\,\mathbf{U}\,Q)$	4.5 %
absence	between	$\mathbf{AG}\,((Q \wedge \neg R) \Rightarrow \mathbf{A}\,((\neg P \vee \mathbf{AG}\,\neg R)\,\mathbf{W}\,R)$	3.2 %
absence	after	$\mathbf{AG}\,(Q \Rightarrow \mathbf{AG}\,\neg P)$	2.1 %
existence	global	$\mathbf{AF}\,P$	2.1 %
			$\approx$ 80 %

more info at: `www.cis.ksu.edu/santos/spec-patterns/`