CS206

Lecture 16

KnuthBendix Algorithm

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Plan for this lecture

- Knuth Bendix Algorothm
- Example

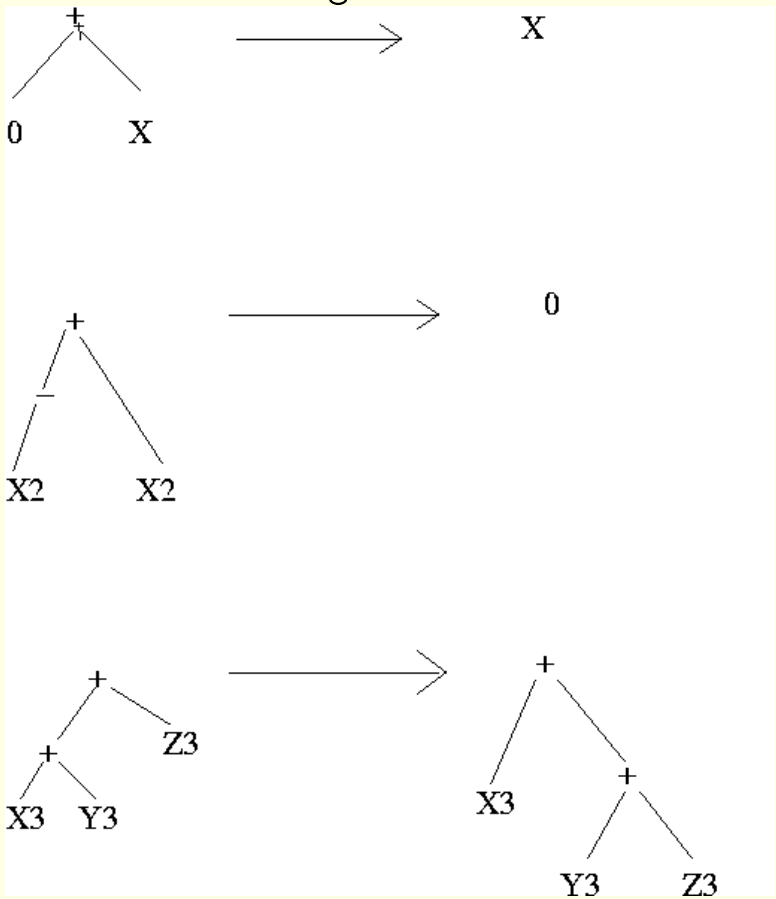
Algorithm

The Knuth-Bendix completion algorithm attempts to transform a set of axioms into a canonical set of rewrite rules.

- Unification
- Rewriting
- Strategy

Basic Rules Assumed

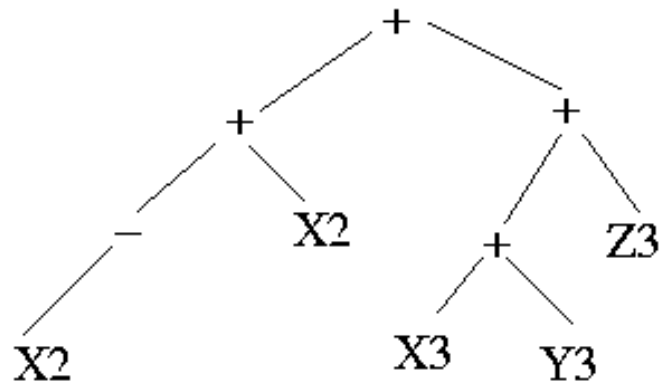
For further discussion the following rules are assumed



Disjoint Overlap

NOW COMBINING R2 AND R3

using disjoint overlap,



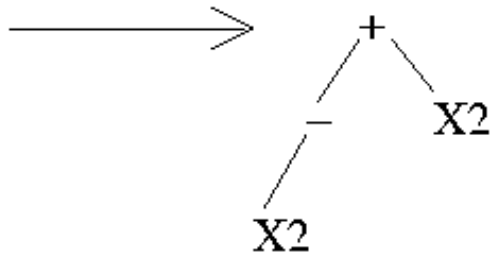
here both ruls are joined by a new functional symbol

This is not of much use,since we cannot simplify this further.

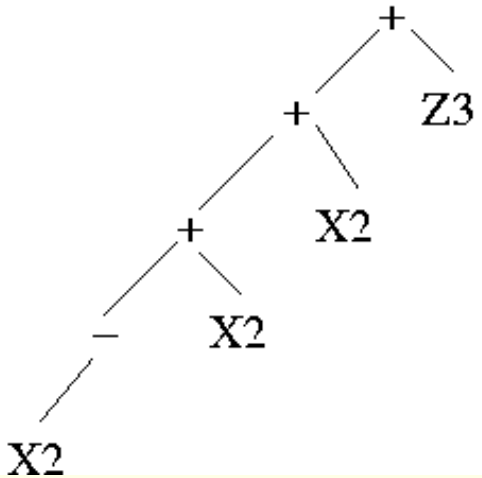
Variable Overlap

NOW APPLYING VARIALBE OVERLAP

Using R2 at one of the variables of R3, bye applying unification

for eg. X3 

We get



Algorithm

Initially, the axiom set contains the initial axioms, and the rule set is empty.

- While the axiom set is not empty do
- Begin Select and remove an axiom from the axiom set
- Normalise the axiom

Algorithm contd.

- If the axiom is not of the form x = x then begin
- Order the axiom using the simplification ordering, », to form a new rule (stop with failure if not possible);
- Place any rules whose left-hand side is reducible by the new rule back into the set of axioms;
- Superpose the new rule on the whole set of rules to find the set of critical pairs;
- Introduce a new axiom for each critical pair; end end.

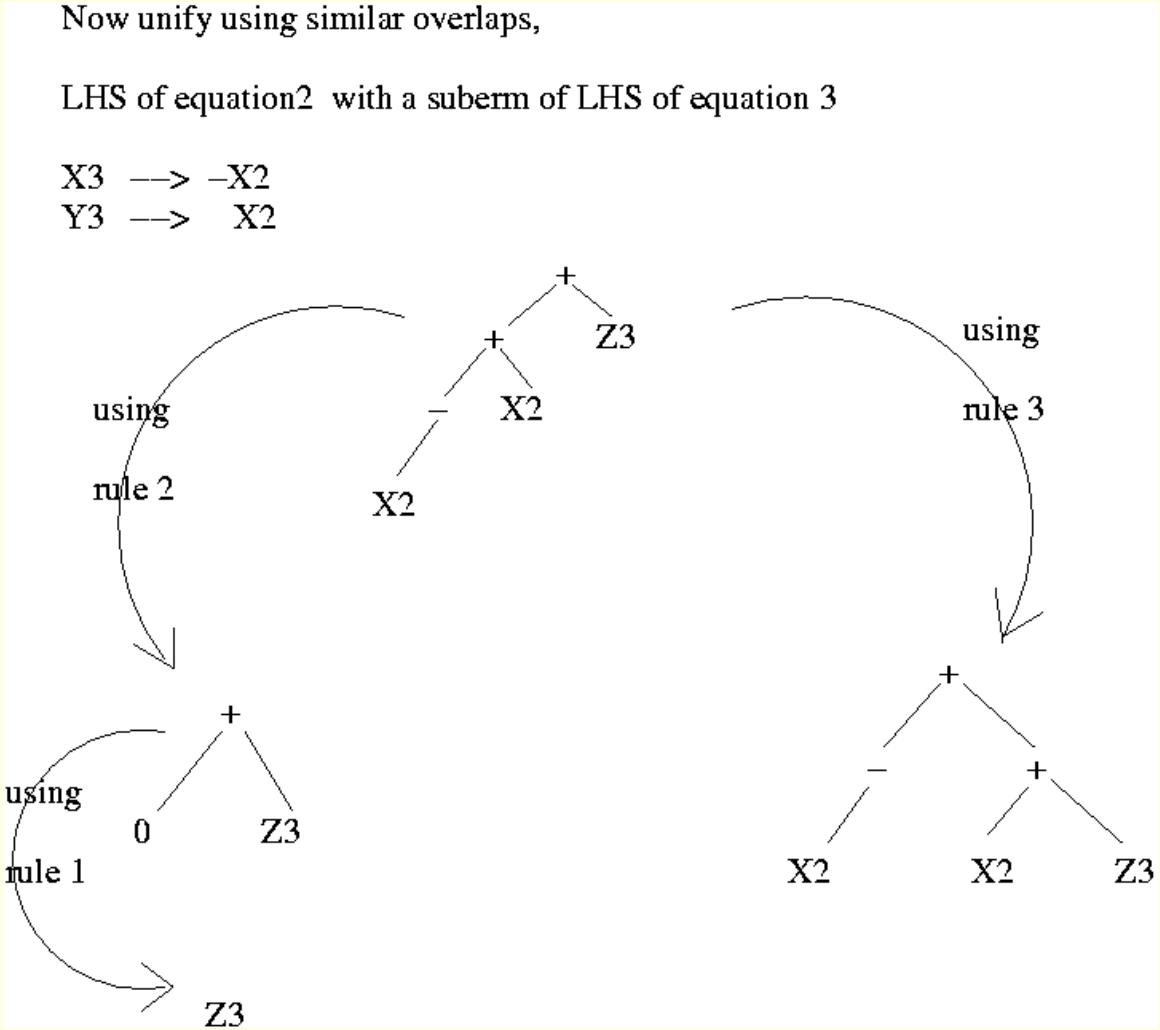
Behaviour of algorithm

The algorithm may behave in one of the following ways :

- terminate with success. A finite canonical set has been found.
- terminate with failure. This occurs at step E if a particular axiom cannot be ordered by «. For example, the ordering we described in section 2 cannot order the commutative axiom, a + b = b + a. If such an axiom is generated, the algorithm fails.
- loop without terminating. Certain canonical sets are infinite, and in attempting to complete them, the algorithm never terminates. .LP Step F ensures that all rewrite rules are normalised with respect to each other. This means that the introduction of a new rule may cause existing rules to disappear. Thus the set of rules does not grow consistently with every iteration.

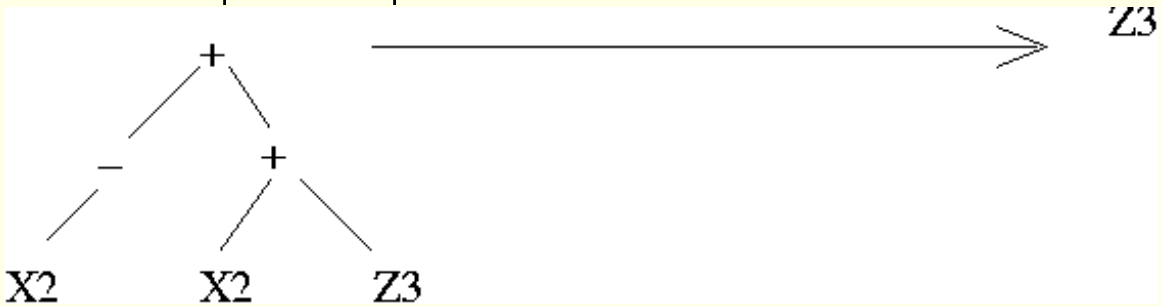
Operation of algorithm

Derived rules :- Derivation :- From Rules applied :- Critical expression LHS RHS

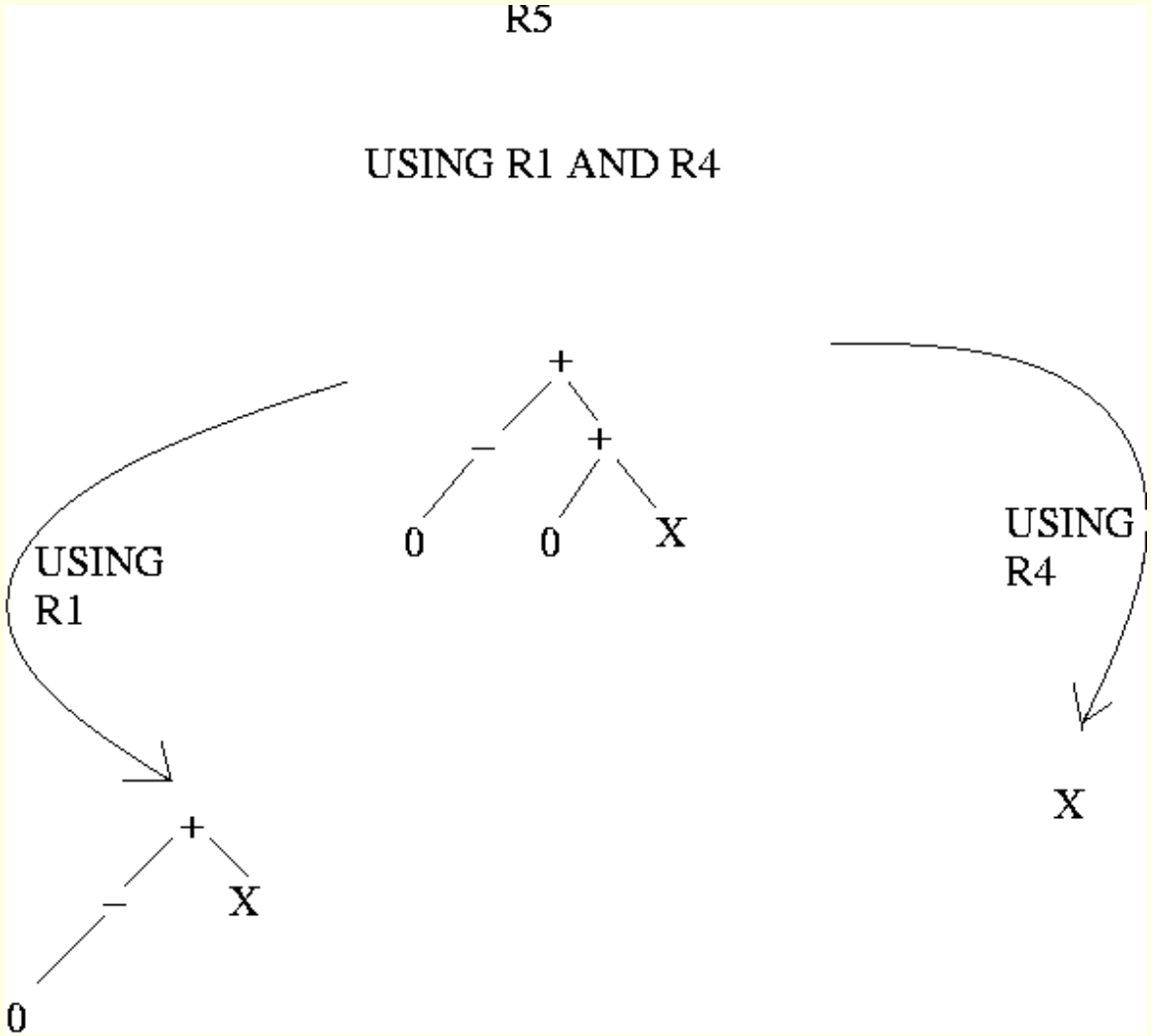


New Rule

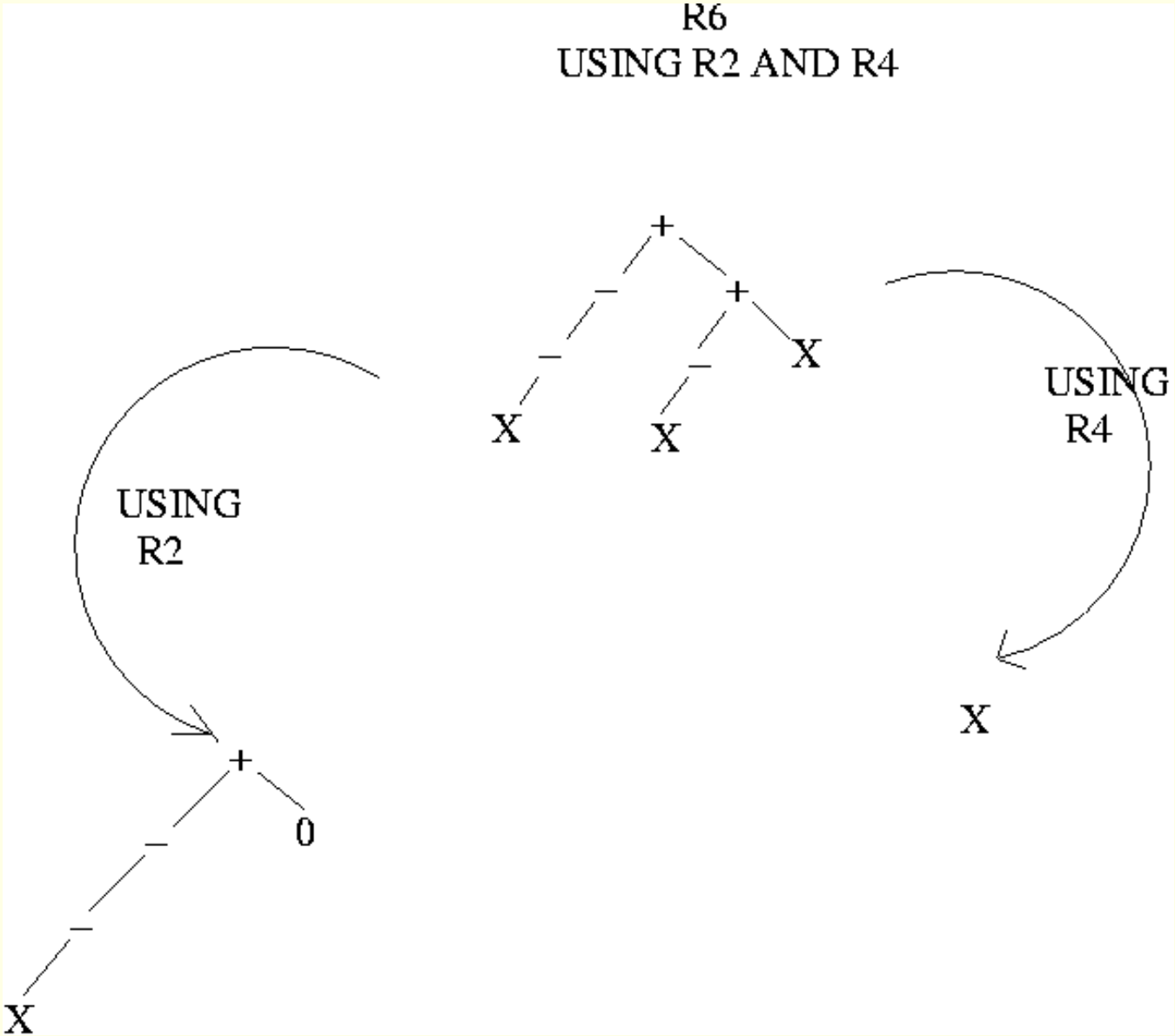
Unification Of Eq2 And Eq3 Yields New Rule We call it R4



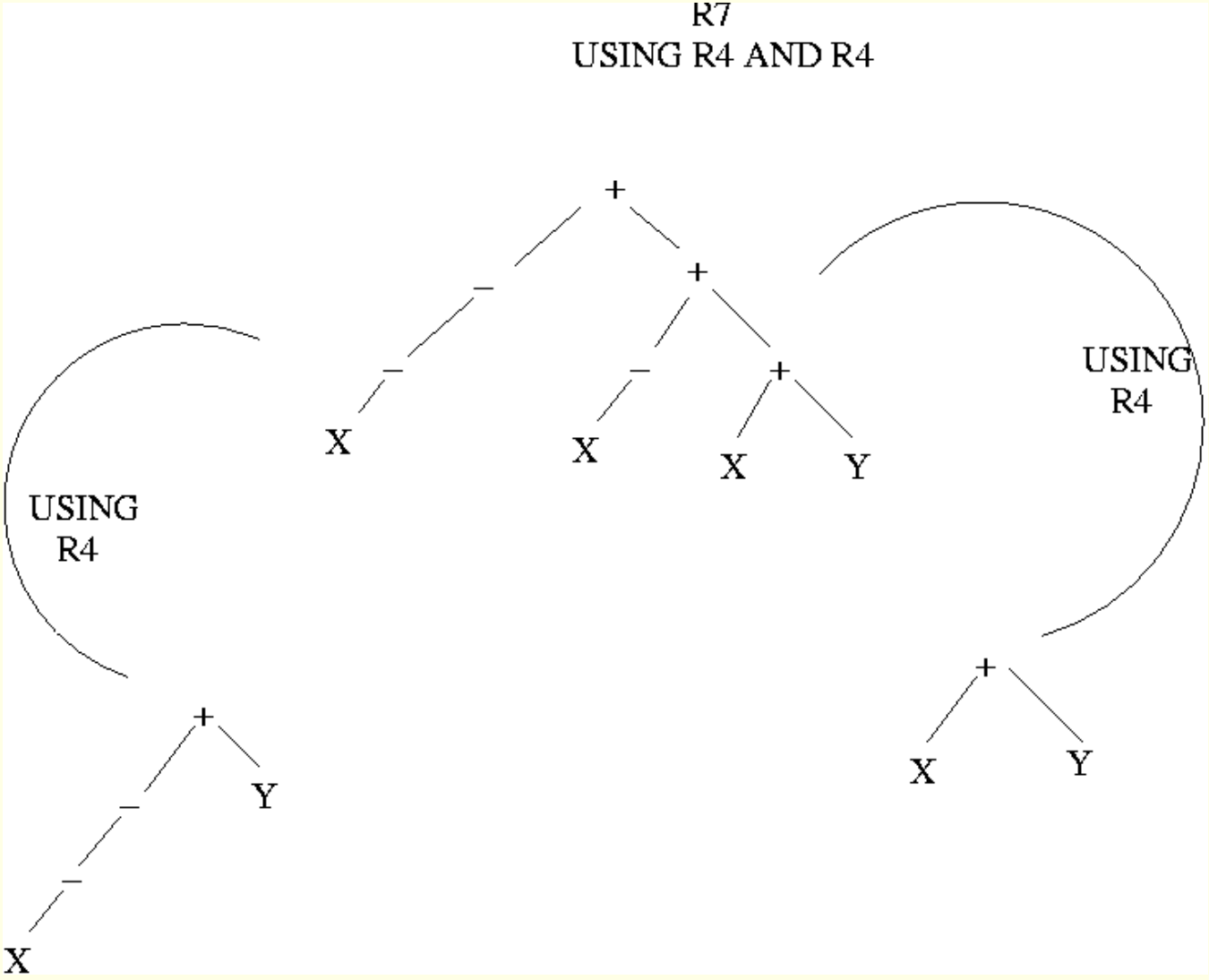
Example



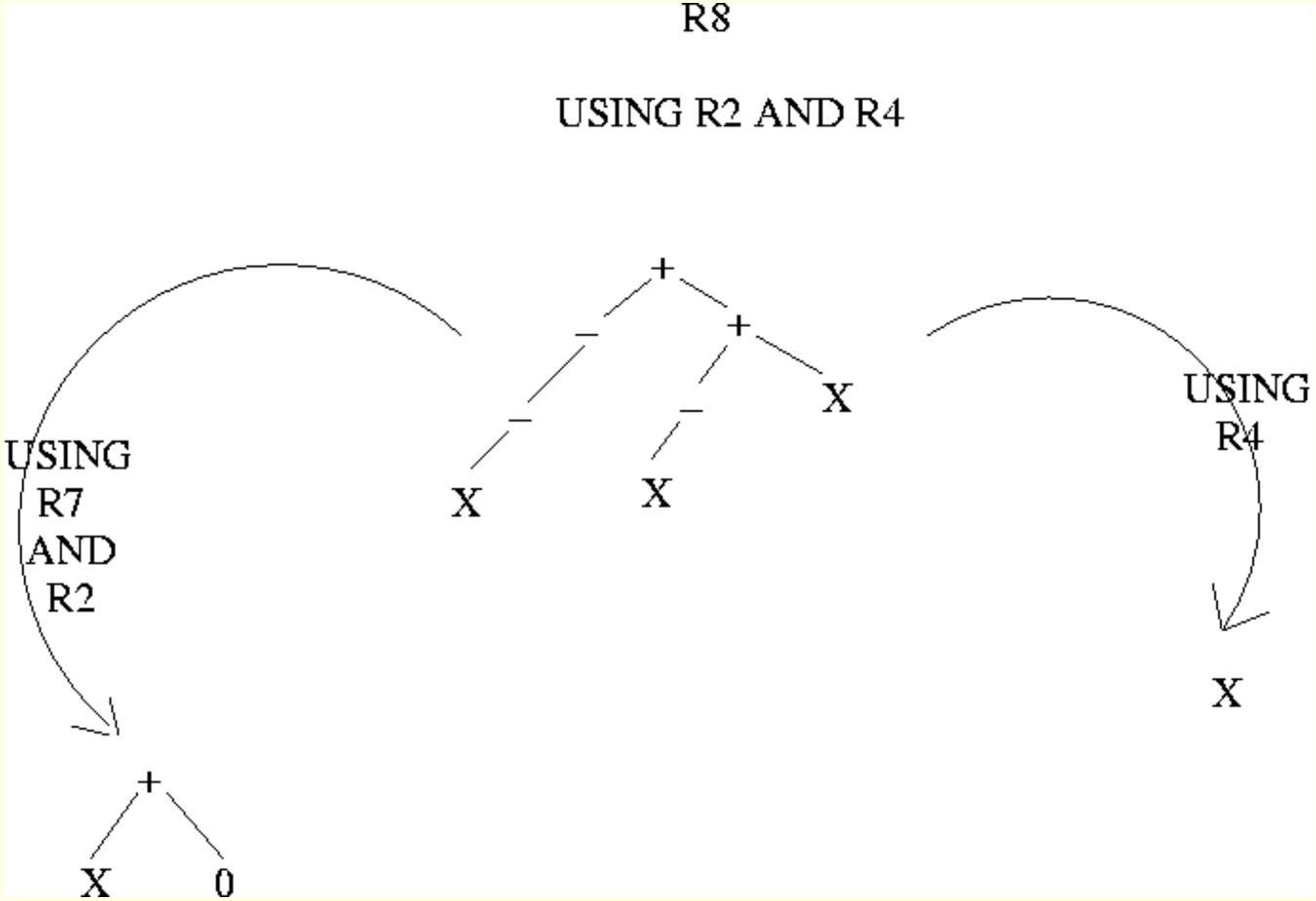
Examlpe Contd.



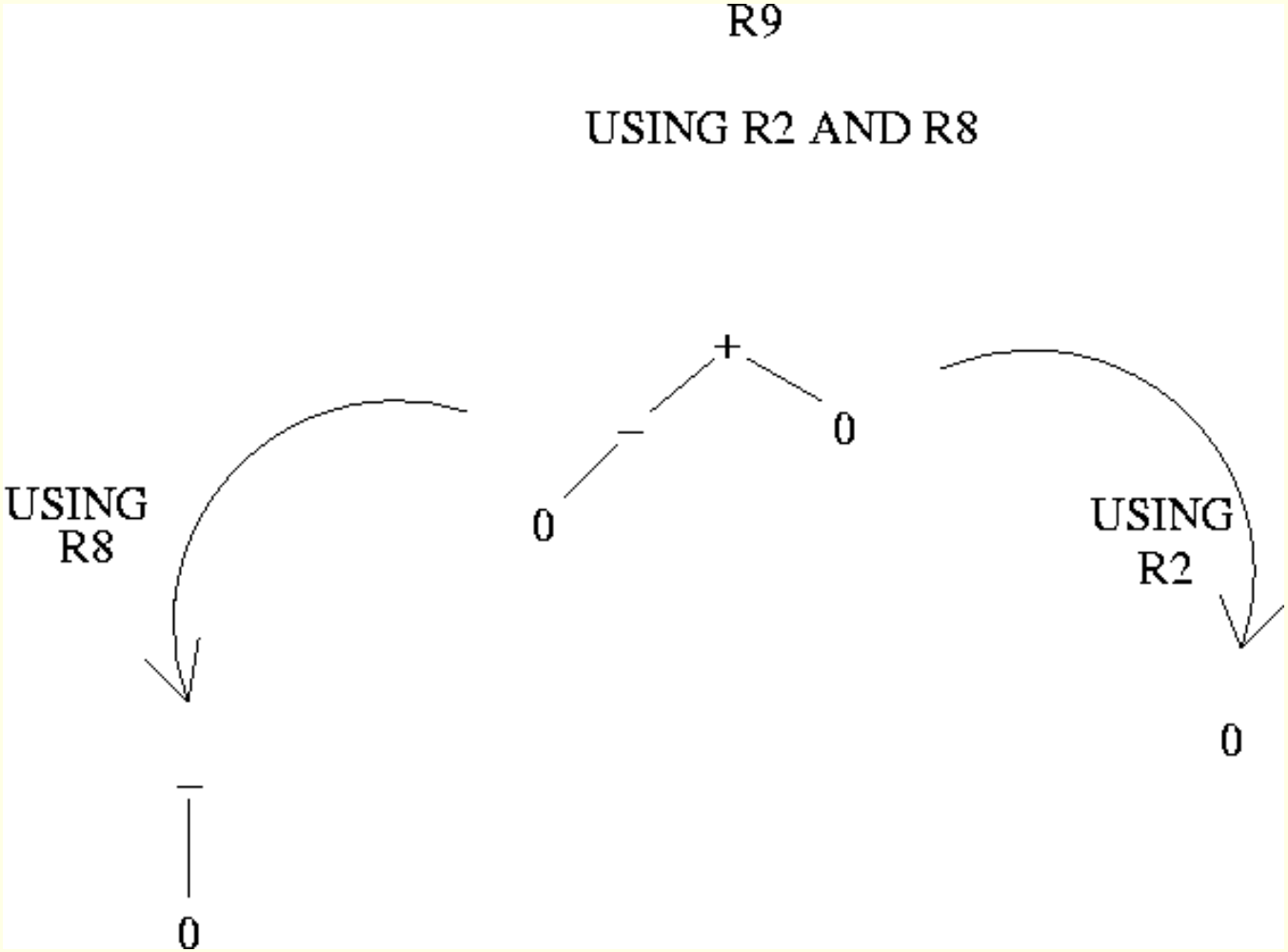
Example Contd.



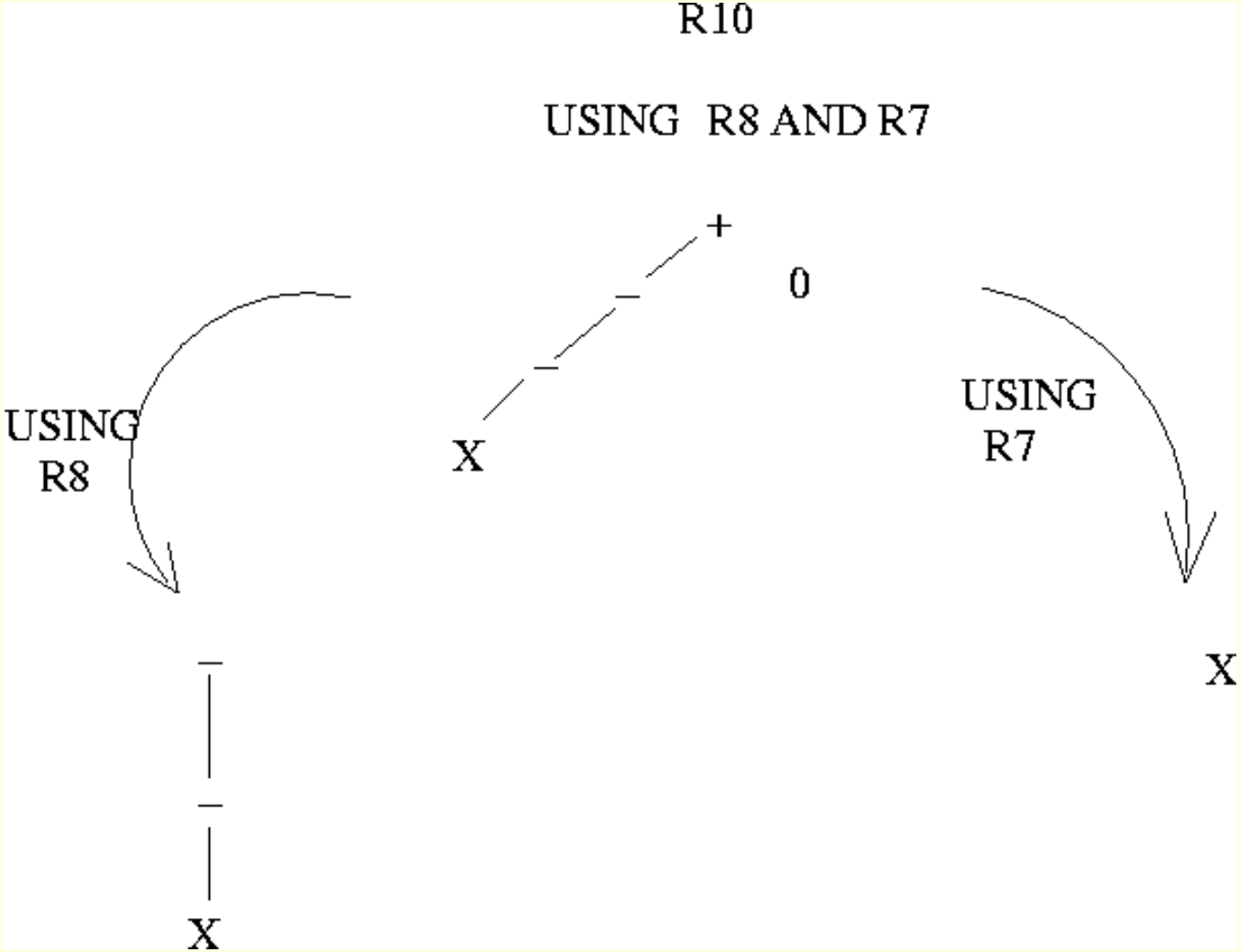
Example Contd.



Example Contd.



Example Contd.



Elimination Of Repetition

During the Unification Procedure as we get new cannonical relation we check whether it can be used with some other previous rule say Rx to give previously Existing Rule say Ry then at that moment we eliminate rule Rx from our Rule list