

CS206 Lecture 01 Formal Methods in Computer Science

G. Sivakumar

Computer Science Department IIT Bombay siva@iitb.ac.in http://www.cse.iitb.ac.in/~siva

Tue, Dec 31, 2002

- Why Formal Methods?
- What is Logic?
- Applications of Logic in CS
 - Specification and Verification
 - Artificial Intelligence
 - Programming Languages
- Broad Course Overview



Home Page
Title Page
Contents
•• ••
Page 2 of 15
1 496 2 01 10
Go Back
Full Screen
Close

Safety Critical Systems

Consider the following systems

- Air Traffic Control
- Space Shuttle Launcher
- Nuclear Power Plant
- ...

Some features:

- Real Time
- Reactive
- Safety Critical

How to design and build such systems?





Traditional Approach





Formal Methods Approach

Constructing a mathematical model of the computer program, hardware, or system concerned, and then using calculation to determine whether the model satisfies desired properties.

 $\mathcal{M} \vdash \mathcal{P}$

- Model using "formal language" (logic)
- Calculation (\vdash) is "formal deduction" (proof)



Traffic Lights Property Specification



Difficulties in using natural language to specify properties

- Ambiguity (Aswattama is dead).
- Incompleteness

• ..



What is Logic?

- Engineer, Mathematician, Logician joke about brown sheep (not Indian cricket team!) in New Zealand.
- What is the Indic word for logic?

For our current purposes, we can live with

Logic is a *formal language* with precise syntax and semantics.

Issues

- Expressive Power (Propositional, First Order, Higher Order, Modal, ..)
- Decidability and Complexity
 - Sound (What I prove is correct)
 - Completeness (If something is correct, I can prove it)



Home Page		
Title Page		
Contents		
Page 8 of 15		
Go Back		
Full Screen		
Close		

Logic in Computer Science

One of the main *pillars* of Computer Science. Along with **concrete mathematics** (Knuth) or *discrete structures*. Some prominent areas (examples follow)

- Specification and Verification
- Artificial Intelligence
 - Knowledge Representation
 - Expert Systems (medical diagnosis)
 - Planning (robot motion)
 - Machine Learning (non-montonicity)
 - ...

- ..

- Programming Languages
 - Algorithms = Logic + Control
 - Logic Programming
 - Compiler Optimizations



Non-classical Logics

• Non-Monotonicity

(As more knowledge comes in, we have to revise earlier beliefs).

- "Birds fly" (general rule inferred from examples)
- "Penguins are birds"
- "Penguins can't fly"
- Resource Constraints
 - If I have 5 Rupees, I can buy coffee
 - If I have 5 Rupees, I can buy tea
 - If I have 5 Rupees, I can buy coffee and I can buy tea



Home Page
Title Page
Contents
44 >>
▲ ▶
Page 10 of 15
Go Back
Full Screen
Close

Functional Programming

Consider a simple function like addition (plus) Functional View Binary function— Two inputs and one output.

Example:

plus(2,3) --> 5, plus(1,4) --> 5 Directionality is associated. Given the two inputs, a program can compute the output.





From Functions to Relations

Suppose we treat plus as a relation taking 3 arguments. That is, we say

```
plus(2,3,5) is true because 2 + 3 = 5.
plus(2,3,4) is false because 2 + 3 \neq 4.
```

and so on

A Logic Program for plus is one that somehow expresses for any three arguments X, Y, Z the *truth value* of plus(X, Y, Z).



Home Page		
Title Page		
Contents		
44 >>		
• •		
Page 12 of 15		
Page 12 of 15		
Page 12 of 15 Go Back		
Page 12 of 15 Go Back		
Page 12 of 15 Go Back Full Screen		
Page 12 of 15 Go Back Full Screen		

Querying a Logic Program

Now we can ask the Logic program interpreter queries like:

```
?- plus(2,3,6).
```

- ?- plus(1,4,3).
- ?- plus(1,1,2).

and it will correctly answer yes or no.





Queries with Variables

Queries can also be of the form:

```
?- plus(2,3,X).
```

```
?- plus(2,4,Y).
```

where uppercase letters denote variables. Now the interpreter will answer

X = 5

for first query and

Y = б

for second query.

This seems to be doing some computing. But nothing new from imperative or functional programming yet.





No Directionality!

But, what about goals like

```
?- plus(X,2,6).
?- plus(2,Y,7).
```

or even

. . .

?- plus(X,Y,5).

The interpreter will generate: X = 4 for first query, and Y = 5 for second query, and many answers for the third query including

```
<X = 1, Y = 4>,
<X = 2, Y = 3>,
```

Question: What about a goal like plus(X,Y,Z)?



•		
Page 1	5 of 15	
Go I	Back	
Full S	Screen	
Clo	ose	

Broad Course Outline

General Plan

- Syntax and Semantics of
 - Propositional Logic
 - First Order (Predicate) Logic
 - Temporal/Modal Logic
- Proof Methods
 - Natural Deduction
 - Refutational Proofs (Resolution)
 - Inductive Proofs
- Applications
 - Logic Programming (Prolog)
 - Specification and Verification
 - * Hardware
 - * Network Protocols
 - * Embeddedd Systems

Web Page with more details soon.