CS206 Lecture 01
Propositional Logic

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Thu, Jan 02, 2003

Plan for Lecture 02

- Syntax
- Semantics

Propositions

A Proposition is a statement with a *truth value.*

- My name is Sivakumar
- Earth is bigger than the Sun.
- $2 + 2 = 4$
- ...
- This statement is false.

Which ones "mean" *true* and which ones *false*?
A typical *puzzle*

> An island is inhabited by two classes of people: knights, who make only true statements, and knaves, who make only false statements. Three inhabitants are conversing. Ashok says, "All of us are knaves." Balu says, "Exactly one of us is a knight." What are Ashok, Balu, and Chandra?

Classical Logic

Has the law of Excluded Middle.

Any proposition has **exactly one** of the two possible **truth values.**

No ArdhaNariswara!.

Allows *proof by contradiction.*

Non-Constructive Proofs: You can prove a number is not a prime without revealing any way to factorize it.

Many other flavours possible.

- Multi-valued logic. *yes, no, maybe*
- Fuzzy Logic *degree of truth*
- Probalistic Logic
- Constructive Logic
- ...

What is Syntax?

- A method to define “legal” Alphabet, Words, Sentences ...
- For any **formal** language, it is possible to give a grammar (set of rules) using which we can construct (and check) all **sentences** (*formulae*) of the language.
- Can be done algorithmically (on computer).
- Typically given as an **inductive** definiton. With **base case** (*atomic* formula) and **inductive case** (*compound* formula).
- Computer program to check is typically called Parser

Propositional Logic Alphabet

- Propositional Symbols
  - Truth Values: $\mathbf{t}$, $\mathbf{f}$ (sometimes 1,0)
  - Propositions: $\mathbf{p}$, $\mathbf{q}$, $\mathbf{r}$
- Logical Connectives
  - Unary (One Argument)
    - Negation (*not*, $\neg$, $\sim$)
  - Binary (Two Arguments)
    - Conjunction (*and*, $\wedge$)
    - Disjunction (*or*, $\vee$)
    - Implication (*implies*, $\longrightarrow$)
    - Exclusive-OR (*xor*, $\oplus$)
    - Equivalence ($\leftrightarrow$)
    - ...

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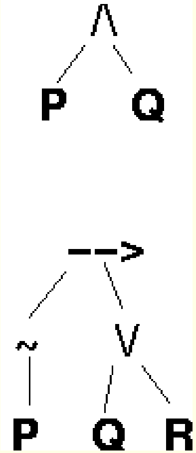
Sample Formulae

- Linear Notation
  - $p \wedge q$
  - $\neg(p) \longrightarrow (q \vee r)$
  - $(p \longrightarrow q) \leftrightarrow (\neg(p) \vee q)$
  - ...
- Tree Notation



Propositional Formulae

Well Formed Formula (wff)

Inductive Definition

- Base case (*atomic* formula) A propositional symbol is a wff..
- Inductive case If f_1 and f_2 are wffs, then so are
  - $\neg(f1)$
  - $(f1) \wedge (f2)$
  - $(f1) \vee (f2)$
  - $(f1) \rightarrow (f2)$
  - $\ldots$
- Closure condition Nothing else is a wff!

Alternative Definition (Tree based) A wff is a tree satisying the following:

- *Leaves* are labelled with propositional symbols.
- *Interior nodes* are labelled with logical connectives of the myred**correct arity**.

Correct arity means that the number of children is 1 for **unary**, 2 for **binary** etc. Tree notation avoids clumsy **paranthesis**.

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Semantics

- Semantics is the **meaning** of a sentence.
- In classical logic, every sentence must mean either **true** or **false**.
- How to assign a meaning to a formula?
  - Base Case

    An interpretation, or truth assignment, is a function from a set of propositional symbols to $\{t, f\}$.
  - Inductive Case

    Truth Tables for Connectives

p	q	$\neg p$	$p \wedge q$	$p \vee q$	$p \longrightarrow q$	$p \leftrightarrow q$
t	t	f	t	t	t	t
t	f	f	f	t	f	f
f	t	t	f	t	t	f
f	f	t	f	f	t	t

- p, q above actually any formulae f_1, f_2.
- Note: Truth table for $\longrightarrow$ (implication) is **non intuitive**.

  If pigs had wings, India will win world cup.

Back to Puzzle

Let us solve the following, using propositional logic.

> An island is inhabited by two classes of people: knights, who make only true statements, and knaves, who make only false statements. Three inhabitants are conversing. Ashok says, "All of us are knaves." Balu says, "Exactly one of us is a knight." What are Ashok, Balu, and Chandra?

More CS-related one (homework)

> Either the program never terminates or the value of n is eventually zero. If the value of n is eventually zero then the value of m will also eventually be zero. The program does terminate. Therefore the value of m will eventually zero. (T: the program terminates; N: the value of n is zero; M: the value of n is zero.)