CS344
Artificial Intelligence

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Inferencing in Fuzzy Logic

- Solving the Fuzzy Rule based system.
- Steps
  1. Read values
  2. Fuzzify
  3. Fire rules
  4. Collate truth values
  5. Defuzzify
  6. Obtain answer
- Lukasiewitz Rule (L Rule)

$$t(p \rightarrow q) = min(1, 1 - t(p) + t(q))$$

Modus Ponens in Fuzzy System

if $t(p) \geq a$ -- low water mark

and $t(p \to q) = c, (0 < a, c \leq 1)$

$t(q) = ?$

- Given: $t(p \to q) = min(1, 1 - t(p) + t(q)), t(p) \geq a$
  - Case 1:
    $c = 1$
    $1 - t(p) + t(q) \geq 1$
    i.e. $t(q) \geq t(p) \geq a$
  - Case 2:
    $c < 1$
    $1 - t(p) + t(q) = c$
    i.e. $t(q) = c + t(p) - 1 \geq c + a - 1$
  - From case 1 and 2, **$0 \leq t(q) \leq 1$**
- **$t(q) \geq max(0, c + a - 1)$** -- *General Expression*

Modus Tolens in Fuzzy System

$if\ (p \rightarrow q) = c, \quad (0 < c \leq 1)$

and $t(q) \leq b$ -- high water mark

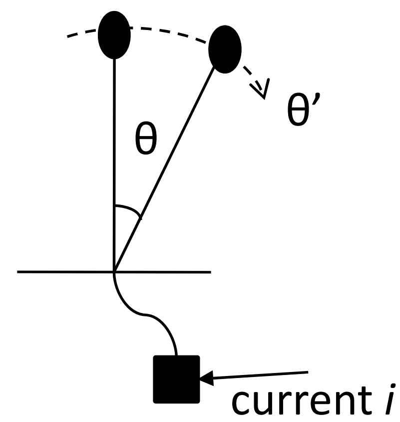
$t(q) = ?$

- Given: $t(p \rightarrow q) = min(1, 1 - t(p) + t(q)) = c,\ t(q) \leq b$
  - Case 1:
    $c = 1$
    $1 - t(p) + t(q) \geq 1$
    $i.e.\ t(p) \leq t(q) \leq b$
  - Case 2:
    $c < 1$
    $1 - t(p) + t(q) = c$
    $i.e.\ t(p) = 1 + t(q) - c \leq 1 + b - c$

  From case 1 and 2, **$0 \leq t(p) \leq 1$**
- **$t(p) \leq min(1, 1 + b - c)$**
- Note - Crisp Modus Tolens: Given $p \rightarrow q$ and $\sim q$ infer $\sim p$

Revisiting the Pendulum Problem

Let $\theta = 1°$, $\theta' = 0.05$ °/sec

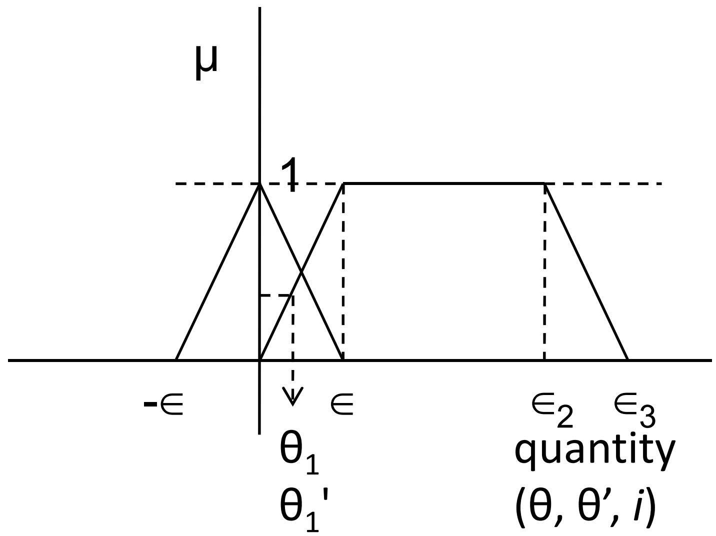


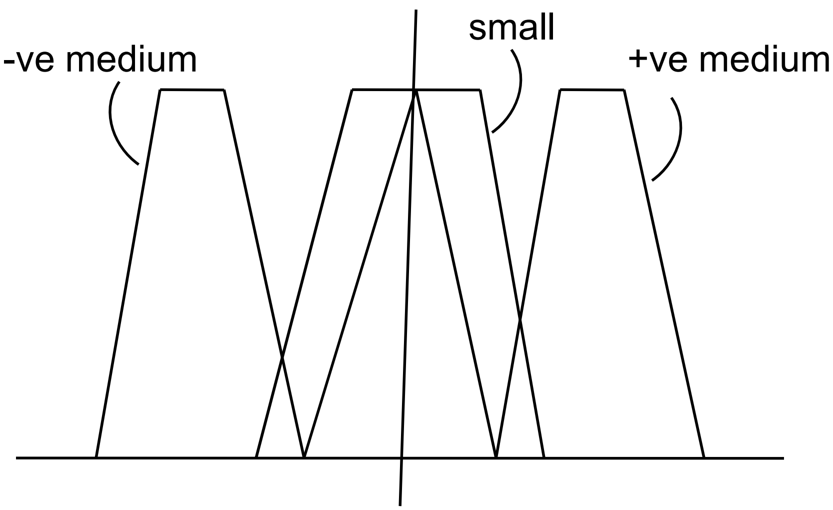
1. If θ is zero and θ’ is zero then *i* is <u>zero</u>.
2. If θ is zero and θ’ is positive small then *i* is <u>negative small</u>.
3. If θ is positive small and θ’ is zero then *i* is <u>negative small</u>.
4. If θ is zero and θ’ is negative small then *i* is <u>positive small</u>.
5. If θ is negative small and θ’ is zero then *i* is <u>positive small</u>.
6. If θ is positive small and θ’ is negative small then *i* is <u>zero</u>.
7. If θ is negative small and θ’ is positive small then *i* is <u>zero</u>.
8. If θ is positive small and θ’ is positive small then *i* is <u>negative medium</u>.
9. If θ is negative small and θ’ is negative small then *i* is <u>positive medium</u>.

θ→ **θ’↓**	Negative small	Zero	Positive small
Negative small		Positive small	
Zero	Positive small	Zero	Negative small
Positive small		Negative small	

Pendulum Problem

- Fuzzify (get μ values from profiles)
  - $\mu_{zero}(1°) = 0.7$
  - $\mu_{+ve\ small}(1°) = 0.3$
  - $\mu_{zero}(0.5°/sec) = 0.8$
  - $\mu_{+ve\ small}(0.5°/sec) = 0.4$
- <u>Rule 1</u>
  - LHS = $\mu_{zero}(1°)$ *and* $\mu_{zero}(0.5°/sec)$
    $= min(1°) = 0.7$
  - RHS = $\mu_{zero}(i) = 0.7$ (assuming $c = 1$ in L. Rule)
- <u>Rule 2</u>
  - RHS = $\mu_{-ve\ small}(i) = 0.3$
- <u>Rule 3</u>
  - RHS = $\mu_{-ve\ small}(i) = 0.4$
- <u>Rule 4</u>
  - RHS = $\mu_{-ve\ medium}(i) = 0.3$





Defuzzification

- To get crisp values
  1. Read values off the Y-axis
  2. Project onto X-axis
  3. Decide the values
- Step 1 and 2 above give rise to two values
- Two strategies to choose values for this step
  a) Max of min
  b) Centroid method
- Centroid method:
  – α-cut is the area below the curve cut at the μ value
  – Required current = X-axis value of centroids of areas A_1, A_2, …
  – Required (X_c, Y_c) is summation of values of centroid

$$X_c = \sum(x.A) / \sum A \quad \text{and} \quad Y_c = \sum(y.A) / \sum A$$