

# Priority Inheritance Protocols: An Approach to Real-Time Synchronization

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**Abstract**—A direct application of commonly used synchronization primitives such as semaphores, monitors, or the Ada rendezvous can lead to uncontrolled priority inversion, a situation in which a higher priority job is blocked by lower priority jobs for an indefinite period of time. In this paper, we investigate two protocols belonging to the class of *priority inheritance protocols*, called the *basic priority inheritance protocol* and the *priority ceiling protocol*. We show that both protocols solve this uncontrolled priority inversion problem. In particular, the priority ceiling protocol reduces the worst case task blocking time to at most the duration of execution of a single critical section of a lower priority task. In addition, this protocol prevents the formation of deadlocks. We also derive a set of sufficient conditions under which a set of periodic tasks using this protocol is schedulable.

**Index Terms**—Priority inheritance, priority inversion, real-time systems, scheduling, synchronization.

## I. INTRODUCTION

THE SCHEDULING of jobs with hard deadlines has been an important area of research in real-time computer systems. Both nonpreemptive and preemptive scheduling algorithms have been studied in the literature [3], [4], [6]–[8], [10], [11]. An important problem that arises in the context of such real-time systems is the effect of blocking caused by the need for the synchronization of jobs that share logical or physical resources. Mok [9] showed that the problem of deciding whether it is possible to schedule a set of periodic processes is NP-hard when periodic processes use semaphores to enforce mutual exclusion. One approach to the scheduling of real-time jobs when synchronization primitives are used is to try to dynamically construct a feasible schedule at run-time. Mok [9] developed a procedure to generate feasible schedules with a kernelized monitor, which does not permit the preemption of jobs in critical sections. It is an effective technique for the case where the critical sections are short. Zhao, Ramamritham, and Stankovic [14], [15] investigated the use

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of heuristic algorithms to generate feasible schedules. Their heuristic has a high probability of success in the generation of feasible schedules.

In this paper, we investigate the synchronization problem in the context of priority-driven preemptive scheduling, an approach used in many real-time systems. The importance of this approach is underscored by the fact that Ada, the language mandated by the U.S. Department of Defense for all its real-time systems, supports such a scheduling discipline. Unfortunately, a direct application of synchronization mechanisms like the Ada rendezvous, semaphores, or monitors can lead to uncontrolled priority inversion: a high priority job being blocked by a lower priority job for an indefinite period of time. Such priority inversion poses a serious problem in real-time systems by adversely affecting both the schedulability and predictability of real-time systems. In this paper, we formally investigate the priority inheritance protocol as a priority management scheme for synchronization primitives that remedies the uncontrolled priority inversion problem. We formally define the protocols in a uniprocessor environment and in terms of binary semaphores. In Section II, we review the problems of existing synchronization primitives, and define the basic concepts and notation. In Section III, we define the basic priority inheritance protocol and analyze its properties. In Section IV, we define an enhanced version of the basic priority inheritance protocol referred to as the priority ceiling protocol and investigate its properties. Section V analyzes the impact of this protocol on schedulability analysis when the rate-monotonic scheduling algorithm is used and Section VI examines the implication considerations as well as some possible enhancements to the priority ceiling protocol. Finally, Section VII presents the concluding remarks.

## II. THE PRIORITY INVERSION PROBLEM

Ideally, a high-priority job  $J$  should be able to preempt lower priority jobs immediately upon  $J$ 's initiation. Priority inversion is the phenomenon where a higher priority job is blocked by lower priority jobs. A common situation arises when two jobs attempt to access shared data. To maintain consistency, the access must be serialized. If the higher priority job gains access first then the proper priority order is maintained; however, if the lower priority job gains access first and then the higher priority job requests access to the shared data, this higher priority job is blocked until the lower priority job completes its access to the shared data. Thus, *blocking* is a form of priority inversion where a higher priority job must wait for the processing of a lower priority job. Prolonged du-

rations of blocking may lead to the missing of deadlines even at a low level of resource utilization. The level of resource utilization attainable before a deadline is missed is referred to as the *schedulability* of the system. To maintain a high degree of schedulability, we will develop protocols that would minimize the amount of blocking. It is also important to be able to analyze the performance of any proposed protocol in order to determine the schedulability of real-time tasks that use this protocol.

Common synchronization primitives include semaphores, locks, monitors, and Ada rendezvous. Although the use of these or equivalent methods is necessary to protect the consistency of shared data or to guarantee the proper use of non-preemptable resources, their use may jeopardize the ability of the system to meet its timing requirements. In fact, a direct application of these synchronization mechanisms can lead to an indefinite period of priority inversion and a low level of schedulability.

*Example 1:* Suppose that  $J_1$ ,  $J_2$ , and  $J_3$  are three jobs arranged in descending order of priority with  $J_1$  having the highest priority. We assume that jobs  $J_1$  and  $J_3$  share a data structure guarded by a binary semaphore  $S$ . Suppose that at time  $t_1$ , job  $J_3$  locks the semaphore  $S$  and executes its critical section. During the execution of job  $J_3$ 's critical section, the high priority job  $J_1$  is initiated, preempts  $J_3$ , and later attempts to use the shared data. However, job  $J_1$  will be blocked on the semaphore  $S$ . We would expect that  $J_1$ , being the highest priority job, will be blocked no longer than the time for job  $J_3$  to complete its critical section. However, the duration of blocking is, in fact, unpredictable. This is because job  $J_3$  can be preempted by the intermediate priority job  $J_2$ . The blocking of  $J_3$ , and hence that of  $J_1$ , will continue until  $J_2$  and any other pending intermediate jobs are completed.

The blocking period in Example 1 can be arbitrarily long. This situation can be partially remedied if a job in its critical section is not allowed to be preempted; however, this solution is only appropriate for very short critical sections, because it creates unnecessary blocking. For instance, once a low priority job enters a long critical section, a high priority job which does not access the shared data structure may be needlessly blocked. An identical problem exists in the use of monitors. The priority inversion problem was first discussed by Lampson and Redell [2] in the context of monitors. They suggest that the monitor be executed at a priority level higher than all tasks that would ever call the monitor. In the case of the Ada rendezvous, when a high priority job (task) is waiting in the entry queue of a server job, the server itself can be preempted by an independent job  $J$ , if job  $J$ 's priority is higher than both the priority of the server and the job which is currently in rendezvous with the server. Raising the server priority to be higher than all its callers would avoid this particular problem but would create a new problem: a low priority job may unnecessarily block the execution of independent higher priority jobs via the use of the server.

The use of *priority inheritance protocols* is one approach to rectify the priority inversion problem in existing synchronization primitives. Before we investigate these protocols, we first define the basic concepts and state our assumptions. A

*job* is a sequence of instructions that will continuously use the processor until its completion if it is executing alone on the processor. That is, we assume that jobs do not suspend themselves, say for I/O operations; however, such a situation can be accommodated by defining two or more jobs. In addition, we assume that the critical sections of a job are *properly* nested and a job will release all of its locks, if it holds any, before or at the end of its execution. In all our discussions below, we assume that jobs  $J_1, J_2, \dots, J_n$  are listed in descending order of priority with  $J_1$  having the highest priority. A *periodic task* is a sequence of the same type of job occurring at regular intervals, and an *aperiodic task* is a sequence of the same type of job occurring at irregular intervals. Each task is assigned a fixed priority, and every job of the same task is initially assigned that task's priority. If several jobs are eligible to run, the highest priority job will be run. Jobs with the same priority are executed in a FCFS discipline. When a job  $J$  is forced to wait for the execution of lower priority jobs, job  $J$  is said to be "blocked." When a job waits for the execution of high priority jobs or equal priority jobs that have arrived earlier, it is not considered as "blocked." We now state our notation.

Notation:

- $J_i$  denotes a job, i.e., an instance of a task  $\tau_i$ .  $P_i$  and  $T_i$  denote the priority and period of task  $\tau_i$ , respectively.
- A binary semaphore guarding shared data and/or resource is denoted by  $S_i$ .  $P(S_i)$  and  $V(S_i)$  denote the indivisible operations *lock* (wait) and *unlock* (signal), respectively, on the binary semaphore  $S_i$ .
- The  $j$ th critical section in job  $J_i$  is denoted by  $z_{i,j}$  and corresponds to the code segment of job  $J_i$  between the  $j$ th  $P$  operation and its corresponding  $V$  operation. The semaphore that is locked and released by critical section  $z_{i,j}$  is denoted by  $S_{i,j}$ .
- We write  $z_{i,j} \subset z_{i,k}$  if the critical section  $z_{i,j}$  is entirely contained in  $z_{i,k}$ .
- The duration of execution of the critical section  $z_{i,j}$ , denoted  $d_{i,j}$ , is the time to execute  $z_{i,j}$  when  $J_i$  executes on the processor alone.

We assume that critical sections are properly nested. That is, given any pair of critical sections  $z_{i,j}$  and  $z_{i,k}$ , then either  $z_{i,j} \subset z_{i,k}$ ,  $z_{i,k} \subset z_{i,j}$ , or  $z_{i,j} \cap z_{i,k} = \emptyset$ . In addition, we assume that a semaphore may be locked at most once in a single nested critical section.

*Definition:* A job  $J$  is said to be blocked by the critical section  $z_{i,j}$  of job  $J_i$  if  $J_i$  has a lower priority than  $J$  but  $J$  has to wait for  $J_i$  to exit  $z_{i,j}$  in order to continue execution.

*Definition:* A job  $J$  is said to be blocked by job  $J_i$  through semaphore  $S$ , if the critical section  $z_{i,j}$  blocks  $J$  and  $S_{i,j} = S$ .

In the next two sections, we will introduce the concept of priority inheritance and a priority inheritance protocol called the priority ceiling protocol. An important feature of this protocol is that one can develop a schedulability analysis for it in the sense that a schedulability bound can be determined. If the utilization of the task set stays below this bound, then the deadlines of all the tasks can be guaranteed. In order to create such a bound, it is necessary to determine the worst case du-

ration of priority inversion that any task can encounter. This worst case blocking duration will depend upon the particular protocol in use.

*Notation:*  $\beta_{i,j}$  denotes the set of all critical sections of the lower priority job  $J_j$  which can block  $J_i$ . That is,  $\beta_{i,j} = \{z_{j,k} | j > i \text{ and } z_{j,k} \text{ can block } J_i\}$ .<sup>1</sup>

Since we consider only properly nested critical sections, the set of blocking critical sections is partially ordered by set inclusion. Using this partial ordering, we can reduce our attention to the set of maximal elements of  $\beta_{i,j}$ ,  $\beta_{i,j}^*$ . Specifically, we have  $\beta_{i,j}^* = \{z_{j,k} | (z_{j,k} \in \beta_{i,j}) \wedge (\sim \exists z_{j,m} \in \beta_{i,j} \text{ such that } z_{j,k} \subset z_{j,m})\}$ .

The set  $\beta_{i,j}^*$  contains the longest critical sections of  $J_j$  which can block  $J_i$  and eliminates redundant inner critical sections. For purposes of schedulability analysis, we will restrict attention to  $\beta^* = \bigcup_{j>i} \beta_{i,j}^*$ , the set of all longest critical sections that can block  $J_i$ .

### III. THE BASIC PRIORITY INHERITANCE PROTOCOL

The basic idea of priority inheritance protocols is that when a job  $J$  blocks one or more higher priority jobs, it ignores its original priority assignment and executes its critical section at the highest priority level of all the jobs it blocks. After exiting its critical section, job  $J$  returns to its original priority level. To illustrate this idea, we apply this protocol to Example 1. Suppose that job  $J_1$  is blocked by job  $J_3$ . The priority inheritance protocol requires that job  $J_3$  execute its critical section at job  $J_1$ 's priority. As a result, job  $J_2$  will be unable to preempt job  $J_3$  and will itself be blocked. That is, the higher priority job  $J_2$  must wait for the critical section of the lower priority job  $J_3$  to be executed, because job  $J_3$  "inherits" the priority of job  $J_1$ . Otherwise,  $J_1$  will be indirectly preempted by  $J_2$ . When  $J_3$  exits its critical section, it regains its assigned lowest priority and awakens  $J_1$  which was blocked by  $J_3$ . Job  $J_1$ , having the highest priority, immediately preempts  $J_3$  and runs to completion. This enables  $J_2$  and  $J_3$  to resume in succession and run to completion.

#### A. The Definition of the Basic Protocol

We now define the basic priority inheritance protocol.

1) Job  $J$ , which has the highest priority among the jobs ready to run, is assigned the processor. Before job  $J$  enters a critical section, it must first obtain the lock on the semaphore  $S$  guarding the critical section. Job  $J$  will be blocked, and the lock on  $S$  will be denied, if semaphore  $S$  has been already locked. In this case, job  $J$  is said to be blocked by the job which holds the lock on  $S$ . Otherwise, job  $J$  will obtain the lock on semaphore  $S$  and enter its critical section. When job  $J$  exits its critical section, the binary semaphore associated with the critical section will be unlocked, and the highest priority job, if any, blocked by job  $J$  will be awakened.

2) A job  $J$  uses its assigned priority, unless it is in its critical section and blocks higher priority jobs. If job  $J$  blocks higher priority jobs,  $J$  inherits (uses)  $P_H$ , the highest priority

<sup>1</sup> Note that the second suffix of  $\beta_{i,j}$  and the first suffix of  $z_{j,k}$  correspond to job  $J_j$ .

of the jobs blocked by  $J$ . When  $J$  exits a critical section, it resumes the priority it had at the point of entry into the critical section.<sup>2</sup>

3) Priority inheritance is transitive. For instance, suppose  $J_1$ ,  $J_2$ , and  $J_3$  are three jobs in descending order of priority. Then, if job  $J_3$  blocks job  $J_2$ , and  $J_2$  blocks job  $J_1$ ,  $J_3$  would inherit the priority of  $J_1$  via  $J_2$ . Finally, the operations of priority inheritance and of the resumption of original priority must be indivisible.<sup>3</sup>

4) A job  $J$  can preempt another job  $J_L$  if job  $J$  is not blocked and its priority is higher than the priority, inherited or assigned, at which job  $J_L$  is executing.

It is helpful to summarize that under the basic priority inheritance protocol, a high priority job can be blocked by a low-priority job in one of two situations. First, there is the *direct* blocking, a situation in which a higher priority job attempts to lock a locked semaphore. Direct blocking is necessary to ensure the consistency of shared data. Second, a medium priority job  $J_1$  can be blocked by a low priority job  $J_2$ , which inherits the priority of a high priority job  $J_0$ . We refer to this form of blocking as *push-through* blocking, which is necessary to avoid having a high-priority job  $J_0$  being indirectly preempted by the execution of a medium priority job  $J_1$ .

#### B. The Properties of the Basic Protocol

We now proceed to analyze the properties of the basic priority inheritance protocol defined above. In this section, we assume that deadlock is prevented by some external means, e.g., semaphores are accessed in an order that is consistent with a predefined acyclical order. Throughout this section,  $\beta_i^*$  refers to the sets of the longest critical sections that can block  $J_i$  when the basic priority inheritance protocol is used.

*Lemma 1:* A job  $J_H$  can be blocked by a lower priority job  $J_L$ , only if  $J_L$  is executing within a critical section  $z_{L,j} \in \beta_{H,L}^*$ , when  $J_H$  is initiated.

*Proof:* By the definitions of the basic priority inheritance protocol and the blocking set  $\beta_{H,L}^*$ , task  $J_L$  can block  $J_H$  only if it directly blocks  $J_H$  or has its priority raised above  $J_H$  through priority inheritance. In either case, the critical section  $z_{L,j}$  currently being executed by  $J_L$  is in  $\beta_{H,L}^*$ . If  $J_L$  is not within a critical section which cannot directly block  $J_H$  and cannot lead to the inheritance of a priority higher than  $J_H$ , then  $J_L$  can be preempted by  $J_H$  and can never block  $J_H$ .

*Lemma 2:* Under the basic priority inheritance protocol, a high priority job  $J_H$  can be blocked by a lower priority job  $J_L$  for at most the duration of one critical section of  $\beta_{H,L}^*$  regardless of the number of semaphores  $J$  and  $J_L$  share.

*Proof:* By Lemma 1, for  $J_L$  to block  $J_H$ ,  $J_L$  must be currently executing a critical section  $z_{L,j} \in \beta_{H,L}^*$ . Once  $J_L$  exits  $z_{L,j}$ , it can be preempted by  $J_H$  and  $J_H$  cannot be blocked by  $J_L$  again.

<sup>2</sup> For example, when  $J$  executes  $V(S_2)$  in  $\{P(S_1), \dots, P(S_2), \dots, V(S_2), \dots, V(S_1)\}$ , it reverts to the priority it had before it executed  $P(S_2)$ . This may be lower than its current priority and cause  $J$  to be preempted by a higher priority task.  $J$  would, of course, still hold the lock on  $S_1$ .

<sup>3</sup> The operations must be indivisible in order to maintain internal consistency of data structures being manipulated in the run-time system.

**Theorem 3:** Under the basic priority inheritance protocol, given a job  $J_0$  for which there are  $n$  lower priority jobs  $\{J_1, \dots, J_n\}$ , job  $J_0$  can be blocked for at most the duration of one critical section in each of  $\beta_{0,i}^*$ ,  $1 \leq i \leq n$ .

*Proof:* By Lemma 2, each of the  $n$  lower priority jobs can block job  $J_0$  for at most the duration of a single critical section in each of the blocking sets  $\beta_{0,i}^*$ .

We now determine the bound on the blockings as a function of the semaphores shared by jobs.

**Lemma 4:** A semaphore  $S$  can cause push-through blocking to job  $J$ , only if  $S$  is accessed both by a job which has priority lower than that of  $J$  and by a job which has or can inherit priority equal to or higher than that of  $J$ .

*Proof:* Suppose that  $J_L$  accesses semaphore  $S$  and has priority lower than that of  $J$ . According to the priority inheritance protocol, if  $S$  is not accessed by a job which has or can inherit priority equal to or higher than that of  $J$ , then job  $J_L$ 's critical section guarded by  $S$  cannot inherit a priority equal to or higher than that of  $J$ . In this case, job  $J_L$  will be preempted by job  $J$  and the lemma follows.

We next define  $\zeta_{i,j,k}^*$  to be the set of all longest critical sections of job  $J_j$  guarded by semaphore  $S_k$  and which can block job  $J_i$  either directly or via push-through blocking. That is,  $\zeta_{i,j,k}^* = \{z_{j,p} | z_{j,p} \in \beta_{i,j}^* \text{ and } s_{j,p} = S_k\}$ .

Let  $\zeta_{i,k}^* = \bigcup_{j>i} \zeta_{i,j,k}^*$  represent the set of all longest critical sections corresponding to semaphore  $S_k$  which can block  $J_i$ .

**Lemma 5:** Under the basic priority inheritance protocol, a job  $J_i$  can encounter blocking by at most one critical section in  $\zeta_{i,k}^*$  for each semaphore  $S_k$ ,  $1 \leq k \leq m$ , where  $m$  is the number of distinct semaphores.

*Proof:* By Lemma 1, job  $J_L$  can block a higher priority job  $J_H$  if  $J_L$  is currently executing a critical section in  $\beta_{H,L}^*$ . Any such critical section corresponds to the locking and unlocking of a semaphore  $S_k$ . Since we deal only with binary semaphores, only one of the lower priority jobs can be within a blocking critical section corresponding to a particular semaphore  $S_k$ . Once this critical section is exited, the lower priority job  $J_L$  can no longer block  $J_H$ . Consequently, only one critical section in  $\beta_i^*$  corresponding to semaphore  $S_k$  can block  $J_H$ . The lemma follows.

**Theorem 6:** Under the basic priority inheritance protocol, if there are  $m$  semaphores which can block job  $J$ , then  $J$  can be blocked by at most  $m$  times.

*Proof:* It follows from Lemma 5 that job  $J$  can be blocked at most once by each of the  $m$  semaphores.

Theorems 3 and 6 place an upper bound on the total blocking delay that a job can encounter. Given these results, it is possible to determine at compile-time the worst case blocking duration of a job. For instance, if there are four semaphores which can potentially block job  $J$  and there are three other lower priority tasks,  $J$  may be blocked for a maximum duration of three longest subcritical sections. Moreover, one can find the worst case blocking durations for a job by studying the durations of the critical sections in  $\beta_{i,j}^*$  and  $\zeta_{i,k}^*$ .

Still, the basic priority inheritance protocol has the following two problems. First, this basic protocol, by itself, does not prevent deadlocks. For example, suppose that at time  $t_1$ , job

$J_2$  locks semaphore  $S_2$  and enters its critical section. At time  $t_2$ , job  $J_2$  attempts to make a nested access to lock semaphore  $S_1$ . However, job  $J_1$ , a higher priority job, is ready at this time. Job  $J_1$  preempts job  $J_2$  and locks semaphore  $S_1$ . Next, if job  $J_1$  tries to lock semaphore  $S_2$ , a deadlock is formed.

The deadlock problem can be solved, say, by imposing a total ordering on the semaphore accesses. Still, a second problem exists. The blocking duration for a job, though bounded, can still be substantial, because a chain of blocking can be formed. For instance, suppose that  $J_1$  needs to sequentially access  $S_1$  and  $S_2$ . Also suppose that  $J_2$  preempts  $J_3$  within the critical section  $z_{3,1}$  and enters the critical section  $z_{2,2}$ . Job  $J_1$  is initiated at this instant and finds that the semaphores  $S_1$  and  $S_2$  have been respectively locked by the lower priority jobs  $J_3$  and  $J_2$ . As a result,  $J_1$  would be blocked for the duration of two critical sections, once to wait for  $J_3$  to release  $S_1$  and again to wait for  $J_2$  to release  $S_2$ . Thus, a blocking chain is formed.

We present in the next section the priority ceiling protocol that addresses effectively both these problems posed by the basic priority inheritance protocol.

#### IV. THE PRIORITY CEILING PROTOCOL

##### A. Overview

The goal of this protocol is to prevent the formation of deadlocks and of chained blocking. The underlying idea of this protocol is to ensure that when a job  $J$  preempts the critical section of another job and executes its own critical section  $z$ , the priority at which this new critical section  $z$  will execute is guaranteed to be higher than the inherited priorities of all the preempted critical sections. If this condition cannot be satisfied, job  $J$  is denied entry into the critical section  $z$  and suspended, and the job that blocks  $J$  inherits  $J$ 's priority. This idea is realized by first assigning a priority ceiling to each semaphore, which is equal to the highest priority task that may use this semaphore. We then allow a job  $J$  to start a new critical section only if  $J$ 's priority is higher than all priority ceilings of all the semaphores locked by jobs other than  $J$ . Example 2 illustrates this idea and the deadlock avoidance property while Example 3 illustrates the avoidance of chained blocking.

**Example 2:** Suppose that we have three jobs  $J_0$ ,  $J_1$ , and  $J_2$  in the system. In addition, there are two shared data structures protected by the binary semaphores  $S_1$  and  $S_2$ , respectively. We define the *priority ceiling* of a semaphore as the priority of the highest priority job that may lock this semaphore. Suppose the sequence of processing steps for each job is as follows.

$$J_0 = \{\dots, P(S_0), \dots, V(S_0), \dots\}$$

$$J_1 = \{\dots, P(S_1), \dots, P(S_2), \dots, V(S_2), \dots, V(S_1), \dots\}$$

$$J_2 = \{\dots, P(S_2), \dots, P(S_1), \dots, V(S_1), \dots, V(S_2), \dots\}.$$

Recall that the priority of job  $J_1$  is assumed to be higher than that of job  $J_2$ . Thus, the priority ceilings of both semaphores  $S_1$  and  $S_2$  are equal to the priority of job  $J_1$ .

The sequence of events described below is depicted in Fig. 1. A line at a low level indicates that the corresponding job

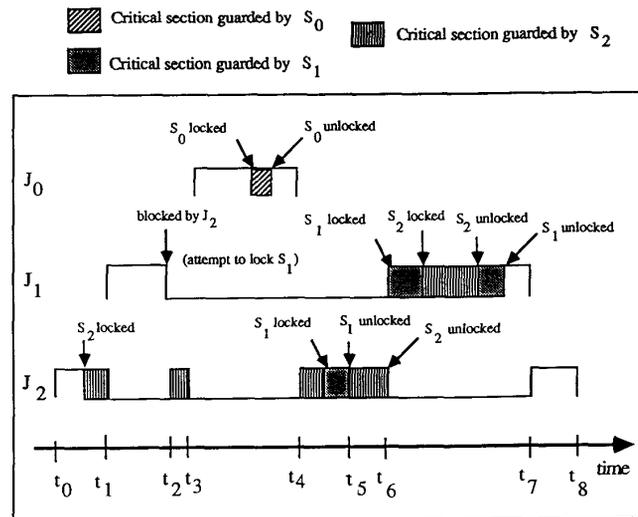


Fig. 1. Sequence of events described in Example 2.

is blocked or has been preempted by a higher priority job. A line raised to a higher level indicates that the job is executing. The absence of a line indicates that the job has not yet been initiated or has completed. Shaded portions indicate execution of critical sections. Suppose that

- At time  $t_0$ ,  $J_2$  is initiated and it begins execution and then locks semaphore  $S_2$ .
- At time  $t_1$ , job  $J_1$  is initiated and preempts job  $J_2$ .
- At time  $t_2$ , job  $J_1$  tries to enter its critical section by making an indivisible system call to execute  $P(S_1)$ . However, the run-time system will find that job  $J_1$ 's priority is *not* higher than the priority ceiling of *locked* semaphore  $S_2$ . Hence, the run-time system suspends job  $J_1$  without locking  $S_1$ . Job  $J_2$  now *inherits* the priority of job  $J_1$  and resumes execution. Note that  $J_1$  is blocked outside its critical section. As  $J_1$  is not given the lock on  $S_1$  but suspended instead, the potential deadlock involving  $J_1$  and  $J_2$  is prevented.
- At time  $t_3$ ,  $J_2$  is still in its critical section and the highest priority job  $J_0$  is initiated and preempts  $J_2$ . Later,  $J_0$  attempts to lock semaphore  $S_0$ . Since the priority of  $J_0$  is higher than the priority ceiling of *locked* semaphore  $S_2$ , job  $J_0$  will be granted the lock on the semaphore  $S_0$ . Job  $J_0$  will therefore continue and execute its critical section, thereby effectively preempting  $J_2$  in its critical section and not encountering any blocking.
- At time  $t_4$ ,  $J_0$  has exited its critical section and completes execution. Job  $J_2$  resumes, since  $J_1$  is blocked by  $J_2$  and cannot execute.  $J_2$  continues execution and locks  $S_1$ .
- At time  $t_5$ ,  $J_2$  releases  $S_1$ .
- At time  $t_6$ ,  $J_2$  releases  $S_2$  and resumes its assigned priority. Now,  $J_1$  is signaled and having a higher priority, it preempts  $J_2$ , resumes execution, and locks  $S_2$ . Then,  $J_1$  locks  $S_1$ , executes the nested critical section, and unlocks  $S_1$ . Later it unlocks  $S_2$  and executes its noncritical section code.
- At  $t_7$ ,  $J_1$  completes execution and  $J_2$  resumes.

- At  $t_8$ ,  $J_2$  completes.

Note that in the above example,  $J_0$  is never blocked because its priority is higher than the priority ceilings of semaphores  $S_1$  and  $S_2$ .  $J_1$  was blocked by the lower priority job  $J_2$  during the intervals  $[t_2, t_3]$  and  $[t_4, t_6]$ . However, these intervals correspond to part of the duration that  $J_2$  needs to lock  $S_2$ . Thus,  $J_1$  is blocked for no more than the duration of one critical section of a lower priority job  $J_2$  even though the actual blocking occurs over disjoint time intervals. It is, indeed, a property of this protocol that any job can be blocked for at most the duration of a single critical section of a lower priority job. This property is further illustrated by the following example.

*Example 3:* Consider the example from the previous section where a chain of blockings can be formed. We assumed that job  $J_1$  needs to access  $S_1$  and  $S_2$  sequentially while  $J_2$  accesses  $S_2$  and  $J_3$  accesses  $S_1$ . Hence, the priority ceilings of semaphores  $S_1$  and  $S_2$  are equal to  $P_1$ . As before, let job  $J_3$  lock  $S_1$  at time  $t_0$ . At time  $t_1$ , job  $J_2$  is initiated and preempts  $J_3$ . However, at time  $t_2$ , when  $J_2$  attempts to lock  $S_2$ , the run-time system finds that the priority of  $J_2$  is *not* higher than the priority ceiling  $P_1$  of the *locked* semaphore  $S_1$ . Hence,  $J_2$  is denied the lock on  $S_2$  and blocked. Job  $J_3$  resumes execution at  $J_2$ 's priority. At time  $t_3$ , when  $J_3$  is still in its critical section,  $J_1$  is initiated and finds that only one semaphore  $S_1$  is locked. At time  $t_4$ ,  $J_1$  is blocked by  $J_3$  which holds the lock on  $S_1$ . Hence,  $J_3$  inherits the priority of  $J_1$ . At time  $t_5$ , job  $J_3$  exits its critical section  $z_{3,1}$ , resumes its original priority, and awakens  $J_1$ . Job  $J_3$ , having the highest priority, preempts  $J_3$  and runs to completion. Next,  $J_2$  which is no longer blocked completes its execution and is followed by  $J_3$ .

Again, note that  $J_1$  is blocked by  $J_3$  in the interval  $[t_4, t_5]$  which corresponds to the single critical section  $z_{3,1}$ . Also, job  $J_2$  is blocked by  $J_3$  in the disjoint intervals  $[t_2, t_3]$  and  $[t_4, t_5]$  which also correspond to the same critical section  $z_{3,1}$ .

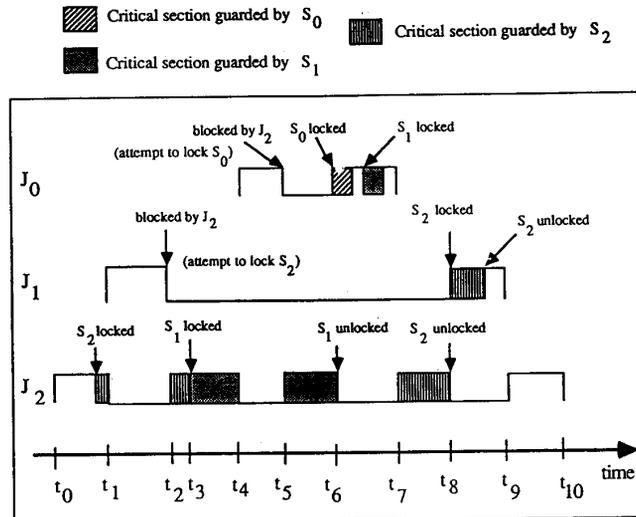


Fig. 2. Sequence of events described in Example 4.

### B. Definition

Having illustrated the basic idea of the priority ceiling protocol and its properties, we now present its definition.

1) Job  $J$ , which has the highest priority among the jobs ready to run, is assigned the processor, and let  $S^*$  be the semaphore with the highest priority ceiling of all semaphores currently locked by jobs other than job  $J$ . Before job  $J$  enters its critical section, it must first obtain the lock on the semaphore  $S$  guarding the shared data structure. Job  $J$  will be blocked and the lock on  $S$  will be denied, if the priority of job  $J$  is not higher than the priority ceiling of semaphore  $S^*$ .<sup>4</sup> In this case, job  $J$  is said to be blocked on semaphore  $S^*$  and to be blocked by the job which holds the lock on  $S^*$ . Otherwise, job  $J$  will obtain the lock on semaphore  $S$  and enter its critical section. When a job  $J$  exits its critical section, the binary semaphore associated with the critical section will be unlocked and the highest priority job, if any, blocked by job  $J$  will be awakened.

2) A job  $J$  uses its assigned priority, unless it is in its critical section and blocks higher priority jobs. If job  $J$  blocks higher priority jobs,  $J$  inherits  $P_H$ , the highest priority of the jobs blocked by  $J$ . When  $J$  exits a critical section, it resumes the priority it had at the point of entry into the critical section.<sup>5</sup> Priority inheritance is transitive. Finally, the operations of priority inheritance and of the resumption of previous priority must be indivisible.

3) A job  $J$ , when it does not attempt to enter a critical section, can preempt another job  $J_L$  if its priority is higher than the priority, inherited or assigned, at which job  $J_L$  is executing.

We shall illustrate the priority ceiling protocol using an example.

<sup>4</sup> Note that if  $S$  has been already locked, the priority ceiling of  $S$  will be at least equal to the priority of  $J$ . Because job  $J$ 's priority is not higher than the priority ceiling of the semaphore  $S$  locked by another job,  $J$  will be blocked. Hence, this rule implies that if a job  $J$  attempts to lock a semaphore that has been already locked,  $J$  will be denied the lock and blocked instead.

<sup>5</sup> That is, when  $J$  exits the part of a critical section, it resumes its previous priority.

*Example 4:* We assume that the priority of job  $J_i$  is higher than that of job  $J_{i+1}$ . The processing steps in each job are as follows:

Job  $J_0$  accesses  $z_{0,0}$  and  $z_{0,1}$  by executing the steps

$$\{\dots, P(S_0), \dots, V(S_0), \dots, P(S_1), \dots, V(S_1), \dots\},$$

job  $J_1$  accesses only  $z_{1,2}$  by executing

$$\{\dots, P(S_2), \dots, V(S_2), \dots\},$$

and job  $J_2$  accesses  $z_{2,2}$  and makes a nested semaphore access to  $S_1$  by executing

$$\{\dots, P(S_2), \dots, P(S_1), \dots, V(S_1), \dots, V(S_2), \dots\}.$$

Note that the priority ceilings of semaphores  $S_0$  and  $S_1$  are equal to  $P_0$ , and the priority ceiling of semaphore  $S_2$  is  $P_1$ .

Fig. 2 depicts the sequence of events described below.

Suppose that

- At time  $t_0$ , job  $J_2$  begins execution and later locks  $S_2$ .
- At time  $t_1$ , job  $J_1$  is initiated, preempts  $J_2$ , and begins execution.
- At time  $t_2$ , while attempting to access  $S_2$  already locked by  $J_2$ , job  $J_1$  becomes blocked. Job  $J_2$  now resumes the execution of its critical section  $z_{2,2}$  at its inherited priority of  $J_1$ , namely  $P_1$ .
- At time  $t_3$ , job  $J_2$  successfully enters its nested critical section  $z_{2,1}$  by locking  $S_1$ . Job  $J_2$  is allowed to lock  $S_1$ , because there is no semaphore  $S^*$  which is locked by other jobs.
- At time  $t_4$ , job  $J_2$  is still executing  $z_{2,1}$  but the highest priority job  $J_0$  is initiated. Job  $J_0$  preempts  $J_2$  within  $z_{2,1}$  and executes its own noncritical section code. This is possible because  $P_0$ , the priority of  $J_0$ , is higher than  $P_1$ , the inherited priority level at which job  $J_2$ 's  $z_{2,1}$  was being executed.
- At time  $t_5$ , job  $J_0$  attempts to enter its critical section  $z_{0,0}$

by locking  $S_0$ , which is not locked by any job. However, since the priority of job  $J_0$  is not higher than the priority ceiling  $P_0$  of the locked semaphore  $S_1$ , job  $J_0$  is blocked by job  $J_2$  which holds the lock on  $S_1$ . This is a new form of blocking introduced by the priority ceiling protocol in addition to the direct and push-through blocking encountered in the basic protocol. At this point, job  $J_2$  resumes its execution of  $z_{2,1}$  at the newly inherited priority level of  $P_0$ .

- At time  $t_6$ , job  $J_2$  exits its critical section  $z_{2,1}$ . Semaphore  $S_1$  is now unlocked, job  $J_2$  returns to the previously inherited priority of  $P_1$ , and job  $J_0$  is awakened. At this point,  $J_0$  preempts job  $J_2$ , because its priority  $P_0$  is higher than the priority ceiling  $P_1$  of  $S_2$ . Job  $J_0$  will be granted the lock on  $S_0$  and will execute its critical section  $z_{0,0}$ . Later, it unlocks  $S_0$  and then locks and unlocks  $S_1$ .
- At time  $t_7$ , job  $J_0$  completes its execution, and job  $J_2$  resumes its execution of  $z_{2,2}$  at its inherited priority  $P_1$ .
- At time  $t_8$ , job  $J_2$  exits  $z_{2,2}$ , semaphore  $S_2$  is unlocked, job  $J_2$  returns to its own priority  $P_2$ , and job  $J_1$  is awakened. At this point, job  $J_1$  preempts job  $J_2$  and  $J_1$  is granted the lock on  $S_2$ . Later,  $J_1$  unlocks  $S_2$  and executes its noncritical section code.
- At time  $t_9$ , job  $J_1$  completes its execution and finally job  $J_2$  resumes its execution, until it also completes at time  $t_{10}$ .

The priority ceiling protocol introduces a third type of blocking in addition to direct blocking and push-through blocking caused by the basic priority inheritance protocol. An instance of this new type of blocking occurs at time  $t_5$  in the above example. We shall refer to this form of blocking as *ceiling* blocking. Ceiling blocking is needed for the avoidance of deadlock and of chained blocking. This avoidance approach belongs to the class of pessimistic protocols which sometimes create unnecessary blocking. Although the priority ceiling protocol introduces a new form of blocking, the worst case blocking is dramatically improved. Under the basic priority inheritance protocol, a job  $J$  can be blocked for at most the duration of  $\min(n, m)$  critical sections, where  $n$  is the number of lower priority jobs that could block  $J$  and  $m$  is the number of semaphores that can be used to block  $J$ . On the contrary, under the priority ceiling protocol a job  $J$  can be blocked for at most the duration of one longest subcritical section.

### C. The Properties of the Priority Ceiling Protocol

Before we prove the properties of this protocol, it is important to recall the two basic assumptions about jobs. First, a job is assumed to be a sequence of instructions that will continuously execute until its completion, when it executes alone on a processor. Second, a job will release all of its locks, if it ever holds any, before or at the end of its execution. The relaxation of our first assumption is addressed at the end of this section. Throughout this section, the sets  $\beta_{i,j}$ ,  $\beta_{i,j}^*$ , and  $\beta_i^*$  refer to the blocking sets associated with the priority ceiling protocol.

*Lemma 7:* A job  $J$  can be blocked by a lower priority job

$J_L$ , only if the priority of job  $J$  is no higher than the highest priority ceiling of all the semaphores that are locked by all lower priority jobs when  $J$  is initiated.

*Proof:* Suppose that when  $J$  is initiated, the priority of job  $J$  is higher than the highest priority ceiling of all the semaphores that are currently locked by all lower priority jobs. By the definition of the priority ceiling protocol, job  $J$  can always preempt the execution of job  $J_L$ , and no higher priority job will ever attempt to lock those locked semaphores.

*Lemma 8:* Suppose that the critical section  $z_{j,n}$  of job  $J_j$  is preempted by job  $J_i$  which enters its critical section  $z_{i,m}$ . Under the priority ceiling protocol, job  $J_j$  cannot inherit a priority level which is higher than or equal to that of job  $J_i$  until job  $J_i$  completes.

*Proof:* Suppose that job  $J_j$  inherits a priority that is higher than or equal to that of job  $J_i$  before  $J_i$  completes. Hence, there must exist a job  $J$  which is blocked by  $J_j$ . In addition,  $J$ 's priority must be higher than or equal to that of job  $J_i$ . We now show the contradiction that  $J$  cannot be blocked by  $J_j$ . Since job  $J_i$  preempts the critical section  $z_{j,n}$  of job  $J_j$  and enters its own critical section  $z_{i,m}$ , job  $J_i$ 's priority must be higher than the priority ceilings of all the semaphores currently locked by all lower priority jobs. Since  $J$ 's priority is assumed to be higher than or equal to that of  $J_i$ , it follows that job  $J$ 's priority is also higher than the priority ceilings of all the semaphores currently locked by all lower priority jobs. By Lemma 7,  $J$  cannot be blocked by  $J_j$ . Hence, the contradiction and the lemma follows.

*Definition:* Transitive blocking is said to occur if a job  $J$  is blocked by  $J_i$  which, in turn, is blocked by another job  $J_j$ .

*Lemma 9:* The priority ceiling protocol prevents transitive blocking.

*Proof:* Suppose that transitive blocking is possible. Let  $J_3$  block job  $J_2$  and let job  $J_2$  block job  $J_1$ . By the transitivity of the protocol, job  $J_3$  will inherit the priority of  $J_1$  which is assumed to be higher than that of job  $J_2$ . This contradicts Lemma 8, which shows that  $J_3$  cannot inherit a priority that is higher than or equal to that of job  $J_2$ . The lemma follows.

*Theorem 10:* The priority ceiling protocol prevents deadlocks.

*Proof:* First, by assumption, a job cannot deadlock with itself. Thus, a deadlock can only be formed by a cycle of jobs waiting for each other. Let the  $n$  jobs involved in the blocking cycle be  $\{J_1, \dots, J_n\}$ . Note that each of these  $n$  jobs must be in one of its critical sections, since a job that does not hold a lock on any semaphore cannot contribute to the deadlock. By Lemma 9, the number of jobs in the blocking cycle can only be two, i.e.,  $n = 2$ . Suppose that job  $J_2$ 's critical section was preempted by job  $J_1$ , which then enters its own critical section. By Lemma 8, job  $J_2$  can never inherit a priority which is higher than or equal to that of job  $J_1$  before job  $J_1$  completes. However, if a blocking cycle (deadlock) is formed, then by the transitivity of priority inheritance, job  $J_2$  will inherit the priority of job  $J_1$ . This contradicts Lemma 8 and hence the theorem follows.

*Remark:* Lemma 1 is true under the priority ceiling protocol.

*Remark:* Suppose that the run-time system supports the

priority ceiling protocol. Theorem 10 leads to the useful result that programmers can write arbitrary sequences of properly nested semaphore accesses. As long as each job does not deadlock with itself, there will be no deadlock in the system.

**Lemma 11:** Let  $J_L$  be a job with a lower priority than that of job  $J_i$ . Job  $J_i$  can be blocked by job  $J_L$  for at most the duration of one critical section in  $\beta_{i,L}^*$ .

*Proof:* First, job  $J_i$  will preempt  $J_L$  if  $J_L$  is not in a critical section  $z_{L,m} \in \beta_{i,L}^*$ . Suppose that job  $J_i$  is blocked by  $z_{L,m}$ . By Theorem 10, there is no deadlock and hence job  $J_L$  will exit  $z_{L,m}$  at some instant  $t_1$ . Once job  $J_L$  leaves this critical section at time  $t_1$ , job  $J_L$  can no longer block job  $J_i$ . This is because job  $J_i$  has been initiated and  $J_L$  is not within a critical section in  $\beta_{i,L}^*$ . It follows from Lemma 1 that job  $J_L$  can no longer block job  $J_i$ .

**Theorem 12:** A job  $J$  can be blocked for at most the duration of at most one element of  $\beta_i^*$ .

*Proof:* Suppose that job  $J$  can be blocked by  $n > 1$  elements of  $\beta_i$ . By Lemma 11, the only possibility is that job  $J$  is blocked by  $n$  different lower priority jobs. Suppose that the first two lower priority jobs that block job  $J$  are  $J_1$  and  $J_2$ . By Lemma 1, in order for both these jobs to block job  $J$ , both of them must be in a longest blocking critical section when job  $J$  is initiated. Let the lowest priority job  $J_2$  enter its blocking critical section first, and let the highest priority ceiling of all the semaphores locked by  $J_2$  be  $\rho_2$ . Under the priority ceiling protocol, in order for job  $J_1$  to enter its critical section when  $J_2$  is already inside one, the priority of job  $J_1$  must be higher than priority ceiling  $\rho_2$ . Since we assume that job  $J$  can be blocked by job  $J_2$ , by Lemma 7 the priority of job  $J$  cannot be higher than priority ceiling  $\rho_2$ . Since the priority of job  $J_1$  is higher than  $\rho_2$  and the priority of job  $J$  is no higher than  $\rho_2$ , job  $J_1$ 's priority must be higher than the priority of job  $J$ . This contradicts the assumption that the priority of job  $J$  is higher than that of both  $J_1$  and  $J_2$ . Thus, it is impossible for job  $J$  to have priority higher than both jobs  $J_1$  and  $J_2$  and to be blocked by both of them under the priority ceiling protocol. The theorem follows immediately.

*Remark:* We may want to generalize the definition of a job by allowing it to suspend during its execution, for instance, to wait for I/O services to complete. The following corollary presents the upper bound on the blocking duration of a generalized job that might suspend and later resume during its execution.

**Corollary 13:** If a generalized job  $J$  suspends itself  $n$  times during its execution, it can be blocked by at most  $n + 1$  not necessarily distinct elements of  $\beta_i^*$ .

## V. SCHEDULABILITY ANALYSIS

Having proved the properties of the priority ceiling protocol, we now proceed to investigate the effect of blocking on the schedulability of a task set. In this section, we develop a set of *sufficient* conditions under which a set of periodic tasks using the priority ceiling protocol can be scheduled by the rate-monotonic algorithm, which assigns higher priorities to tasks with shorter periods and is an optimal static priority algorithm when tasks are independent [8]. To this end, we will use a simplified scheduling model. First, we assume that

all the tasks are periodic. Second, we assume that each job in a periodic task has deterministic execution times for both its critical and noncritical sections and that it does not synchronize with external events, i.e., a job will execute to its completion when it is the only job in the system. Finally, we assume that these periodic tasks are assigned priorities according to the rate-monotonic algorithm. Readers who are interested in more general scheduling issues, such as the reduction of aperiodic response times and the effect of task stochastic execution times, are referred to [4] and [12].

We quote the following theorem also due to Liu and Layland which was proved under the assumption of independent tasks, i.e., when there is no blocking due to data sharing and synchronization.

**Theorem 14:** A set of  $n$  periodic tasks scheduled by the rate-monotonic algorithm can always meet their deadlines if

$$\frac{C_1}{T_1} + \dots + \frac{C_n}{T_n} \leq n(2^{1/n} - 1)$$

where  $C_i$  and  $T_i$  are the execution time and period of task  $\tau_i$ , respectively.

Theorem 14 offers a sufficient (worst case) condition that characterizes the rate-monotonic schedulability of a given periodic task set. The following exact characterization was proved by Lehoczky, Sha, and Ding [5]. An example of the use of this theorem will be given later in this section.

**Theorem 15:** A set of  $n$  periodic tasks scheduled by the rate-monotonic algorithm will meet all their deadlines for all task phasings if and only if

$$\forall i, 1 \leq i \leq n, \quad \min_{(k,l) \in R_i} \sum_{j=1}^i C_j \frac{1}{T_k} \left\lceil \frac{lT_k}{T_j} \right\rceil \\ = \min_{(k,l) \in R_i} \sum_{j=1}^i U_j \frac{T_j}{T_k} \left\lceil \frac{lT_k}{T_j} \right\rceil \leq 1$$

where  $C_j$ ,  $T_j$ , and  $U_j$  are the execution time, period, and utilization of task  $\tau_j$ , respectively, and  $R_i = \{(k,l) | 1 \leq k \leq i, l = 1, \dots, \lfloor T_i/T_k \rfloor\}$ .

When tasks are independent of one another, Theorems 14 and 15 provide us with the conditions under which a set of  $n$  periodic tasks can be scheduled by the rate-monotonic algorithm.<sup>6</sup> Although these two theorems have taken into account the effect of a task being preempted by higher priority tasks, they have not considered the effect of a job being blocked by lower priority jobs. We now consider the effect of blocking. Each element in  $\beta_i$  is a critical section accessed by a lower priority job and guarded by a semaphore whose priority ceiling is higher than or equal to the priority of job  $J_i$ . Hence,  $\beta_i^*$  can be derived from  $\beta_i$ . By Lemma 7 and Theorem 12, job  $J_i$  of a task  $\tau$  can be blocked for at most the duration of a single element in  $\beta_i^*$ . Hence, the worst case blocking time for  $J$  is at most the duration of the longest element of  $\beta_i^*$ . We denote this worst case blocking time of a job in task  $\tau_i$  by  $B_i$ . Note that given a set of  $n$  periodic tasks,  $B_n = 0$ , since there is no lower priority task to block  $\tau_n$ .

<sup>6</sup> That is, the conditions under which all the jobs of all the  $n$  tasks will meet their deadlines.

Theorems 14 and 15 can be generalized in a straightforward fashion. In order to test the schedulability of  $\tau_i$ , we need to consider both the preemptions caused by higher priority tasks and blocking from lower priority tasks along with its own utilization. The blocking of any job of  $\tau_i$  can be in the form of direct blocking, push-through blocking, or ceiling blocking but does not exceed  $B_i$ . Thus, Theorem 14 becomes

**Theorem 16:** A set of  $n$  periodic tasks using the priority ceiling protocol can be scheduled by the rate-monotonic algorithm if the following conditions are satisfied:

$$\forall i, 1 \leq i \leq n, \quad \frac{C_1}{T_1} + \frac{C_2}{T_2} + \dots + \frac{C_i}{T_i} + \frac{B_i}{T_i} \leq i(2^{1/i} - 1).$$

*Proof:* Suppose that for each task  $\tau_i$  the equation is satisfied. It follows that the equation of Theorem 14 will also be satisfied with  $n = i$  and  $C_i$  replaced by  $C_i^* = (C_i + B_i)$ . That is, in the absence of blocking, any job of task  $\tau_i$  will still meet its deadline even if it executes for  $(C_i + B_i)$  units of time. It follows that task  $\tau_i$ , if it executes for only  $C_i$  units of time, can be delayed by  $B_i$  units of time and still meet its deadline. Hence, the theorem follows.

*Remark:* The first  $i$  terms in the above inequality constitute the effect of preemptions from all higher priority tasks and  $\tau_i$ 's own execution time, while  $B_i$  of the last term represents the worst case blocking time due to all lower priority tasks for any job of task  $\tau_i$ . To illustrate the effect of blocking in Theorem 16, suppose that we have three harmonic tasks:  $\tau_1 = (C_1 = 1, T_1 = 2)$ ,  $\tau_2 = (C_2 = 1, T_2 = 4)$ ,  $\tau_3 = (C_3 = 2, T_3 = 8)$ . In addition,  $B_1 = B_2 = 1$ . Since these tasks are harmonic, the utilization bound becomes 100%. Thus, we have " $C_1/T_1 + B_1/T_1 = 1$ " for task  $\tau_1$ . Next, we have " $C_1/T_1 + C_2/T_2 + B_2/T_2 = 1$ " for task  $\tau_2$ . Finally, we have " $C_1/T_1 + C_2/T_2 + C_3/T_3 = 1$ " for task  $\tau_3$ . Since all three equations hold, these three tasks can meet all their deadlines.

**Corollary 17:** A set of  $n$  periodic tasks using the priority ceiling protocol can be scheduled by the rate-monotonic algorithm if the following condition is satisfied:

$$\frac{C_1}{T_1} + \dots + \frac{C_n}{T_n} + \max\left(\frac{B_1}{T_1}, \dots, \frac{B_{n-1}}{T_{n-1}}\right) \leq n(2^{1/n} - 1).$$

*Proof:* Since  $n(2^{1/n} - 1) \leq i(2^{1/i} - 1)$  and  $\max(B_1/T_1, \dots, B_{n-1}/T_{n-1}) \geq B_i/T_i$ , if this equation holds then all the equations in Theorem 16 also hold.

Similar to the sufficient condition in Theorem 16, the conditions in Theorem 15 can be easily generalized. Specifically,

**Theorem 18:** A set of  $n$  periodic tasks using the priority ceiling protocol can be scheduled by the rate-monotonic algorithm for all task phasings if

$$\forall i, 1 \leq i \leq n,$$

$$\min_{(k,l) \in R_i} \left[ \sum_{j=1}^{i-1} U_j \frac{T_j}{lT_k} \left\lceil \frac{lT_k}{T_j} \right\rceil + \frac{C_i}{lT_k} + \frac{B_i}{lT_k} \right] \leq 1$$

where  $C_i$ ,  $T_i$ , and  $U_i$  are defined in Theorem 15, and  $B_i$  is the worst case blocking time for  $\tau_i$ .

*Proof:* The proof is identical to that of Theorem 16.

*Remark:* The blocking duration  $B_i$  represents the worst case conditions and hence the necessary and sufficient conditions of Theorem 15 become sufficient conditions in Theorem 18.

The following example helps clarify the use of Theorem 18. Consider the case of three periodic tasks:

- Task  $\tau_1$ :  $C_1 = 40$ ;  $T_1 = 100$ ;  $B_1 = 20$ ;  $U_1 = 0.4$
- Task  $\tau_2$ :  $C_2 = 40$ ;  $T_2 = 150$ ;  $B_2 = 30$ ;  $U_2 = 0.267$
- Task  $\tau_3$ :  $C_3 = 100$ ;  $T_3 = 350$ ;  $B_3 = 0$ ;  $U_3 = 0.286$ .

Task  $\tau_1$  can be blocked by task  $\tau_2$  for at most 20 units, while  $\tau_2$  can be blocked by task  $\tau_3$  for at most 30 time units. The lowest priority task,  $\tau_3$ , cannot be blocked by any lower priority tasks. The total utilization of the task set ignoring blocking is 0.952, far too large to apply the conditions of Theorem 16. Theorem 18 is checked as follows:

- 1) Task  $\tau_1$ : Check  $C_1 + B_1 \leq 100$ . Since  $40 + 20 \leq 100$ , task  $\tau_1$  is schedulable.
- 2) Task  $\tau_2$ : Check whether either

$$C_1 + C_2 + B_2 \leq 100 \quad 80 + 30 > 100$$

$$\text{or} \quad 2C_1 + C_2 + B_2 \leq 150 \quad 120 + 30 \leq 150.$$

Task  $\tau_2$  is schedulable and in the worst case phasing will meet its deadline exactly at time 150.

- 3) Task  $\tau_3$ : Check whether either

$$C_1 + C_2 + C_3 \leq 100 \quad 40 + 40 + 100 > 100$$

$$\text{or} \quad 2C_1 + C_2 + C_3 \leq 150 \quad 80 + 40 + 100 > 150$$

$$\text{or} \quad 2C_1 + 2C_2 + C_3 \leq 200 \quad 80 + 80 + 100 > 200$$

$$\text{or} \quad 3C_1 + 2C_2 + C_3 \leq 300 \quad 120 + 80 + 100 = 300$$

$$\text{or} \quad 4C_1 + 3C_2 + C_3 \leq 350 \quad 160 + 120 + 100 > 350.$$

Task  $\tau_3$  is also schedulable and in the worst case phasing will meet its deadline exactly at time 300.

## VI. APPLICATIONS OF THE PROTOCOL AND FUTURE WORK

In this section, we briefly discuss the implementation aspects of the protocol as well as the possible extensions of this work.

### A. Implementation Considerations

The implementation of the basic priority inheritance protocol is rather straightforward. It requires a priority queuing of jobs blocked on a semaphore and indivisible system calls *Lock\_Semaphore* and *Release\_Semaphore*. These system calls perform the priority inheritance operation, in addition to the traditional operations of locking, unlocking, and semaphore queue maintenance.

The implementation of the priority ceiling protocol entails further changes. The most notable change is that we no longer

maintain semaphore queues. The traditional ready queue is replaced by a single job queue  $Job\_Q$ . The job queue is a priority-ordered list of jobs ready to run or blocked by the ceiling protocol. The job at the head of the queue is assumed to be currently running. We need only a single prioritized job queue because under the priority ceiling protocol, the job with the highest (inherited) priority is always eligible to execute. Finally, the run-time system also maintains  $S\_List$ , a list of currently locked semaphores ordered according to their priority ceilings. Each semaphore  $S$  stores the information of the job, if any, that holds the lock on  $S$  and the ceiling of  $S$ . Indivisible system calls *Lock\_Semaphore* and *Release\_Semaphore* maintain  $Job\_Q$  and  $S\_List$ . An example of the implementation can be seen in [13].

The function *Lock\_Semaphore* could also easily detect a self-deadlock where a job blocks on itself. Since the run-time system associates with each semaphore the job, if any, that holds the lock on it, a direct comparison of a job requesting a lock and the job that holds the lock determines whether a self-deadlock has occurred. If such a self-deadlock does occur, typically due to programmer error, the job could be aborted and an error message delivered.

Suppose monitors are used for achieving mutual exclusion. We again assume that a job does not suspend until its completion when it executes alone on the processor. We also assume that the job does not deadlock with itself by making nested monitor calls. A job inside a monitor inherits the priority of any higher priority job waiting on the monitor. To apply the priority ceiling protocol, each monitor is assigned a priority ceiling, and a job  $J$  can enter a monitor only if its priority is higher than the highest priority ceiling of all monitors that have been entered by other jobs. Since the priority ceiling protocol prevents deadlocks, nested monitor calls will not be deadlocked. The implications of priority ceiling protocol to Ada tasking are more complicated and are beyond the scope of this paper. Readers who are interested in this subject are referred to [1].

### B. Future Work

The priority ceiling protocol is an effective real-time synchronization protocol for it prevents deadlock, reduces the blocking to at most one critical section, and is simple to implement. Nonetheless, it is still a suboptimal protocol in that it can cause blocking to a job that can be avoided by enhancements to the protocol. Although a formal treatment of possible enhancements is beyond the scope of this paper, we would like to present the ideas of some possible enhancements to stimulate more research on this subject.

For example, we can define the *priority floor* of a semaphore, analogous to its priority ceiling, as the priority of the lowest priority job that may access it. Then, a job  $J$  can lock a semaphore  $S$  if its priority is higher than the priority ceiling of  $S$  or if the following conditions are true. The lock on  $S$  can also be granted if the priority of  $J$  is equal to the priority ceiling of  $S$  and the priority floor of  $S$  is greater than the highest priority preempted job. This latter condition, called the *priority floor condition*, ensures that neither a preempted job nor a higher priority job accesses  $S$ . This guaran-

tees that deadlocks and chaining will be avoided. This protocol is called the *priority limit protocol*. The priority limit protocol eliminates the ceiling blocking that  $J_0$  encounters at time  $t_5$  in Example 4. Moreover, this protocol requires identical information as does the priority ceiling protocol and can be implemented with equal ease. However, the priority limit protocol does not improve the worst case behavior and hence the schedulability.

It is also possible to enhance the priority limit protocol by replacing the priority floor condition by the following condition. A job  $J$  can also be allowed to lock a semaphore  $S$  if the priority of  $J$  is equal to the priority of  $S$  and no preempted lower priority job accesses the semaphore  $S$ . This condition also guarantees avoidance of deadlock and chaining. This protocol is called the *job conflict protocol* and is better than the priority ceiling and priority limit protocols.<sup>7</sup> The job conflict protocol is, however, still a suboptimal protocol. It will be an interesting exercise to develop an optimal priority inheritance protocol, and then compare it to the priority ceiling protocol for both performance and implementation complexity.

### VII. CONCLUSION

The scheduling of jobs with hard deadlines is an important area of research in real-time computer systems. In this paper, we have investigated the synchronization problem in the context of priority-driven preemptive scheduling. We showed that a direct application of commonly used synchronization primitives may lead to uncontrolled priority inversion, a situation in which a high priority job is indirectly preempted by lower priority jobs for an indefinite period of time. To remedy this problem, we investigated two protocols belonging to the class of *priority inheritance protocols*, called the *basic priority inheritance protocol* and the *priority ceiling protocol* in the context of a uniprocessor. We showed that both protocols solve the uncontrolled priority inversion problem. In particular, the priority ceiling protocol prevents deadlocks and reduces the blocking to at most one critical section. We also derived a set of sufficient conditions under which a set of periodic tasks using this protocol is schedulable by the rate-monotonic algorithm. Finally, we outlined implementation considerations for and possible extensions to this protocol.

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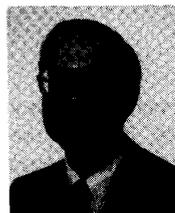
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