CS 435 : Linear Optimization Fall 2006

Lecture 9: Duality theorem, how to solve dual using solver for primal

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1 Duality

Theoram(Duality): If both primal and dual are feasible and bounded, then their optimums must be equal.
Proof:
If x_0 and y_o are optimum then $c^T x_o = y_o^T b$
For any feasible x and y,

$$c^T x = y^T A x \leq y^T b$$
$$\mathrm{c}^T x \leq y^T b$$

Let x_o be optimum, using the previos discussion

$$c^T x_0 = y^T A x_0 = y^T b'$$

Extend y^T by putting all other y's as 0(zero) to get $c^T x_0 = y^T b$

2 Rephrasing

$$min\ y^T b$$
$$\mathrm{A}^T y = c$$
$$\mathrm{y}_i \geq 0$$

can be rephrased as

$$max\ -b^T y$$
$$\mathrm{A}^T y \leq c$$
$$\mathrm{A}^T y \geq c$$
$$\mathrm{y} \geq 0$$

For reverse part

$$max\ c^T x$$
$$\mathrm{Ax} \leq b$$

can be rephrased as

$$min\ -x^T c$$
$$\mathrm{Ax+x'} = \mathrm{b}$$
$$\mathrm{x'} \geq 0$$