.

CS 435 : Linear Optimization | Fall 2006

Lecture 22.: Using LP techniques to design algorithms for combinatorial problems : Deriving Kruskal's MST algorithm using LP duality

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1 Integer Linear Program(ILP)

An Integer linear program is a linear program with an extra constraint that all x_i are integral.
Formally

$$\max\ c^T x$$
$$\mathrm{A}x \le b$$

where x_i is intergral $\forall i$

Now we allready know that that for

Primal	Dual
$\max\ c^T x$	$\min\ y^T b$
$Ax \le b$	$A^T y = c$
	$y \ge 0$

if x, y are feasiable and
$y_i > 0 \Longrightarrow \mathrm{A}_i\mathrm{x}=b_i$
then the solution is optimal.

The second question in quiz 2 was an attempt to make the above process iterative.

$\max\ c^T x$	$\max\ c^T y$
$Ax \le b$	$A_1 y \le 0$
for an x_o	
$A_1 x_o = b_1$	This can be solved much more easily
$A_2 x_o < b_2$	

Now $x = x_o + \epsilon y$
where ϵ depends on A.

2 Formulation of Minimum Spanning Tree as an ILP

Variables: X_e , one per edge
Cost: min $\mathrm{C}^T X$

where C_e is cost of edge e
Constraint: $0 \leq X_e \leq 1$ and X_e is an integer

$$\forall \text{ partition } \pi$$
$$\sum_{e crosses \pi} X_e \geq \#(\pi) - 1$$
where $\#(\pi)$is the number of parts in a partition.

Next we drop the constraint that X_e is an integer and also that $X_e \leq 1$ Now writing the dual of the MST linear program we get

$$max \sum_{\pi} y_\pi (\#(\pi) - 1)$$

$$\forall \text{ e } \sum_{e crosses \pi} y_\pi \leq C_e$$

$$y_\pi \geq 0$$