CS 435 : Linear Optimization — Fall 2006

Lecture 31: Network Flow

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1 Inroduction

Let G = (V,E) be a directed graph.For all edges e,define capacity $C_e \geq 0$.Let the variable f_e denote amount which flows along edge e.

$$\textit{feasibility: } f_e \leq C_e$$

In addition ,There will always be one vertex s with the property that all arcs containing s are directed away from s and one vertex t with the property that all arcs containing t are directed toward t .

constraints over networks.

- For each edge e ,the amount of flow f which flows along edge must be less than or equal to capacity of that edge.
  feasibility: $f_e \leq C_e$
- *Conservation rule* for Network flows,
  For each u,v ∈ V, $f_{uv} + f_{vu} = 0$
  At each vertex u,v ≠ s,t $\sum f_{uv} = 0$

Our problem is to calculate maximum value of a flow for a given network.

$$\max \sum f_{su}$$

2 Primal-Dual Method for Max Flow

Step1: Initialize with any feasible dual solution.Let $f_{uv} = 0$ for all (u,v) ∈ E

Step2: for all edges e for which equality

$$f_e = C_e$$

holds.

Let E' is set of such edges.Then DRP becomes,

$$\max \sum f'_{su}$$
$$f'_{uv} \leq 0 \text{ , for each edge (u,v) } \in \text{E'}$$
$$f'_{uv} + f'_{vu} = 0$$
$$\sum f'_{uv} = 0 \text{ for all u} \neq \text{s,t}$$
$$f_{uv} \leqslant 1$$

We can observe that (DRP) has the following interpretation. Find a path from s to t (with a flow of value 1) that uses only the following arcs in the following ways: saturated arcs in the backward direction; arcs with zero flow in the forward direction; and other arcs in either direction.We need to find a path in the residual graph.