Coordinated Scheduling of Thermostatically Controlled Real-Time Systems under Peak Power Constraint

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Abstract—A substantial fraction of the energy demand of buildings comes from air-conditioners (ACs), refrigerators, etc., which do not need human interaction for their continuous operations. So long as desirable temperature levels, which we refer to as thermal comfort-bands, are maintained by such background loads, users will not be concerned about when they perform their assigned functions, i.e., when they consume the energy required to function. This paper addresses the problem of maintaining thermal comfort-bands associated with background loads under Peak energy consumption constraint. Based on our empirical observations pertaining to energy consumption profiles of thermal comfort-band maintaining ACs, a) we present a feasibility criterion for maintaining the thermal comfort-band of a given set of background loads under constraints on the peak available power and (b) we propose TCBM, a new algorithm for scheduling such background loads under peak power constraint. In addition to limiting peak power demand, the TCBM algorithm avoids undesirable switching (on and off) of electrical appliances to improve efficiency of the equipment and reduce failures. Results from simulation and real-life implementation demonstrate that our algorithm is superior to the existing work on background load scheduling. We also show how TCBM can adapt to changes in ambient parameters and provide the basis for efficient demand-response systems.

Index Terms—TCBM; Smart Home; Green home; Real-Time

I. INTRODUCTION

A significant fraction of home as well as large buildings' energy demand comes from air-conditioners, refrigerators and room-heaters. These Thermostatically Controlled Electrical Devices (TCEDs or TCE devices) maintain the temperature of the environment under their control according to user-specified set-points. They do not run continuously, but follow a pattern of on-off cycles to maintain the desired temperature. The TCE devices, which we also refer to as background loads, do not need human interaction for their continuous operation. So long as desirable temperature levels, which we refer to as thermal comfort-bands, are maintained by such background loads, users will not be concerned about when they perform their assigned functions, i.e., when they consume the energy required to perform these functions.

Under un-coordinated individual control, it is possible that all the above devices run simultaneously during some time intervals. This can result in higher peak power demand. Today many commercial buildings are subject to very high tariff under peak power demand pricing [1]. In the near future, residential buildings are also likely to be subject to such electricity pricing schemes. Further, predictably lower peak demand benefits power utilities by reducing the cost and complexity of handling power grid stability, load-shedding and blackouts. Hence, the design and evaluation of techniques for the coordinated scheduling of background loads form the crux of this paper. The objective of any TCE device, $A_i$, is to maintain the temperature $T_i$ of the environment under its control within a desirable thermal comfort-band $[T^U_i, T^L_i]$. Under peak power constraint, not all devices may be able to run concurrently at any point of time. Therefore, under peak power demand constraint, we need to switch power between the devices intelligently, while ensuring that the desired temperature is maintained by each. Suppose $W_i$ denotes the wattage of device $A_i$ and $PPL$ denotes the Peak Power Limit, then the devices that can be powered at any point of time will be governed by the following constraints:

$$\forall t, \sum_{i=1}^{n} x_i(t) \times W_i \leq PPL$$

where, $\forall i, x_i(t) \in \{0, 1\}$ and $T^L_i \leq T_i \leq T^U_i$. $x_i(t)$ is the state ($1 = \text{ON}$ and $0 = \text{OFF}$) of the $i^{th}$ device at time $t$.

We focus on the background load due to ACs, as analysis of an AC system will be applicable to other TCE devices, like refrigerators, dehumidifier, room heaters, etc. All these devices maintain the comfort-band of the environment that they control and work on the same basic principles of heat transfer.

The functioning of a TCE device $A_i$ can be modelled as a periodic activity having an on-time and off-time within its duty cycle equivalent to execution time ($C_i$) and laxity ($L_i$) respectively; the duty cycle is hence equivalent to period $P_i = C_i + L_i$. For an AC unit that maintains temperature $T_i$ within a comfort band $[T^U_i, T^L_i]$, we can consider $C_i$ as the time duration the device runs to bring down $T_i$ from $T^U_i$ to $T^L_i$ and $L_i$ as the time duration it can be switched off, i.e., when $T_i$ rises from $T^L_i$ to $T^U_i$. Any individual TCE device exhibits this periodic nature when controlled thermostatically.

Not surprisingly, existing literature [2] [3][4] suggests modelling background electrical loads as real-time tasks and

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applying traditional real-time scheduling algorithms EDF[5], LSF[6], etc. The main assumption in real-time schedulability analysis is that a task is considered schedulable, if it is executed for $C_i$ units of processor time before it deadline $D_i (C_i \leq D_i \leq P_i)$; when this $C_i$ time is received within $P_i$ along the time scale is not a consideration.

We now show that in order to schedule thermostatically controlled electrical devices, it is not just enough to allocate power for $C_i$ units of time within $P_i$; it is also important when this $C_i$ is allocated within $P_i$. It can be seen in Figure 1, that if the TCED task is executed (powered-On) for $C_i$ units of time during the beginning of its period from $t_0$ to $t_b$, the environmental temperature $T_i$ remains within the comfort-band. But, if the task is preempted after its execution till $t_a$ and is powered-On for the remaining execution time just at the end of the period $[t_0, t_0 + P_i]$, $T_i$ goes beyond the upper limit $T^U$ of the comfort-band $[T^L, T^U]$.

Thus, whereas traditional real-time scheduling algorithms consider the deadline $D_i$, maximum laxity $L_i$ and duty cycle $P_i$ as constants for a task, in the case of TCE devices these parameters are dynamic, because execution of such devices depend on the existing temperature of the very environment controlled by them. From Figure 1, it can be observed that for the same TCED task, the duty cycle should change dynamically from $P_i$ to $(t_c - t_0)$ when preempted at $t = t_a$. Also, the maximum Laxity $L_i$ changes from $(P_i - t_b)$ to $(t_c - t_a)$. Therefore, in case of TCED tasks these parameters are required to be calculated dynamically in order to apply any scheduling algorithm effectively.

Furthermore, preemption decisions must be taken based on the environmental parameter, which is to be maintained within the comfort-band $[T^L, T^U]$. For example, so not to violate the need for $T_i \leq T^U$ of AC $A_i$, another AC $A_j$, whose zonal temperature $T_j$ is $< T^U$ might have to be preempted to divert power to $A_i$.

Preemption decisions must also consider the fact that minimal switching of TCE devices, except for resistive heating loads, is desirable for the following reasons:

1) Compressor driven devices have a specified delay ($\sim 3$ min) before they can be restarted. The delay allows the pressures in the system to equalize so that the compressor does not start under a load. If no restart-delay is provided, the compressor may not start due to an overload or it can even damage the equipment.

2) High starting current [7] with every switch-on of induction motors (used in ACs, refrigerators, etc.) causes additional power loss.

These considerations motivated us to develop a new algorithm, Thermal Comfort-Band Maintenance (TCBM), to schedule thermostatically controlled devices (presented in Section III). The basic intuition behind TCBM is that since the main goal is to maintain the temperature within a desirable $[High, Low]$ band, we need not do switching (ON/OFF) of any load, if the temperature is within the comfort-band associated with it. This helps in reducing the number of undesirable switching of devices. TCBM is also designed to support feasibility analysis, presented in Section III-D, using which it will be possible to determine, at the time a user sets the desired comfort-band, whether that comfort band can be maintained given peak power constraint.

Besides TCBM and the associated feasibility analysis, this paper makes the following additional contributions.

- A conceptual model of TCE devices as a special class of real-time tasks (in Section II). The model is corroborated up by our empirical study of the thermal characteristics of ACs, which demonstrates that the temperature of the environment controlled by an AC rises exponentially, when the device is switched Off and falls exponentially when it is switched On. The exponential nature of the characteristic equations underlies the design of the TCBM algorithm as well as the analysis of feasibility of maintaining the specified comfort-band under peak demand constraint.

- Refining the concepts of Laxity ($L_i$), Deadline ($D_i$) and period ($P_i$) to make them cognizant of the environmental parameter (temperature $T_i$) controlled by a TCE device. This sets the stage for adapting scheduling algorithms from the real-time literature for TCED scheduling.

- A performance study, involving simulation as well as prototype implementation (presented in Section IV) that demonstrates the superior performance of our algorithm compared to approaches adapted from the literature.

- Algorithms for on-line adaptive determination of comfort-bands of TCE devices based on our feasibility analysis and insights obtained from experimental observations (discussed in Section V).

Related work is discussed in Section VI and is followed by conclusion. Important parameters and notations used in the paper are listed in Table I.
room with a single AC or a large room is assumed to be divided into thermally de-coupled zones whose temperatures are maintained by one AC in each zone. We have a set of $n$ ACs for $n$ zones, but peak power constraint permits at most $m$ ACs out of $n$ to be on at any point of time. The ACs are to be scheduled such that the temperature $T_i$, $i = 1, 2, \cdots n$ in individual zones lies within the comfort-band $[T^U, T^L]$.

As portrayed in Fig. 1, when an AC is on, i.e., the compressor is running, it cools down the room temperature $T_i$; the temperature rises from the instant the AC is switched off because of heat loads and losses. Based on elementary principles of heat transfer [8], we develop a model, simplified by means of using overall heat transfer co-efficient of calculating heat transfer from the terminal temperatures, i.e., the temperature of the two bodies between which heat transfer takes place. It may be noted that various heat transfer co-efficients, pertaining to different heat transfer modes (conduction, convection and radiation), are combined into an overall heat transfer co-efficient for simplification of the heat transfer problem as is done in practice [9].

Using this (simplified) model we first derive the warming and cooling rate of an AC. The time taken by an AC to cool a zone from $T_i$ to $T^L$ and the time it takes for $T_i$ to rise up to $T^U$, when the AC is off are then derived, based on experimental data.

A. Modeling Rate of Change in Temperature

- **Warming when AC is off:** Let $Q_{hi}$ be the heat input rate in zone $Z_i$, $1 \leq i \leq n$ and $T_i$ be the temperature of the zone. Then, the rate of change of $T_i$ can be expressed as

$$S_i \frac{dT_i}{dt} = Q_{hi} + h_0(T_a - T_i) \quad (2)$$

where $S_i$ denotes the thermal capacity of the zone and $h_0$ denotes the overall heat transfer co-efficient of the room. It follows that

$$\frac{dT_i}{dt} = \frac{Q_{hi} + h_0 T_a}{S_i} - \frac{h_0}{S_i} T_i = \alpha' - \beta' T_i \quad (3)$$

in which $\alpha' = \frac{Q_{hi} + h_0 T_a}{S_i}$ and $\beta' = \frac{h_0}{S_i}$.

- **Cooling when AC is on:** When an AC is switched on, it removes heat from the room. With heat load $Q_{hi}$ and ambient temperature $T_a$ remaining constant, Equation 2 can be modified to capture the heat removal by AC as

$$S_i \frac{dT_i}{dt} = Q_{hi} - h_1(T_i - T^C) + h_0(T_a - T_i) \quad (4)$$

where, $T^C$ is the temperature of the heat-transfer coil of the AC and $h_1$ is the overall heat transfer co-efficient of the AC. It follows that

$$\frac{dT_i}{dt} = \frac{Q_{hi}}{S_i} + \frac{h_1 T^C + h_0 T_a}{S_i} - \alpha - \beta T_i \quad (5)$$

in which $\alpha = \frac{Q_{hi} T^C + h_0 T_a}{S_i}$ and $\beta = \frac{h_1 h_0}{S_i}$.

B. Experimentally Determining AC Thermal Characteristics

We measured the changing temperatures of a zone controlled by an AC using Pt100 RTD (Resistance Temperature Detectors) and recorded them every 10 sec. in a digital recorder (Eurotherm Chessel 5000). The recorder provides a facility to generate alarm(s), when the input parameter(s) goes beyond specified limit(s). We generated cooling curve data by switching on an AC when the room temperature $T_i$ was equal to the ambient temperature $T_a = 27.5^\circ C$ and kept it running till $T_i$ saturated to the lowest value ($20.5^\circ C$ here). Drop in temperature gets bounded because, at some point, the rate of cooling matches with the rate of heating due to heat loads and losses. The data of warming curve was generated by switching off the AC at $T_i = 20.5^\circ C$ and keeping it off till $T_i$ went up to $27.5^\circ C$. From the experimental data, we did curve-fitting and obtained the thermal characteristic equations of ACs:

- **Cooling Down (AC Switched ON):**

$$T_i(t) = ae^{-bt+c} + d \quad (6)$$

where, $a, b, c \& d$ are constants specific to a particular AC for a particular ambient temperature, the values of which obtained by curve-fitting from our experimental data for two ACs are shown in Table II.

Solving Equation 6, we get

$$t = \frac{1}{b} \left( c - ln \frac{T_i - d}{a} \right) \quad (7)$$

Therefore, the time for $T_i$ to reach from $T^U$ to $T^L$, i.e.,

$$C_i = \frac{1}{b} \left( c - ln \frac{T^U - d}{a} - (c - ln \frac{T^L - d}{a}) \right) \quad (8)$$

$$C_i = \frac{1}{b} \ln \frac{T^L - d}{T^U - d} \quad (9)$$

Also, from Equation 6 we get

$$\frac{dT_i}{dt} = -abe^{-bt+c} = -abe^{-bt+c} - bd + bd.$$
Therefore, the time for
where  are constants specific to a particular AC at
for a particular ambient temperature obtained by curve-fitting
from our experimental data. Solving Equation 11, we get
(10)

\[
T_i(t) = a'e^{-b'i+c'} + d'
\]

where  are constants specific to a particular AC at
the position of the comfort-band is shifted along the
curve. Therefore, an AC will run for lesser time if
observed that the higher the temperature, the faster is the rate
of change, b. The difference in the cooling time  and in the
utilization  is significant.

These observations are in compliance with Eq. (9). The
increase in  can be explained from the fact that
ln(A - d)/ln(B - d) > ln A/ln B, if A < B,
0 < d < A and 0 < d < B. The scenario is
the same with the Eq. (9), when the comfort-band is changed from
[26.0°C, 23.0°C] to [25.5.0°C, 22.5°C] or to [25.0°C, 22.0°C].

- Comfort-Bands with varying  : We increased the  values by 0.5°C, keeping the  values same, as shown in
Table IV(b) and observed that by increasing  by 0.5°C, the decrease in  is larger, as expected.

- Comfort-Bands with varying  : When the values of
were decreased by 0.5°C, keeping the values of  the same, as shown in
Table IV(c), we observed that decreasing the lower limit  of the comfort-band, as
expected, increases the value of  .

From Figures 2 (a), (b) & (c) and from Equation 10, it can be
observed that the higher the temperature, the faster is the rate of cooling,
because of the exponential nature of the cooling curve. Therefore, an AC will run for lesser time if  is
shifted up as compared to the same when  is lower. This results in reduced power consumption.

We have shown here i) how to obtain the constants  and
and  by experimentally, to characterize the ACs ii) how manipulating the comfort-band can affect the AC power consumption. These observations are used in Section V for adaptive demand-response control.

### III. TCBM Scheduling

The primary goal of TCBM is to maintain the comfort-band. TCBM achieves this while turning on ACs judiciously because of the peak power constraint. Secondly, turning on an AC has to be done taking into account the fact that once turned-off, it can not be turned-on for a minimum period of time, typically 3 min. Thirdly, the number of switchings (on
The scheduling (on-off) decisions are taken periodically. Therefore, if a switching decision is made only after the zonal temperature reaches $T^U$, comfort-band can be violated before the next scheduling decision is taken. To avoid the possibility of comfort-band violation, an AC is considered for switching off only in time before its temperature reaches $T^L$. Switching on of an AC may also be delayed if more than $m$ ACs reach $T^U$ simultaneously.

**Definition 1:** $B^U$ is the upper-limit of $T_i$ only above which, the $i^{th}$ AC is considered for switching on. The value of $B^U$ is calculated as $B^U = T^U - \Delta U$, where $\Delta U$ denotes the rise in $T_i$ due to the maximum possible delay in switching off the AC after it reaches $B^U$.

**Definition 2:** $B^L$ is the lower-limit of $T_i$ below which, the $i^{th}$ AC is switched off only if it becomes necessary to power on some other AC to maintain the comfort-band. The value of $B^L$ is calculated as $B^L = T^L + \Delta L$, where $\Delta L$ denotes the fall in $T_i$ due to the maximum possible delay in switching off the AC after it reaches $B^L$.

### A. The TCBM Algorithm

Initially, $T_i = T_a$ and an AC operates to bring down the $T_i$ from $T_a$ to a value that lies within the comfort-band. Specifically, the TCBM algorithm starts by switching on arbitrarily chosen ACs, where the value of $m$ is determined by the peak demand constraint, so that

$$\sum_{i=1}^{m} W_i \leq PPL$$  \hspace{1cm} (16)$$

where, $W_i$ denotes the power required by $i^{th}$ AC and the sum is obtained from the first $m$ of the $n$ ACs arranged in descending order of their wattage.

Once $T_i$ is within the comfort band, the ACs are controlled to maintain the respective $T_i$s within the comfort band, by applying the following rules every $I^S$ units of time.

1) **Rule # 1:** Turn off $AC_i$ if it is on at time $t$ and if
   a) $T_i \leq B^L$ OR
   b) there is an $AC_j (i \neq j)$ with $T_j \geq B^U$ AND no. of on-ACs $\geq m$ AND $T_i \leq T_j^U$ AND $AC_i$ is the coolest one among on-ACs.

2) **Rule # 2:** Turn on $AC_i$ if
   a) $T_i \geq B^U$ AND
   b) No. of on-ACs $< m$

### B. Calculating $\Delta U$

Since an AC is switched on only if its zonal temperature reaches $B^U$, the worst delay in switching-on an AC will arise when all ACs are off and the zonal temperatures $T_i$ of all the ACs reaches $B^U$ at a time, say $t = t_s$.

Now, at $t_s$, only $m$ ACs can be switched-on and therefore, the $T_i$ corresponding to $(n-m)$ ACs will rise beyond $B^U$. Our goal is to switch-on these $n-m$ ACs within a reasonable time so that none of the $T_i$s will cross $T^U$. Let $I^S$ be the scheduling period. Therefore, after $t_s + I^S$ time, the next set of $m$ ACs will be running, either by switching-on ACs which were off.
during the previous scheduling period or allowing ACs to run, which were already on. Therefore, the best case will be, if at $t = t_s + I^S$ the next set of $m$ ACs are selected from the set of ACs which were off and in every subsequent scheduling period, a new set of $m$ ACs are selected in the same manner till all the ACs are switched on at least once after $t_s$. Thus the AC, which will be switched on last after $t_s$, will face a delay $t_d$, where

$$t_d = \lceil \frac{n}{m} \rceil \times I^S$$

(17)

Now, by definition, $\Delta U$ is the temperature rise from $B^U$ to $T^U$. Therefore, switching on of an AC, should be allowed at least $\Delta U$ units of temperature ahead of $T^U$ to ensure that the AC will not cross $T^U$ because of the delay in scheduling decision, as discussed. Further, a minimum $\Delta U$ ensures minimum number of AC switching.

From Equation 15, we get the temperature rising slope at $B^U$ as

$$\frac{dT_i}{dt} \mid _{T_i = B^U} = b'(d' - B^U)$$

(18)

Substituting $B^U$ with $T^U - \Delta U$, [Definition 1]

$$\frac{dT_i}{dt} \mid _{T_i = B^U} = b'(d' - T^U + \Delta U)$$

(19)

Considering the worst case of linear rise of temperature from $B^U$ to $T^U$, we get

$$\Delta U = b'(d' - T^U) - b'(T^U - \Delta U)$$

Substituting the value of $t_d$ from Equation 17, we get

$$\Delta U = \frac{b'(d' - T^U)}{1 - b'|I^S|} \times \lceil \frac{n}{m} \rceil I^S$$

(20)

C. Calculating $\Delta L$

Since any number of ACs can be turned off at an instance, there is no delay involved beyond $I^S$.

Considering the temperature falling slope according to Equation 10 and following a reasoning similar to that described in Section III-B, we get

$$\Delta L = \frac{b(d - T_L)}{1 + b'I^S} \times I^S$$

(21)

D. Feasibility Analysis

In this subsection, we discuss the feasibility of scheduling $m$ out of $n$ ACs that ensures a given comfort-band $[T^U, T^L]$. Theorem 1: A given comfort-band $[T^U, T^L]$ will be maintained by scheduling $m$ out of $n$ ACs, if

$$\sum_{i=1}^{m} \text{abs}(b_i(d_i - B^U)) \geq \sum_{i=m+1}^{n} b'_i(d'_i - B^U)$$

(22)

where, $b_i, d_i$ and $b'_i, d'_i$ are the constants pertaining to the cooling curve and the warming curve respectively corresponding to $i^{th}$ AC, as discussed in Section II-B and ii) sum on the left is obtained from the first $m$ of the $n$ ACs arranged in ascending order of their falling slope and iii) sum on the right is obtained from the first $(n - m)$ of the $n$ ACs arranged in descending order of their rising slope.

Proof Sketch: In Section III-B, it has been shown that, in order to ensure minimum switching, scheduling decision to switch on an AC is taken when its zonal temperature reaches $B^U$. Therefore, we check for feasibility considering the worst case when, say at time $t_0$, the zonal temperatures ($T_i$) of all the ACs reach $B^U$ because it is a scenario when all the $n$ ACs are ready to be switched on.

Under peak demand constraint, at time $t_0$, only $m$ ACs will be switched on and in every successive scheduling period, a new set of $m$ ACs will be selected to run. Now, in order to maintain the comfort-band it should always be the case that the cumulative temperature rise of the $(n - m)$ ACs (off) is less than the cumulative temperature fall of the $m$ ACs (on) at $T_i = B^U$.

The rise and fall in temperature of an ACs during a scheduling decision interval $I^S$ can be expressed as $I^S \times S_f^i$ and $I^S \times S_t^i$ respectively. So, a sufficient condition for selecting any set of $m$ ACs for the purpose of comfort-band maintenance is

$$m \times \text{abs}(S_f^i) \geq (n-m) \times S_t^i$$

(23)

where, the cooling slope $S_f = \min(S_f^i, |i| = 1, 2 \cdots n)$ and the warming slope $S_r = \max(S_r^i, |i| = 1, 2 \cdots n)$ at $T_i = B^U$. Note that if the Equation 23 is valid at $T_i = B^U$, it is also valid for any $T_i > B^U$, because of the exponential nature of the thermal characteristics of ACs.

Equation 23 is pessimistic, because temperature rising and falling slopes will be different for different ACs. The weaker sufficient condition is:

$$\sum_{i=1}^{m} \text{abs}(S_f^i) \geq \sum_{i=1}^{n-m} S_r^i$$

(24)

Where, i) the sum on the left is obtained from the first $m$ of the $n$ ACs arranged in ascending order of their falling slope and ii) the sum on the right is obtained from the first $n - m$ of the $n$ ACs arranged in descending order according to their rising slope.

From Equation 10, we get

$$S_f^i \mid _{T_i = B^U} = \frac{dT_i}{dt} \mid _{T_i = B^U} = b_i(d_i - B^U)$$

(25)

where, $b_i, d_i$ are constants corresponding to the $i^{th}$ AC. Similarly from Equation 15, we get

$$S_t^i \mid _{T_i = B^U} = \frac{dT_i}{dt} \mid _{T_i = B^U} = b'_i(d'_i - B^U)$$

(26)

where, $b'_i, d'_i$ are constants corresponding to the $i^{th}$ AC.

Substituting the values of $S_f^i$ and $S_t^i$ in Equation 24, the proof follows.

IV. SIMULATION & PROTOTYPE IMPLEMENTATION OF TCBM

Here we report on the performance of our algorithm and compare the results with that of several candidate algorithms adapted from the real-time domain for AC scheduling. We also implemented TCBM algorithm in a prototype set-up and report on observations made from it.
Simulation studies were carried out based on thermal characteristics of ACs generated by curve-fitting using the empirical data presented in Section II-B. The load consisted of 5 ACs. We assume that peak power limit permits no more than 3 ACs to be ON at a time. Feasibility is checked according to Equation 22 for a comfort band with \( T^U = 26^\circ C \) & \( T^L = 23^\circ C \), \( \Delta^U (0.24) \) and \( \Delta^L (0.19) \) values are calculated according to the methods discussed in Section III-B and III-C respectively.

### A. Candidate scheduling algorithms

We first applied global EDF (Earliest Deadline First) scheduling [10], Least Slack First (LSF) scheduling [2], and value-based scheduling [11] [12] from the real-time domain. The periodic task model as described in Section I, is used for AC tasks in this simulation with \( C_i \) as the time duration an AC needs to run to bring down \( T_i \) from \( T^U \) to \( T^L \) and \( L_i \) as the time duration it can be switched OFF, i.e., when \( T_i \) rises from \( T^L \) to \( T^U \). The period \( P_i \) is its duty-cycle \((C_i + P_i)\).

In EDF [5], the task which has the earliest absolute deadline is scheduled first. Global EDF [10] extends the EDF scheduling of tasks to uniform multi-processors, where \( m \) out of \( n \) tasks can run on \( m \) processors at any point of time. Global EDF (gEDF) scheduling appears to be a natural candidate for AC scheduling as the constraint that only \( m \) out of \( n \) ACs can run simultaneously can be straight-away mapped to scheduling \( n \) tasks on \( m \) processors.

In LSF or Least Slack-Time First (LST) [6], the task which has the least slack (or laxity) is scheduled first, where at any time \( t \), slack of a task having a deadline \( D_i \) is defined as \((D_i - t)\) minus the time required to complete the remaining portion of the task. In our case, slack at any time is the remaining length of time it can be OFF.

Value-based scheduling [11] [12] is another promising algorithm for scheduling TCE devices, as a TCED task can be associated with a value \( V_i(T_i) \), depending on the existing temperature \( T_i \) of the environment under its control. Specifically, we can assign a value to the AC task according to the amount of cooling it can give per unit time when powered on and the desired comfort-band. The state-dependent attribute \( V_i \) has the highest value at \( T_i = T^U \) and it becomes negative at \( T_i > T^U \). \( V_i \) goes on decreasing as \((T^U - T_i)\) decreases and again attains negative value at \( T_i < T^L \). We can summarize it as follows.

\[
V_i(T_i) = \begin{cases} 
T^U - T_i, & \text{if } T_i > T^U, \\
T_i - T^L, & \text{if } T_i \leq T^U
\end{cases}
\]  

### B. Relative performance of the algorithms

Figures 3, 4, 5 and 6 show the simulation results of LSF, global EDF, Value-based and TCBM scheduling respectively, when ACs are started with a room temperature \( T_{i0} = 27.5^\circ C \). The observations are summarized in Table V. It is important to note that LSF, Value-based and TCBM scheduling take less than \( \leq 10 \) minutes, which can be considered to be a reasonable amount of time, to bring down the room temperatures within \([26^\circ C, 23^\circ C]\) from the initial temperature of \(27.5^\circ C \) and maintain it thereafter. In contrast, global EDF takes 31 minutes.

Global EDF does not always maintain the room temperature in the comfort-band, which is obviously unacceptable. Even under LSF scheduling, \( T_i \) goes beyond \( T^L \) on some occasions.

It can also be observed from Figure 3 that all ACs are switched OFF at different points of time causing higher \( T_i \), because LSF policy keeps a load OFF irrespective of \( T_i \), if it has consumed power (i.e., remains ON) for \( C_i \) amount of time within its period \( P_i \). Similarly, in case of gEDF scheduling, in Figure 4, loads are switched OFF because they consumed power for \( C_i \) amount of time within their period \( P_i \), without taking into consideration the value of \( T_i \). This also explains the high values of discomfort duration shown in Table V under LSF and gEDF scheduling.

In case of TCBM, it can be observed in Figures 6 that \( m \) ACs are always ON, as long as they are required to maintain the comfort-bands. During the initial phase, when the zonal temperature is high \((27.5^\circ C)\), \( T_i \) of some ACs increases only because it is not possible to run more than \( m \) ACs at a time.

The number of \( ON \) \& \( OFF \) switchings of ACs in LSF and Value-based scheduling is very high, 400\% and 1050\% more respectively, compared to our TCBM scheduling. Switching in global EDF scheduling is 150\% higher compared to TCBM scheduling. We calculated the expected number of switching of 5 ACs in 1500 min. of simulation considering 2 \((ON \& OFF)\) switchings per period when run only under thermostat control. The number of excess switching against the minimum value of 492 is shown in Table V and it can be observed that the no. of excess switching under TCBM is only 70 as against 5507, 1705 & 384 under value-based, LSF and gEDF respectively.

In case of value based scheduling, it can be observed from Figure 5 and Table V that the number of AC switchings is extremely high \((5999 \, \text{for 5 ACs})\). It can be explained from the fact that as an AC is switched ON, its corresponding room temperature decreases, which in turn decreases the value of supplying power to it. In the subsequent scheduling period(s), the same AC is likely to be switched OFF because of its reduced value and hence later its value will rise again. Therefore, such cyclical rise and fall of values causes excessive switchings.

In Figure 7, we focus on a single AC. It highlights the superior performance of our TCBM algorithm with respect to different performance metrics.

### C. Prototype Experimental Studies

The results of real-life control of 2 ACs in a room using TCBM algorithm, after both ACs reach the comfort-band, are shown in Figure 8. It may be noted here that, when an AC is switched ON, its compressor and fan run and when it reaches the lower value of its set point, the AC is switched to fan-mode (i.e., compressor is OFF, but the fan is running).

Feasibility analysis according to Equation 24 indicates that comfort-band of \([26^\circ C - 23^\circ C]\) can be maintained with peak power permitting only one AC to be ON at a time. Figure

\[\text{2} \text{The power requirement in the fan-mode is very low as compared to the same when AC is in cooling-mode (compressor & fan both running). Hence, for simplicity of explanation, we ignore the power consumed by the fan-mode.}\]
8 demonstrates that both ACs function alternatively, while maintaining the comfort-band. The values of $\Delta U$ and $\Delta L$ obtained using the methods discussed in Section III-B and III-C are $0.24^\circ$C and $0.18^\circ$C respectively.

In can be observed from Figure 8 that whenever an AC is switched on, it can affect the temperature of the other zone marginally. For example, at time $t=40$, when AC1 is switched on while the AC2 was off, the temperature of the zone corresponding to AC2 falls by about $0.2^\circ$C and then goes up again. This is because our assumption that the zones controlled by the two ACs are thermally de-coupled may not hold true in reality, because they are in the same room. How to consider such dependencies is part of our ongoing work.

In summary, none of the candidate scheduling algorithms from the real-time domain namely, LSF, global EDF and Value-based scheduling, is suitable for TCED scheduling because they suffer from the following disadvantages:

- Maintaining room temperature within comfort zone $[T^U, T^L]$ is not guaranteed in LSF and gEDF scheduling, as the basic criterion for these algorithms is to provide resource for $C_i$ units of time within every period $P_i$, irrespective of when this $C_i$ time is allocated. This lacuna can cause the controlled environmental parameter to go beyond its limit even though the task gets resource (power in case of TCED) for $C_i$ unit of time within every $P_i$, as explained in Section I.

- Excessive and undesirable switching of ACs since LSF, gEDF and Value-based policies decide to switch ON/OFF ACs irrespective of whether room $T_i$ is within the comfort zone $[T^U, T^L]$.

Since it is important to avoid unnecessary switching (ON/OFF) as discussed in Section I, it is preferable that switching of an AC occurs only when its environmental parameter $T_i$ is outside the specified comfort-band $[T^U, T^L]$.

It may be argued that excessive switching under LSF, gEDF and Value-Based scheduling can be avoided if we simply apply those scheduling policies only when $T_i$ is outside the comfort-band. But, switching (ON or OFF) of a device becomes mandatory if temperature goes beyond the comfort-band associated with the device. In other words, such a scheme leaves no scope for applying LSF, gEDF or Value-based scheduling once the $T_i$ is outside the comfort band.

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**TABLE V**

**SUMMARY OF PERFORMANCE OF DIFFERENT SCHEDULING POLICIES**

<table>
<thead>
<tr>
<th>Metric</th>
<th>LSF</th>
<th>gEDF</th>
<th>Value</th>
<th>TCBM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time to reach Comfort-band</td>
<td>2197</td>
<td>876</td>
<td>50999</td>
<td>562</td>
</tr>
<tr>
<td>No. of Switching</td>
<td>1705</td>
<td>384</td>
<td>5507</td>
<td>70</td>
</tr>
<tr>
<td>Excess Switching</td>
<td>56</td>
<td>293</td>
<td>14</td>
<td>33</td>
</tr>
</tbody>
</table>

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In summary, none of the candidate scheduling algorithms from the real-time domain namely, LSF, global EDF and Value-based scheduling, is suitable for TCED scheduling because they suffer from the following disadvantages:

- Maintaining room temperature within comfort zone $[T^U, T^L]$ is not guaranteed in LSF and gEDF scheduling, as the basic criterion for these algorithms is to provide resource for $C_i$ units of time within every period $P_i$, irrespective of when this $C_i$ time is allocated. This lacuna can cause the controlled environmental parameter to go beyond its limit even though the task gets resource (power in case of TCED) for $C_i$ unit of time within every $P_i$, as explained in Section I.

- Excessive and undesirable switching of ACs since LSF, gEDF and Value-based policies decide to switch ON/OFF ACs irrespective of whether room $T_i$ is within the comfort zone $[T^U, T^L]$.

Since it is important to avoid unnecessary switching (ON/OFF) as discussed in Section I, it is preferable that switching of an AC occurs only when its environmental parameter $T_i$ is outside the specified comfort-band $[T^U, T^L]$. It may be argued that excessive switching under LSF, gEDF and Value-Based scheduling can be avoided if we simply apply those scheduling policies only when $T_i$ is outside the comfort-band. But, switching (ON or OFF) of a device becomes mandatory if temperature goes beyond the comfort-band associated with the device. In other words, such a scheme leaves no scope for applying LSF, gEDF or Value-based scheduling once the $T_i$ is outside the comfort band.
### B. Shifting/Relaxation of Comfort-Band

From Figure 2 (a) & (b), it can be observed that the duration for which AC remains on is reduced if, i) the comfort-band is shifted up along the temperature scale or if ii) $T^U$ is shifted up. This happens because of the fact that cooling rate of AC is more at higher temperatures. Now, from the feasibility condition (Equation 24), it can be observed that if the cooling slope $S_f$ is increased (by shifting $T^U$ up) feasibility condition will be satisfied with a lower value of $m$. Suppose, peak demand constraint allows power for 3 ACs at a time and we have 5 to control using TCBM algorithm. We consider that out of 5 ACs, two ACs have the same thermal characteristics as that of AC1 and three ACs have the same characteristics as that of AC2. The constants of their characteristic equations are shown in Table II and III. The data related to feasibility of these 5 ACs under TCBM are generated using Equations 25 & 26 are shown in Table VI.

It can be observed from Table VI, that it is feasible to maintain a comfort-band of $[23^0C - 25^0C]$ by running 3 ACs at a time. But, the same comfort-band cannot be maintained, if peak demand constraint allows powering-on of at most 2 ACs at a time.

Now, let’s take two cases; i) comfort band is shifted from $[23^0C - 25^0C]$ to $[24^0C - 26^0C]$ and ii) upper limit of the comfort band is changed from $25^0C$ to $26^0C$. It can be observed from Table VI that in both the cases, comfort-band can be maintained by running 2 out of 5 ACs at a time.

Therefore, we conclude that under varying peak demand limit, comfort-band $[T^L, T^U]$ can be suitably adjusted, as discussed above, to meet the peak demand constraint.

This has implications for adaptive demand-response in dynamic energy pricing/availability scenarios:

1) Given an externally-imposed peak demand constraint, the comfort-band can be adjusted dynamically and the user informed about it.

2) If a user insists on staying with a pre-set comfort-band, he can be warned about the implications of violating the peak power consumption limit ahead of time.

### VI. RELATED WORK

Reduction in peak power demand for grid stability, reduction of occurrences of blackouts as well as equipment failures and improvement of energy efficiency have drawn the attention of many researchers. These cover the gamut from minimizing peak demand through buffering of renewable resources [13] to the use of model predictive control for energy efficiency.
in building climate control [14]. The objective of our work is different since we focus on i) reduction of peak power demand by means of proper scheduling of background electrical loads keeping in mind practical considerations and ii) providing a feasibility criterion based on the requirement of thermal comfort-band maintenance.

Reduction in peak power demand using a global scheduler has been addressed in [15], [2] and [16]. Peak-power reduction by means of scheduling appliances using a combination of admission & curtailment control has been discussed in [15], but it does not discuss schedulability. A comprehensive study on power-usage in residential buildings has been discussed in [2] and Least Slack First (LSF) is proposed for background load scheduling. Lazy scheduling approach is proposed in [16] to control HVAC&R (heating, ventilation, air-conditioning & Refrigeration) devices for peak demand reduction, but it does not guarantee meeting the peak demand constraint. Modelling electrical loads, categorized under cyber-physical energy systems (CPES), as real-time tasks is discussed in [4] and [3] and application of traditional real-time scheduling algorithms like EDF are suggested. But, we have shown that they are not suitable for thermostatically controlled electrical devices (TCED), which may be considered as a special subclass of CPES. Work on global EDF (gEDF) scheduling [10] is also related as scheduling of background loads can be directly mapped into scheduling of \( m \) out of \( n \) real-time tasks running in parallel on \( m \) uniform multiprocessors. Similarly, value-based scheduling discussed in [11] and [12] is related as value can be attached dynamically to a TCED task, based on the environmental parameter feedback. But, we showed that none of the LSF, gEDF and value-based policies are suitable to our problem as is, as either they involve unnecessary and undesirable switching of TCE devices, or do not guarantee maintenance of temperature within the comfort-band.

VII. Conclusion

In this paper, based on our empirical study of the functioning of ACs and by applying heat transfer principles, we developed a conceptual model of power consumption and thermal comfort-band maintenance. From the insights gained from this study, a feasibility analysis technique was proposed for maintaining thermal comfort-band under peak power demand constraint. We showed how this analysis can be utilized for adaptive demand-response control of TCE devices under time varying peak power demand constraint. Driven by the goal of maintaining the comfort-band for as many devices as possible with minimal number of switching of power between appliances, we presented the TCBM approach for selecting the subset of appliances to power at a given point in time.

Our performance study demonstrates the superior performance characteristics of our algorithm compared to algorithms adapted from the literature. It showed that existing scheduling algorithms for timely task execution are not suitable for scheduling TCE devices because, either i) they do not prevent undesirable switching (preemption) of the devices even when the temperature of the environment under their control is within the associated comfort-band or ii) they do not guarantee maintenance of comfort-band. In contrast, our TCBM algorithm offers a minimum of 400\% less switching as compared to the candidate algorithms.

In this paper, ACs were chosen as a representative of the TCE devices. Generalizing this work, applying the lessons learned for other TCE devices and studying the thermal dependencies of zones controlled by multiple TCE devices are part of our ongoing work.

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REFERENCES