Description Logic

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Contents

• Motivation
• Introduction to Description Logic
• Knowledge Representation in DL
• Relationship with First-Order Predicate Logic
• Inferencing in DL
• Current Research
• References
Motivation

• Need to represent relationships between concepts.

• Apply reasoning to infer facts and relationships between concepts using the knowledge.

• Expedite inferencing procedure or provide tractability.
Introduction

• What are DLs?
  – Family of formal(logic-based) knowledge representation languages.

• Used to?
  – Represent the knowledge of an application domain (known as terminological knowledge) in a structured and formally well-understood way.
  – Model concepts, roles, individuals, and their relationships
  – Reason about the knowledge.
Example

• Suppose we want to define the concept of:
  – “A cricketer that is married to an actor and all whose sons are cricketers.”

• This concept can be described with the following concept description:
  – Cricketer \( \sqcap \exists \)married.Actor \( \sqcap (\forall \text{hasChild.}(\text{Cricketer} \sqcap \text{Male})) \).

• Concept descriptions like above can be used to build statements in a DL knowledge base.
Introduction contd...

- Fundamental building blocks of basic DL such as ALC (Attributive concept Language with Complements)
  - Individuals (Pataudi, Tagore, Khan)
  - Concepts (Human, Player, Cricketer, Actor)
  - Roles (married, parentOf, childOf)
Constructors of basic DL, ALC

<table>
<thead>
<tr>
<th>Constructor</th>
<th>Syntax</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atomic concept</td>
<td>A</td>
<td>Cricketer</td>
</tr>
<tr>
<td>Top, Bottom</td>
<td>T, ⊥</td>
<td></td>
</tr>
<tr>
<td>Atomic Role</td>
<td>R</td>
<td>PlaysFor</td>
</tr>
<tr>
<td>Conjunction</td>
<td>C∧D</td>
<td>Bowler ⊓ Batsman</td>
</tr>
<tr>
<td>Disjunction</td>
<td>C⊔D</td>
<td>Batsman ⊔ Bowler</td>
</tr>
<tr>
<td>Negation</td>
<td>¬C</td>
<td>¬Cricketer</td>
</tr>
<tr>
<td>Exists Restriction</td>
<td>∃R.C</td>
<td>∃PlaysFor.IPLTeam</td>
</tr>
<tr>
<td>Value Restriction</td>
<td>∀R.C</td>
<td>∀MatchBW.Winner</td>
</tr>
</tbody>
</table>
Knowledge Representation

• A DL knowledge base (KB) is made up of two parts, a terminological part (TBox) and an assertional part (ABox), each part consisting of a set of axioms.

• The TBox corresponds to rules in traditional systems.

• The ABox corresponds to facts in traditional systems.
Knowledge Base

- TBox
- ABox
- Knowledge Base
  - Rules
  - Facts
- Inference System
TBox

- TBox contains sentences describing concept hierarchies (i.e., relations between concepts). It contains
  - standalone concepts (C)
    - Player
  - complex concepts (C \(\sqcap\) D)
    - Bowler \(\sqcap\) Batsman
  - equivalence relations (C \(\equiv\) D)
    - Allrounder \(\equiv\) Bowler \(\sqcap\) Batsman
  - definitions (C \(\equiv\) D)
    - Allrounder \(\equiv\) Bowler \(\sqcap\) Batsman
  - General Concept Inclusion, GCI (C \(\sqsubseteq\) D)
    - Player \(\sqsubseteq\) \(\exists\text{married.Actor}\)
Diagrammatic Representation

- Player
  - Batsman
  - Bowler
  - All-Rounder

Is a relationship exists between all the positions.
ABox

- ABox contains ground sentences stating where in the hierarchy individuals belong (i.e., relations between individuals and concepts/roles)

- An assertional axiom is of the form $x : C$ or $(x, y) : R$, where $C$ is a concept, $R$ is a role and $x$ and $y$ are individual names.
  - Pataudi : Cricketer
  - Tagore : Actor
  - <Khan, Tagore> : childOf
  - <Pataudi, Khan> : parentOf
Relationship with First-Order Predicate Logic

- A basic Description Logic is a sub-set of first-order predicate logic.
- DLs are generally the decidable fragments of first-order predicate logic.[1]
- Tractability of inferencing is the main thrust behind the development of Description Logic.
- Description Logic has equivalent representations only for predicates with at most two variables.
Relationship with First-Order Predicate Logic

• Consider the following predicate logic statement:
  \[ \forall x (\text{opening-batsman}(x) \rightarrow \text{cricketer}(x)) \]

• The translation to English would be:
  For all x, if x is an opening batsman, then x is a cricketer.

• But do we generally use such arbitrary variables in regular conversation?

• In spoken English, this line would be
  Opening Batsmen are cricketers
Relationship with First-Order Predicate Logic

• First order predicate logic is over-expressive.[3]

• The variables are arbitrary and aren’t useful for inferencing.

• Description logics don’t allow the use of any variables, thereby expressing concepts in a concise manner without the loss of formality.

• The same statement can be expressed in DL as
  \[\text{OpeningBatsman} \sqsubseteq \text{Cricketer}\].
Relationship with First-Order Predicate Logic

• $x : C$ \quad C(x)
• $x : C \sqcap D$ \quad C(x) \land D(x)
• $x : C \sqcup D$ \quad C(x) \lor D(x)
• $x : \neg C$ \quad \neg C(x)
• $x : \exists R. C$ \quad \exists y (R(x,y) \land C(y))
• $x : \forall R. C$ \quad \forall y (R(x,y) \rightarrow C(y))
• $C \subseteq D$ \quad \forall x (C(x) \rightarrow D(x))
Inferencing in DL

• Inferencing in DL knowledge basically means using the TBox and Abox to arrive at implied assertions or implied concept subsumptions.

• For example:
  – Given in TBox
    • Cricketer ⊑ Man, Man ⊑ Human
  – We can infer from above
    • Cricketer ⊑ Human
Important Inference Problems

• Inferencing:
  – Consistency of a Knowledge Base
  – Satisfiability of a concept
  – Concept Subsumption
  – Concept Equivalence
  – Role or Concept assertions

• For Basic DLs providing all the Boolean operators like ALC, all above reasoning problems can be reduced to KB consistency.
  – \((TBox, ABox) \vDash a : C\)
    iff
    \((TBox, Abox \cup \{a : \neg C\})\) is inconsistent
Inferencing Technique—*Tableau method*[3]

• This method can be used for proving concept assertions or role assertions.

• In this method, the expressions in ABox are progressively decomposed until
  – No more completion rules can be applied
  – Inconsistency surfaces in the form \((x:A, x:\neg A)\)

• This may necessitate the use of concept definitions and axioms in the TBox.
Completion Rules

• AND-rule
  – If $x:C_1 \sqcap C_2$ exists in the KB, then
    • add $x:C_1$ and $x:C_2$ to the KB unless both are already in KB.

• OR-rule
  – If $x:C_1 \sqcup C_2$ exists in the KB, then
    • add $x:E$, where $E$ is $C_1$ or $E$ is $C_2$ (non determinism)
      If neither is in the KB.
Completion Rules

• FORALL-rule
  – If $x$: $\forall R.C$ and $<x,y>:R$ exist in KB, then
    • Add $y:C$
      unless $y:C$ is already in the KB

• EXISTSS-rule
  – If $x$: $\exists R.C$ exists in KB, then
    • add $y:C$ and $<x,y>:R$, where $y$ is a new variable
      provided, there is no $z$ such that both $<x,z>:R$ and $z:C$ 
      exist in KB
Completion Rules

• SUBSUMPTION-rule
  – If \( x : C_1 \) and \( C_1 \subseteq C_2 \) exist in KB, then
    • add \( x : C_2 \), by subsumption and inheritance
      provided \( x : C_2 \) not already in KB
Example

• Consider the following assertional axiom
  – Pataudi : Player \( \land \forall \text{parentOf.Actor} \)

• It states Pataudi is a player and all his children are actors.

• We have to check if Pataudi has a child who is not an actor. It can be represented as
  – Pataudi : \( \exists \text{parentOf.\neg Actor} \)
Example contd.

1. Pataudi : Player $\land \forall$parentOf.Actor
2. Pataudi : $\exists$parentOf.$\neg$Actor
3. Pataudi : Player (AND-rule on 1)
4. Pataudi : $\forall$parentOf.Actor (AND-rule on 1)
5. $<$Pataudi, x$>$ : parentOf (Exists-rule on 2)
6. x : $\neg$Actor (Exists-rule on 2)
7. x : Actor (FORALL-rule on 4 and 5)
Current Research[1]

• Adding other operators and extensions
  – For example: n-ary predicates, (reflexive-) transitive closure, etc

• Combining DLs with other KR formalisms

• Formal properties of DLs
  – Implementation and optimisation of DL systems

• Apart from satisfiability and subsumption, testing other reasoning problems
Conclusion

• DL is a family of KR languages that allow us to capture the relationships between concepts in a formal manner

• DLs are generally decidable fragments of FOL and designed to expedite the process of inferencing.
References


