CS623: Introduction to Computing with Neural Nets *(lecture-14)*

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Finding weights for Hopfield Net applied to TSP

- Alternate and more convenient $E_{problem}$
- $E_P = E_1 + E_2$

where

E_1 is the equation for n cities, each city in one position and each position with one city.

E_2 is the equation for distance

Expressions for E_1 and E_2

$$E_1 = \frac{A}{2}[\sum_{\alpha=1}^{n}(\sum_{i=1}^{n} x_{i\alpha} - 1)^2 + \sum_{i=1}^{n}(\sum_{\alpha=1}^{n} x_{i\alpha} - 1)^2]$$

$$E_2 = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{\alpha=1}^{n} d_{ij} \cdot x_{i\alpha} \cdot (x_{j,\alpha+1} + x_{j,\alpha-1})$$

Explanatory example

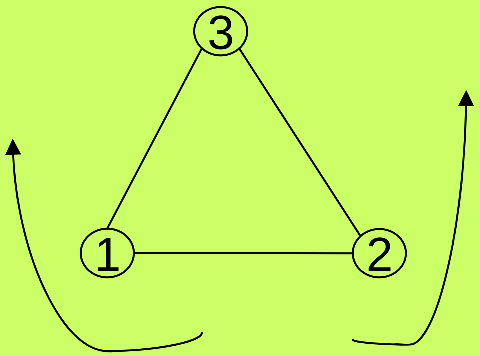

Fig. 1

Fig. 1 shows two possible directions in which tour can take place

pos →

city ↓

	1	2	3
1	x_{11}	x_{12}	x_{13}
2	x_{21}	x_{22}	x_{23}
3	x_{31}	x_{32}	x_{33}

For the matrix alongside, $x_{i\alpha} = 1$, if and only if, i^{th} city is in position α

Kinds of weights

- Row weights:

$w_{11,12}$	$w_{11,13}$	$w_{12,13}$
$w_{21,22}$	$w_{21,23}$	$w_{22,23}$
$w_{31,32}$	$w_{31,33}$	$w_{32,33}$

- Column weights

$w_{11,21}$	$w_{11,31}$	$w_{21,31}$
$w_{12,22}$	$w_{12,32}$	$w_{22,32}$
$w_{13,23}$	$w_{13,33}$	$w_{23,33}$

Cross weights

$w_{11,22}$	$w_{11,23}$	$w_{11,32}$	$w_{11,33}$
$w_{12,21}$	$w_{12,23}$	$w_{12,31}$	$w_{12,33}$
$w_{13,21}$	$w_{13,22}$	$w_{13,31}$	$w_{13,32}$
$w_{21,32}$	$w_{21,33}$	$w_{22,31}$	$w_{23,33}$
$w_{23,31}$	$w_{23,32}$		

Expressions

$$E_{problem} = E_1 + E_2$$

$$\begin{aligned} E_1 = \frac{A}{2}[& (x_{11} + x_{12} + x_{13} - 1)^2 \\ & + (x_{21} + x_{22} + x_{23} - 1)^2 \\ & + (x_{31} + x_{32} + x_{33} - 1)^2 \\ & + (x_{11} + x_{21} + x_{31} - 1)^2 \\ & + (x_{12} + x_{22} + x_{32} - 1)^2 \\ & + (x_{13} + x_{23} + x_{33} - 1)^2] \end{aligned}$$

Expressions (contd.)

$$E_2 = \frac{1}{2}[d_{12} x_{11}(x_{22} + x_{23}) +$$
$$d_{12} x_{12}(x_{23} + x_{21}) +$$
$$d_{12} x_{13}(x_{21} + x_{22}) +$$
$$d_{13} x_{11}(x_{32} + x_{33}) +$$
$$d_{13} x_{12}(x_{33} + x_{31}) +$$
$$d_{13} x_{13}(x_{31} + x_{32}) \ldots]$$

$E_{network}$

$$E_{network} = -[w_{11,12}x_{11}x_{12} + w_{11,13}x_{11}x_{13} + w_{12,13}x_{12}x_{13}$$
$$+ w_{11,21}x_{11}x_{21} + w_{11,22}x_{11}x_{22} + w_{11,23}x_{11}x_{23}$$
$$+ w_{11,31}x_{11}x_{31} + w_{11,32}x_{11}x_{32} + w_{11,33}x_{11}x_{33}\ldots]$$

Find row weight

- To find, $w_{11,12}$
  = -(co-efficient of $x_{11}x_{12}$) in E_{network}
- Search $x_{11}x_{12}$ in $E_{problem}$

$w_{11,12}$ = -A ...from E_1. E_2 cannot contribute

Find column weight

- To find, $w_{11,21}$

  $= -(\text{co-efficient of } x_{11}x_{21}) \text{ in } E_{network}$
- Search co-efficient of $x_{11}x_{21}$ in $E_{problem}$

$w_{11,21} = -A$ ...from E_1. E_2 cannot contribute

Find Cross weights

- To find, $w_{11,22}$
  $= -(\text{co-efficient of } x_{11}x_{22})$
- Search $x_{11}x_{22}$ from $E_{problem.}$ E_1 cannot contribute
- Co-eff. of $x_{11}x_{22}$ in E_2
  $(d_{12} + d_{21}) / 2$

Therefore, $w_{11,22} = -((d_{12} + d_{21}) / 2)$

Find Cross weights

- To find, $w_{11,33}$
  $= -(\text{co-efficient of } x_{11}x_{33})$
- Search for $x_{11}x_{33}$ in $E_{problem}$

  $w_{11,33} = -((d_{13} + d_{31}) / 2)$

Summary

- Row weights = $-A$
- Column weights = $-A$
- Cross weights = $-((d_{ij} + d_{ji}) / 2),\ j = i \pm 1$