

# Homework Exercise 4

Due on 11<sup>th</sup> October, 2009

1. Let  $h : \Re^k \rightarrow \Re$  and  $g : \Re^n \rightarrow \Re^k$ . Define  $f(x) = h(g(x))$ . Prove that  $f$  is convex if
  - $g_i$  is convex,  $h$  is convex and nondecreasing in each argument
  - or  $g_i$  is concave,  $h$  is convex and nonincreasing in each argument

(2 Marks)

2. Figure 1 shows 4 rectangular level sets (in blue) of a function  $f(x)$ , viz.,  $ABCD$ ,  $EFGH$ ,  $IJKL$  and  $MNOP$ . The function values (in red) corresponding to the four level sets are: 100, 90, 80 and 70 respectively. Can anything be said about the convexity or concavity of the function? Justify your answer.

(2 Marks)

3. Show that the function  $f : \Re_{++}^n \rightarrow \Re$  is convex for  $k < 1$  such that  $k \neq 0$ .

$$f(\mathbf{x}) = - \left( \sum_{i=1}^n x_i^k \right)^{\left(\frac{1}{k}\right)}$$

(2 Marks)

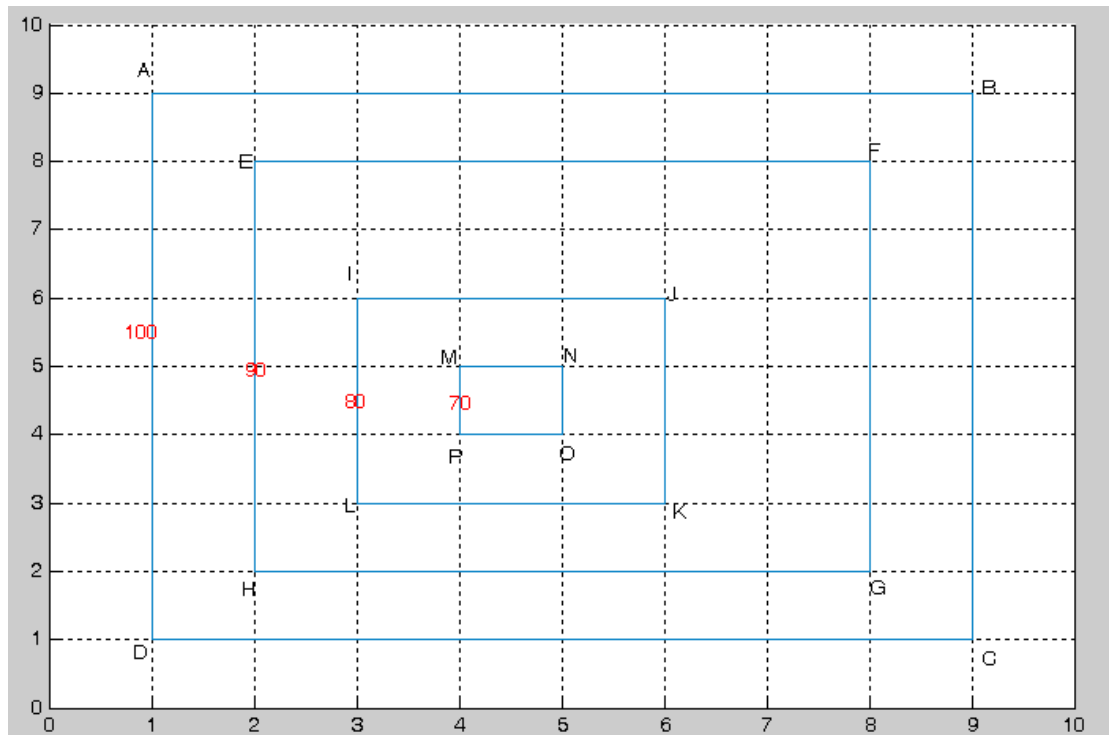


Figure 1: Level sets of a function  $f(x)$ .