Homework Exercise 5

Due on 24^{th} **October, 2009**

1. Consider the half space defined by $\mathcal{H} = \left\{\mathbf{x} \in \Re^n | \mathbf{a}^T\mathbf{x} + \alpha \geq 0\right\}$ where $a \in \Re^n$ and $\alpha \in \Re^n$ are given. Formulate and solve the optimization problem for finding the point $\mathbf{x}$ in $\mathcal{H}$ that has the smallest Euclidean norm.

   (**2 Marks**)

2. Let $\mathcal{I}(\mathbf{x}^*) = \{i_1, i_2, \ldots, i_m\}$ be the active index set at $\mathbf{x}^*$ for the constraints g_i's in the primal problem discussed in class (I guess equation (4.85) in the notes, but please confirm). Show that the set

   $$\mathcal{S} = \left\{\mathbf{s} \mid \mathbf{s}^T\nabla f(\mathbf{x}^*) < 0, \mathbf{s}^T\nabla h_j(\mathbf{x}^*) = 0 \ for\ j = 1 \ldots k,\ and\ \mathbf{s}^T\nabla g_i(\mathbf{x}^*) \leq 0 \ for\ i \in \mathcal{I}(\mathbf{x}^*)\right\}$$

   is empty if and only if there exist multipliers λ_j^* for $1 \leq j \leq k$ and $\mu_j^* \geq 0$, such that

   $$\nabla f(\mathbf{x}^*) = \sum_{j=1}^{k} \lambda_j^* \nabla h_j(\mathbf{x}^*) - \sum_{i \in \mathcal{I}(\mathbf{x}^*)} \mu_i^* \nabla g_i(\mathbf{x}^*)$$

   This lemma is known as the Extension of Farkas lemma.

   (**4 Marks**)