Assignment 1

August 4, 2009

1. What happens if, in spite of all exchanges, elimination still results in a 0 in any one of the pivot positions? Then we consider the process to have failed, and the equations do not have a solution. EXPLAIN WHY.
2. Now, for square matrices, it is easy to see that the left and right inverses are the same: PROVE.
3. Therefore, if the inverse exists, then elimination must produce an upper triangular matrix with non-zero pivots. In fact, the condition works both ways? If elimination produces non-zero pivots then the inverse exists. Otherwise, the matrix is not invertible, or singular. PROVE.
4. Prove: A matrix will be invertible if its "determinant "is 0. Can the condition be hardened to "if and only if "? Why ?(1 mark)
5. Prove that if there are solutions other than x = 0 to Ax = 0, then the matrix A is singular.(1 mark)
6. PROVE: "A matrix is singular if the columns (or rows) are not linearly independent."
7. If A is full row rank and n > m, then AA^T is a full rank m × m matrix. PROVE.
8. Prove: A matrix will be singular if its columns or rows are not linearly independent. Can the condition be hardened to "if and only if ? "Why ?
9. Justify the following statement: "In essence, what elimination does is change the matrix A and consequently its column space, while leaving the null space of A intact ".

10. Find the inverse of the following matrix (1 mark)

$$A = \begin{pmatrix} 13 & 7 & 5 & 18 \\ 6 & 23 & 17 & 19 \\ 23 & 19 & 29 & 11 \\ 31 & 23 & 11 & 3 \end{pmatrix}$$

11. Solve the system Ax = b where (1 mark)

$$A = \begin{pmatrix} 5 & 4 & 6 & 2 \\ 4 & 6 & 1 & 7 \\ 3 & 5 & 2 & 5 \\ 2 & 3 & 4 & 7 \end{pmatrix}$$

$$b = \begin{pmatrix} 5 \\ 4 \\ 3 \\ 2 \end{pmatrix}$$

12. Prove: The right null space $N(A^T A)$ is the same as $N(A)$.(1 mark)

13. Prove: The eigenvectors $v_1, v_2, \dots, v_n$ of a matrix A are linearly independent if all its eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_n$ are different(1 mark)

14. Prove the equivalence of the Pythagoras condition and the orthogonality condition(1 mark)

    This assignment will carry 7 marks