Homework Exercise 2

Due on 24th August, 2009

1. Classify the following matrices as positive definite, positive semidefinite, indefinite[1], *etc.*, while providing justification and outlining all the steps:

   (a)

$$A = \begin{bmatrix} 5 & 3 & 1 \\ 3 & 4 & 2 \\ 1 & 2 & 6 \end{bmatrix}$$

   (**1.5 Marks**)

   (b)

$$A = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & 0 \\ 4 & 0 & 18 \end{bmatrix}$$

   (**1.5 Marks**)

2. (a) Solve the following minimization problem by using a graphical method to get a precise numerical solution. You could use programming if required.

$$\begin{array}{ll} \text{minimize} & f(\mathbf{x}) = \frac{1}{4}(x_1 - 6)^2 + (x_2 - 4)^2 \\ \text{subject to} & g_1(\mathbf{x}) = \frac{80}{7} - x_2 - \frac{8}{7}x_1 \geq 0, \\ & g_2(\mathbf{x}) = x_2 \geq 0, \\ & h_1(\mathbf{x}) = x_1 - 3 = 0, \\ \text{variable } \mathbf{x} = (x_1, x_2) & \end{array} \tag{1}$$

   (**2 Marks**)

   (b) Indicate the feasible region.
   (**1 Marks**)

[1]Note that a matrix A is indefinite if neither A nor $-A$ is positive semi-definite.