Homework Exercise 6

Due on 8^{th} November, 2009

1. Consider the objective function

$$f(\mathbf{x}) = (x_1 + 10x_2)^2 + 5(x_3 - x_4)^2 + (x_2 - 2x_3)^4 + 100(x_1 - x_4)^4$$

   (a) Assume throughout that, for the algorithm in Figure 1, $\rho = 0.1$, $\sigma = 0.1$, $\tau = 0.1$ and $\xi = 0.75$. Is the line seach in Figure 1, (i) exact (ii) approximate (iii) inexact using Wolfe conditions (iv) inexact using Goldstein conditions or (v) none of these? Reason it out.

   (b) Solve the problem using the steepest-descent method with stopping criterion $||\alpha_k \mathbf{d}^k|| < \epsilon$ where $\epsilon = 10^{-6}$, using the line search in Figure 1. Report using both initial points $[-2 \quad -1 \quad 1 \quad 2]^T$ and $[200 \quad -200 \quad 100 \quad -100]^T$.

   (c) Solve the problem using the modified Newton method:

$$\widehat{H}^k = \frac{\nabla^2 f(\mathbf{x}^k) + \beta I}{1 + \beta}$$

   where

$$\beta = \begin{cases} 0 & \text{if} \quad \lambda_{min}\left(\nabla^2 f(\mathbf{x}^k)\right) > 0 \\ 0.25 - \lambda_{min}\left(\nabla^2 f(\mathbf{x}^k)\right) & \text{if} \quad \lambda_{min}\left(\nabla^2 f(\mathbf{x}^k)\right) \leq 0 \end{cases} \quad (1)$$

   with the same termination tolerance and initial points as in (b). You should use the line search in Figure 1 where needed.

   (d) Solve the problem using the Gauss-Newton method with the same termination tolerance and initial points as in (b). You should use the line search in Figure 1 where needed.

   (e) Based on the results of (b)-(d), compare the computational efficiency and solution accuracy of the three methods.

(8 Marks)

Step 1
Input $\mathbf{x}^k$, $\mathbf{d}^k$.
Initialize algorithm parameters ρ, σ, τ, and ξ.
Set $\alpha_L = 0$ and $\alpha_U = 10^{99}$.
Step 2
Compute $f_L = f(\mathbf{x}^k + \alpha_L \mathbf{d}^k)$.
Compute $f_L' = \nabla^T f(\mathbf{x}^k + \alpha_L \mathbf{d}^k)\mathbf{d}^k$.
Step 3
Estimate α_0 by exact line search on the quadratic approximation for $g(\alpha) = f(\mathbf{x}^k + \alpha \mathbf{d}^k)$, as was discussed in class.
Step 4
Compute $f_0 = f(\mathbf{x}^k + \alpha_0 \mathbf{d}^k)$.
Step 5 (Interpolation)
if $f_0 > f_L + \rho(\alpha_0 - \alpha_L) f_L^{'}$ **then**
If $\alpha_0 < \alpha_U$, then set $\alpha_U = \alpha_0$.
$\widehat{\alpha}_0 = \alpha_L + \frac{(\alpha_0 - \alpha_L)^2 f_L^{'}}{2[f_L - f_0 + (\alpha_0 - \alpha_L) f_L^{'}]}$.
If $\widehat{\alpha}_0 < \alpha_L + \tau(\alpha_U - \alpha_L)$ then set $\widehat{\alpha}_0 = \alpha_L + \tau(\alpha_U - \alpha_L)$.
If $\widehat{\alpha}_0 > \alpha_U - \tau(\alpha_U - \alpha_L)$ then set $\widehat{\alpha}_0 = \alpha_U - \tau(\alpha_U - \alpha_L)$.
Set $\alpha_0 = \widehat{\alpha}_0$ and go to **Step 4**.
end if
Step 6
Compute $f_0^{'} = \nabla^T f(\mathbf{x}^k + \alpha_0 \mathbf{d}^k)\mathbf{d}^k$.
Step 7 (Extrapolation)
if $f_0^{'} < \sigma f_L^{'}$ **then**
Compute $\Delta\alpha_0 = \frac{(\alpha_0 - \alpha_L) f_0^{'}}{(f_L^{'} - f_0^{'})}$.
If $\Delta\alpha_0 < \tau(\alpha_0 - \alpha_L)$, then set $\Delta\alpha_0 = \tau(\alpha_0 - \alpha_L)$.
If $\Delta\alpha_0 > \xi(\alpha_0 - \alpha_L)$, then set $\Delta\alpha_0 = \xi(\alpha_0 - \alpha_L)$.
Compute $\widehat{\alpha}_0 = \alpha_0 + \Delta\alpha_0$.
Set $\alpha_L = \alpha_0$, $\alpha_0 = \widehat{\alpha}_0$, $f_L = f_0$, $f_L^{'} = f_0^{'}$, and go to **Step 4**.
end if
Step 8
Output α_0 and $f_0 = f(\mathbf{x}^k + \alpha_0 \mathbf{d}^k)$, and stop.

Figure 1: Line search.