

CS 717 Statistical Relational Learning

January 27, 2010

A depth-first search inspired algorithm for listing all hypotheses satisfying a quality criterion

Assumptions

The hypotheses form a finite lattice

The quality criterion is polynomially computable [Sanity check : No point developing a polynomial algorithm if the check takes exponential time]

In the example discussed in class, the quality criterion was anti-monotonic. We develop the algorithm for the anti-monotonic case [the only place this is used is in the optimizations, not the essence of the search algorithm].

We shall assume the Downward Refinement operator as defined in class :

$$\rho : S \longrightarrow 2^S$$

such that for all \mathbf{h}' in $\rho(\mathbf{h})$, $\mathbf{h}' \leq \mathbf{h}$

Note : S is the set of all hypotheses

We define $\rho(\mathbf{h})$ to be the “unvisited” elements of the complete set of downward closures of \mathbf{h}

[If this does not make sense, we can explicitly enforce this by maintaining “visited” information with each node in the lattice, and verifying that a particular node is not “visited” before visiting it.]

The Algorithm

Initialize

stack S = Empty Stack

set Answer = \emptyset

S.push(\emptyset) *// Not to be confused with the Answer set. This is the **T** of the Hypothesis Lattice*

While (S is not empty) :

$h' = S.pop()$

if (h' qualifies) :

Answer = Answer $\cup \{h'\}$

set Children = $\rho(h')$

for each element $\{h''\}$ of Children :

S.push(h'')

else :

// Trying to optimize here. Very very hand-wavy

set All_Descendents = $\rho(h')$

for each element $\{h''\}$ of All_Descendents :

[Set node h'' as visited]

All_Descendents = All_Descendents $\cup \rho(h'')$

return Answer

A hill-climbing search inspired algorithm for listing at most one hypothesis satisfying a quality criterion

Assumptions as in DFS algorithm

We shall assume the Upward Refinement operator as defined in class :

$$\rho : S \longrightarrow 2^S$$

such that for all h' in $\rho(h)$, $h \leq h'$

We define $\rho(h)$ to be the complete set of upward closures of h

We also define a heuristic function “utility”, that takes a hypothesis and outputs a real number (score).

$$\mu : S \longrightarrow \mathbb{R}$$

We shall “climb a hill” assuming the utility μ gives the slope. We shall be greedy about it.

The Algorithm

Initialize

$h = \Sigma$ // h is the current hypothesis being examined

 // Σ is the $_I_$ of the hypothesis lattice

While (not h qualifies AND not $h == \phi$) :

set Parents = $\rho(h)$

$h = \operatorname{argmax}_{h'' \text{ in Parents}} \mu(h'')$

return h // What if T of hypothesis lattice does not qualify? We shall interpret ϕ as unable

 // to find any qualifying hypothesis. ϕ is a degenerate case anyway