Incorporating Background knowledge

Plotkin's relative subsumption $\succeq_B$

C, D = clauses. B = set of clauses

$C \succeq_B D$ if $\exists\, \theta$ s.t. $B \vDash \underbrace{(C\theta \rightarrow D)}_{\text{may not be a clause}}$

Byldn $\vDash$ $B \cup \{C\theta\} \vDash D$

pure subsumtion

$C \succeq D$ if $\exists\, \theta$ s.t. $C\theta \subseteq D$

For prev ex: $B_1 \succeq_{(B_1, H_B)} D_2$

$S(x) \leftarrow C(x)$

$P(x) \leftarrow C(x)$

$C\,P(x) \leftarrow S(x), F(x), P(x)$

$C\,P(x) \leftarrow C(x), F(x)$

Properties of $\succeq_B$

B is fixed

① Reflexive & transitive $\Rightarrow$ Induces a quasi order & $\therefore$ also a partial order

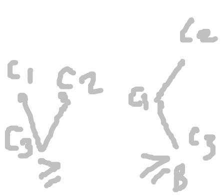
Each B induces its own p.o

$\{q(a)\}$ $\quad$ $\{q(a), p(b)\}$

$\therefore$ Diff p.o!

② Strictly stronger than subsumption (connects more clauses)

ⓐ ie if $C \succeq D$, the $C \succeq_B D$ for any B



$\exists\ \theta$ s.t $C\theta \subseteq D \Rightarrow C\theta \models D \Rightarrow C\theta \to D$ is a tautology. By dedn thm

$\therefore$ For any B, $B \models (C\theta \to D)$

(b) But Not vice versa: $C \succeq_B D \not\Rightarrow C \succeq_\theta D$

① P, Q, R be props

$C = P \qquad D = Q \qquad B = \{Q \leftarrow P\}$ } propositions ~ preds with 0 arity

Then $B \models (C \to D)$?

② More general example

$B = \phi, \quad D \equiv \top$ (tautology), & Given any C

then $\forall C, \exists \theta = \epsilon$ st $B \cup \{C\theta\} \models D$

But does $C \succeq_\theta D$?

$Q \qquad P \leftarrow P$

③ If C & D are non-tautologous clauses & B is a finite set of ground literals with $B \cap D = \phi$, then

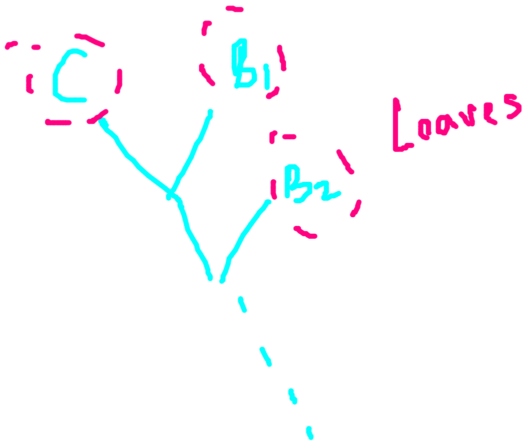
$C \succeq_B D$ iff $C \succeq_\theta (D \cup B)$

LS Given

5 Procedural viewpoint

$C \succcurlyeq_B D$ iff $\{C\} \cup B \vdash_{Res} D$

C occurs at most once as a leaf.



Verify: If $C = Q(x) \leftarrow P(x)$
$D = Q(a)$
$B = \{P(a)\}$

Think of resolution tree as an inverted tree

⑥ LGG does not exist always for $\geq_B$.

$C_1 = Q(x) \leftarrow P(x, f(x))$

B not needed; $\not\geq_B$

$C_2 : Q(x) \leftarrow P(x, f(x)), P(x, f^2(x))$

$\geq$

⋮

$C_i : Q(x) \leftarrow P(x, f(x)), P(x, f^2(x)) \ldots P(x, f^i(x))$

$D(x)$

LGRS of D_1, D_2 does not exist

$D_1 = Q(a)$ $\quad D_2 = Q(b)$

$B = \{P(a, y), P(b, y)\}$

$\forall i,\quad C_i \geq_B D_1 \quad C_i \geq_B D_2$

(3) We can restrict $\mathcal{B}$ to guarantee existence & computability of LGRS

(1) Language can be horn

(2) $\mathcal{B} \subseteq \mathcal{C}$ is a finite set of ground literals

$\Downarrow$

$\forall\ S \subseteq C \quad (S \subseteq \mathcal{H})$

LGRS(S) exists.

Proof:-

If (i) D is a tautology or $B \cap D = \phi$

$B \models D$?

We can find an int I making every literal in B true (model) & a var assignment so that D is true

For this simplified case, $C \underset{B}{\succ} D \quad \forall C$

$(\because B \not\models (D \Leftarrow C))$

Remove from S → all tautologies; → all D s.t. $D \cap B = \phi$ } S'

If $S' = \{\ \}$, any tautology is an LGRS of S

Proof: (only when B is ground)

If $B \cap D = \phi$ then $C \succeq_B D$ iff $C \succeq (D \vee \bar{B})$

Rsub Sub.

$\Rightarrow$ Suppose $C \succeq_B D \dashrightarrow B \models C\theta \to D$ for some θ.

Suppose $C\theta \not\subseteq D \cup \bar{B}$. Then $\exists$ $L \in C\theta$ s.t. $L \notin D$ & $L \notin \bar{B}$

Since $B \cap D = \phi$, $\exists$ I s.t. I makes every lit in B true & var assignment s.t. L ($\because C\theta$) is true under I & U

while no lit in D is true $\Rightarrow$ $C\theta \to D$ is false under I! $\therefore B \not\models C\theta \to D \Rightarrow C\theta \subseteq D \vee \bar{B}$

$\Leftarrow$ Now, if $C \geq (D \cup \bar{B}) \Rightarrow \exists \theta$ s.t

$C\theta \subseteq D \cup \bar{B}$

Let M be a model of B & V be a var assignment s.t $C\theta$ is true under M & V. ($\Rightarrow$ every lit in $\bar{B}$ is false under M)

$L \in D \cup \bar{B}$

To prove:- D is also true under M & V.

Now atleast one $L \in C\theta$ is true under M&V.

$\Rightarrow L \in D \Rightarrow D$ is true under M&V.

$\underbrace{B}_{M} \models (\underbrace{\theta}_{V,M} \rightarrow D)$

Let $S' = \{D_1 \ldots D_n\}$

By construction, each D_i is neither a tautology

Nor has $D_i \cap B = \phi$

We are now interested in the least element C s.t $C \succeq_B D_i \;\; \forall i$

$\Updownarrow$

By "sublemma"

We are interested in the least element C s.t $C \subseteq \bar{B} \vee D_i \;\; \forall i$ } see what you have got

$$LGRS(S) = LGRS(S') = LGS\{(\bar{B} \vee D_1), (\bar{B} \vee D_2), \ldots (\bar{B} \vee D_i)\}$$

Q: If S is only horn clauses?

If B = set of ground Atoms - - - -

then each $\bar{B} \vee D_i$ is horn if D_i is horn

$\Downarrow$

$LGRS(S)$ = horn = $LGS\{(\bar{B}_1 \vee D_1), (\bar{B} \vee D_2) \ldots\}$

Golem - - - uses this idea.

(8) Since lsub is strictly stronger than sub, non existence of finite chains of covers carries over.

Relative subsumption & ILP.

We are interested in theory/hypothesis H.

$H \succeq_B e$

$$
\begin{array}{ll}
e: & gfather(henry, john) \leftarrow \\
B: & father(henry, jane) \leftarrow \\
 & father(henry, joe) \leftarrow \\
 & parent(jane, john) \leftarrow \\
 & parent(joe, robert) \leftarrow \\
C: & gfather(X, Y) \leftarrow father(X, Z), parent(Z, Y)
\end{array}
$$

But $C \not\succeq e$

For this B, C, e with $\theta = \{X/henry, Y/john,\ Z/jane\}$, $B \cup \{C\theta\} \models e$

$\equiv C \succeq_B e \quad \equiv \quad B \models (e \leftarrow C\theta).$

$\equiv B \cup \{\bar{e}\} \models \overline{C\theta} \equiv C \subseteq \bar{B} \vee e$

Since B is only ground lits

Let $a_1 \wedge a_2 \ldots \wedge a_m$ be ground literals true in all models of $\left(\overline{\overline{B} \vee e}\right) = (B \wedge \bar{e})$

ie $B \wedge \bar{e} \models a_1 \wedge a_1 \ldots \wedge a_m$

$\Rightarrow a_i$, for $i = 1$ to m $\in MM(G(B \wedge \bar{e}))$

Let $\perp(B,e) = \overline{MM(G(B \wedge \bar{e}))}$

$\overline{a_1 \wedge a_2 \ldots \wedge a_m} = \perp(B,e) \models \overline{B} \vee e$

Note $C \succeq \overline{B} \vee e$

If $C \succeq \perp(B,e)$