INPUT: A clause C.
OUTPUT: A reduction D of C.
Set $D = C$, $\theta =$;
repeat
  Set D to $D\theta$;
  Find a literal $\mathbf{l} \in D$ and a substitution θ such that $D\theta \subseteq D \setminus \{\mathbf{l}\}$;
until Such a $(\mathbf{l}, \theta)$ does not exist;
return D.

Plotkin's reduction algorithm

Theorem 26 *Let C be a clause. If for some θ, $C\theta \subseteq C$, then there is a reduced clause $D \subseteq C\theta$ such that $C \equiv D$.*

Proof: $C_1 = C\theta \Rightarrow C\theta = C_1 \models C$ & $C \models C\theta = C_1$

$\therefore C \equiv C\theta = C_1$

If C_1 is reduced, $D = C_1$

If not, $\underbrace{,}_{\text{by defn of reduced clause}}$ $\exists\, \theta_1$ s.t. $C_2 = C_1\theta_1 \subset C_1 \Rightarrow C_2 \equiv C$

... can go on ... since C has finite # lits this will converge to reduced clause

Lattice structure of clauses "$\mathcal{L}$"

Every finite set 'S' of clauses (normal / Horn)

has a

↳ GLB ... GSS

↳ $GSS(S) = \bigcup_{c \in S} c$ } for general clauses

↳ for horn clauses?

↳ LUB .... LGG

→ $GSS(\{f(x), g(y), \neg h(a)\}, \{h(z), \neg f(b)\})$

$\perp = ?$ for clauses

$\perp = ?$

GSS(S) .. trickier

GSS of horn clauses $S \subseteq \mathcal{H}^{+}$.

$\mathcal{H}^{+} = \mathcal{H} \cup \{\perp\}$

Theorem 27 *Let $\mathcal{H}$ be the Horn language $\mathcal{H}$, with an additional bottom element $\perp \in \mathcal{H}$. Then for every finite non-empty $\mathcal{S} \subseteq \mathcal{H}$, there exists a GSS (glb) of $\mathcal{S}$ in $\mathcal{H}$.*

↳ Assume clauses standardized apart

$S = \{\underbrace{D_1 \ldots D_k}_{\text{Definite programs}}, \underbrace{D_{k+1} \ldots D_n}_{\text{Definite goals}}\}$

$D_1^+ \ldots D_k^+$ are not unifiable ... $\perp$
else let σ be an mgu

Union is GSS
$gss(S) = \bigcup_{D_i \in S} D_i\sigma$

To show that D is a GSS of S in $\mathcal{H}$, suppose $C \in \mathcal{H}$ is some clause such that $D_i \succeq C$ for every $1 \leq i \leq n$. For every $1 \leq i \leq n$, let θ_i be such that $D_i\theta_i \subseteq C$, and θ_i only acts on variables in D_i. Let $\theta = \theta_1 \cup \ldots \cup \theta_n$. For every $1 \leq i \leq k$, $D_i^+\theta = D_i^+\theta_i = C^+$, so ~~$theta$~~ θ is a unifier for $\{D_1^+, \ldots, D_k^+\}$. But σ is an mgu for this set, so there is a γ such that $\theta = \sigma\gamma$. Now $D\gamma = D_1\sigma\gamma \cup \ldots \cup D_n\sigma\gamma = D_1\theta \cup \ldots \cup D_n\theta = D_1\theta_1 \cup \ldots \cup D_n\theta_n \subseteq C$. Hence $D \succeq C$, so D is a GSS of S in $\mathcal{H}$. See Figure 2.2 for illustration of the case where $n = 2$.

For example, $D = P(a) \leftarrow P(f(a)), Q(y)$ is a GSS of $D_1 : P(x) \leftarrow P(f(x))$ and $D_2 : P(a) \leftarrow Q(y)$. Note that D can be obtained by applying $\sigma = \{x/a\}$ (the mgu for the heads of D_1 and D_2) to $D_1 \cup D_2$, the GSS of D_1 and D_2 in C.