(i) Assume that before resolution, factoring has been performed on clauses

} Factor of unificable literals $L_i$'s in $C$ is $L_i\theta$ where $\theta$ is their mgu.

Factors: $C_1 \models$ its factor

But not complete

The rule of resolution remains $\boxed{\text{sound}}$ for clauses in the predicate logic. That is, if $C_1$ and $C_2$ are clauses and $R$ is a resolvent, then $\{C_1, C_2\} \models R$. The presence of variables and substitutions makes the proof of this a little more involved.

**Theorem 19** *Suppose $R$ is the result of resolving on literal $L$ in $C_1$ and $M$ in $C_2$. Let $\theta$ be the most general unifier of $L$ and $\neg M$ that is used to obtain $R$. Then, the soundness of a single step of resolution means $\{C_1, C_2\} \models (C_1 - \{L\})\theta \cup (C_2 - \{M\})\theta$.*

*Proof:* Let $\bar{M}$ be a model for $C_1$ and $C_2$. Now, we know that either (a) $L\theta$ is true and $M\theta$ is false in $\bar{M}$; or (b) $L\theta$ is false and $M\theta$ is true in $\bar{M}$. Suppose the former. Since $\bar{M}$ is a model for $C_2$, it is a model for $C_2\theta$ (based on theorem 18). Therefore, at least one other literal $(C_2 - \{M\})\theta$ must be true in $\bar{M}$. In other words, $\bar{M}$ is a model for $(C_1 - \{L\})\theta \cup (C_2 - \{M\})\theta$. Case (b) similarly results in $\bar{M}$ being a model for $(C_1 - \{L\})\theta$ and hence for $R$. So, a single resolution step is sound - the soundness of a proof consisting of several resolutions steps can be shown quite easily using the technique of induction. □

Also Refutation complete

SUBSUMPTION

defn.

Clause $C$ subsumes $D$ iff $C\theta \subseteq D$ for some $\theta$.

[In prop logic :- $C$ subsumes $D$ iff $C \subseteq D$]
$\Rightarrow C \models D$

Q: If $C\theta \subseteq D$ does $C \models D$?

$\triangleleft$ - converse does not hold as for prop

Trivially :- $C \models C\theta$

$C\theta \subseteq D \Rightarrow C \models D$ (Since $C$ is univ. quant)

$f(x) \vee g(y)$ $\quad$ $f(x) \vee g(y) \vee p(z)$

If $\Sigma \models D$ then $\exists$ $C$ s.t. $\Sigma \vdash_R C$ & $C \subseteq D$

subsumption thm

Here are a pair of clauses $C$ and $D$ such that $C$ subsumes $D$:

$$C : Primate(x) \leftarrow Ape(x)$$

$$D : Primate(Henry) \leftarrow Ape(Henry), Human(Henry)$$

$R \to C$ in $D$

$\theta = ?$

$$C : Human(x) \leftarrow Human(father(x))$$

$$D : Human(y) \leftarrow Human(father(father(y)))$$

- $\top \succeq \mathbf{l}$ for all $\mathbf{l} \in \mathcal{A}^+$
- $\mathbf{l} \succeq \perp$ for all $\mathbf{l} \in \mathcal{A}^+$
- $\mathbf{l} \succeq m$ iff there is a substitution $\theta$ such that $\mathbf{l}\theta = m$, for $\mathbf{l}, \mathbf{m} \in \mathcal{A}$

We will represent a list of elements $e_1, \ldots, e_n$ as the(as the language Prol does) by $[e_1, \ldots, e_n]$, and let $\mathbf{l} = Mem(x, [x, y])$ and $\mathbf{m} = Mem(1, [1, 2])$ th $\mathbf{l} \succeq \mathbf{m}$ with $\theta = \{x/1, y/2\}$. It is easy to see that $\succeq$ is a quasi-order over $\mathcal{A}$ clearly $\mathbf{l} \succeq \mathbf{l}$, with the empty substitution $\theta = \emptyset$ (that is, $\succeq$ is reflexive). No let $\mathbf{l} \succeq \mathbf{m}$ and $\mathbf{m} \succeq \mathbf{l}$. That is, there are some substitutions $\theta_1$ and $\theta_2$ such th $\mathbf{l}\theta_1 = \mathbf{m}$ and $\mathbf{m}\theta_2 = \mathbf{l}$. That is, $(\mathbf{l}\theta_1) \circ \theta_2 = n$. With $\theta = \theta_1 \circ \theta_2$ it follows th $\mathbf{l} \succeq \mathbf{l}$.