

INPUT: A clause C .

OUTPUT: A reduction D of C .

Set $D = C$, $\theta =$;

repeat

Set D to $D\theta$;

Find a literal $l \in D$ and a substitution θ such that $D\theta \subseteq D \setminus \{l\}$;

until Such a (l, θ) does not exist;

return D .

Plotkin's reduction algorithm

Theorem 26 Let C be a clause. If for some θ , $C\theta \subseteq C$, then there is a reduced clause $D \subseteq C\theta$ such that $C \equiv D$.

Proof: $C_1 = C\theta \Rightarrow C\theta = C_1 \subseteq C$ & $C \subseteq C\theta = C_1$
 $\therefore C \equiv C\theta = C_1$

if C_1 is reduced, $D = C_1$

if not, $\exists \theta$, s.t. $C_2 = C_1\theta \subset C_1 \Rightarrow C_2 \equiv C$

by def'n of reduced clause

... can go on ... Since C has finite # lits this will converge to reduced clause

Lattice structure of clauses "L"

Every finite set 'S' of clauses (normal / horn)

has a

$\hookrightarrow$ GLB $\dots$ GSS

$\hookrightarrow GSS(S) = \bigcup_{C \in S} C$ } for general clauses

$\hookrightarrow$ for horn clauses?

$\hookrightarrow$ LUB $\dots$ LGG

$\rightarrow GSS(\{f(x), g(y), \neg h(a)\}, \{h(z), \neg f(b)\})$

$\perp = ?$ for clauses

$\perp = ?$

$GSS(S) \dots$ trickier

$\leftarrow$

GSS of horn clauses $S \subseteq \mathcal{H}^+$.

Theorem 27 Let $\mathcal{H}$ be the Horn language $\mathcal{H}$, with an additional bottom element $\perp \in \mathcal{H}$. Then for every finite non-empty $S \subseteq \mathcal{H}$, there exists a GSS (glb) of S in $\mathcal{H}$.

↳ Assume clauses standardized apart

$$S = \{ \underbrace{D_1 \dots D_k}_{\text{Definite programs}}, \underbrace{D_{k+1} \dots D_n}_{\text{Definite goals}} \}$$

Definite programs

Definite goals



$D_1^t \dots D_k^t$ are not unifiable

Union is GSS

 else let σ be an mgu

$$gss(S) = \bigcup_{D_i \in S} D_i \sigma$$

To show that D is a GSS of S in $\mathcal{H}$, suppose $C \in \mathcal{H}$ is some clause such that $D_i \succeq C$ for every $1 \leq i \leq n$. For every $1 \leq i \leq n$, let θ_i be such that $D_i\theta_i \subseteq C$, and θ_i only acts on variables in D_i . Let $\theta = \theta_1 \cup \dots \cup \theta_n$. For every $1 \leq i \leq k$, $D_i^+\theta = D_i^+\theta_i = C^+$, so θ is a unifier for $\{D_1^+, \dots, D_k^+\}$. But σ is an mgu for this set, so there is a γ such that $\theta = \sigma\gamma$. Now $D\gamma = D_1\sigma\gamma \cup \dots \cup D_n\sigma\gamma = D_1\theta \cup \dots \cup D_n\theta = D_1\theta_1 \cup \dots \cup D_n\theta_n \subseteq C$. Hence $D \succeq C$, so D is a GSS of S in $\mathcal{H}$. See Figure 2.2 for illustration of the case where $n = 2$.

For example, $D = P(a) \leftarrow P(f(a)), Q(y)$ is a GSS of $D_1 : P(x) \leftarrow P(f(x))$ and $D_2 : P(a) \leftarrow Q(y)$. Note that D can be obtained by applying $\sigma = \{x/a\}$ (the mgu for the heads of D_1 and D_2) to $D_1 \cup D_2$, the GSS of D_1 and D_2 in C .

