Incorporating Background knowledge

Plotkin's relative subsumption $\succeq_B$

C, D = clauses. B = set of clauses

may not be a clause

$C \succeq_B D$ if $\exists\, \theta$ s.t. $B \vdash (\underbrace{C\theta \to D})$

By dedn $\models$ $B \cup \{C\theta\} \vdash D$

pure subsumption

$C \succeq D$ if $\exists\, \theta$ s.t. $C\theta \subseteq D$

For prev ex: $B_3 \succeq D_2$ (B_1, H_B)

$S(x) \leftarrow C(x)$

$P(x) \leftarrow C(x)$

$CP(x) \leftarrow S(x), F(x), P(x)$

$CP(x) \leftarrow C(x), F(x)$

Properties of $\succeq_B$

① Reflexive & transitive $\Rightarrow$ Induces a quasi order & $\therefore$ also a partial order

B is fixed

Each B induces its own p.o

$\{p(a)\}$ $\{p(a), p(b)\}$

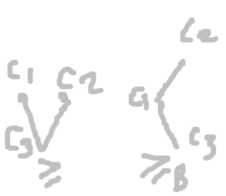
... Diff p.o's!

② Strictly stronger than subsumption (connects more clauses)

ⓐ ie if $C \succeq D$, the $C \succeq_B D$ for any B

$\exists\, \theta$ s.t $C\theta \subseteq D \Rightarrow C\theta \models D \Rightarrow C\theta \rightarrow D$ is a tautology. By deduction thm

$\therefore$ For any B, $B \models (C\theta \rightarrow D)$



(b) But Not vice versa: $C \succeq_B D \not\Rightarrow C \succeq_{\theta} D$

(1) P, Q, R be props

$C = P \qquad D = Q \qquad B = \{Q \leftarrow P\}$ } propositions ~ preds with 0 arity

Then $B \models (C \to D)$?

(2) More general example

$B = \phi$, $D \equiv T$ (tautology), & Given any C

then $\forall C, \exists \theta = \epsilon$ st $B \cup \{C\theta\} \models D$

But does $C \succeq D$?

$Q \qquad P \leftarrow P$

(3) If C & D are non-tautologous clauses & B is a finite set of ground literals with $B \cap D = \phi$, then

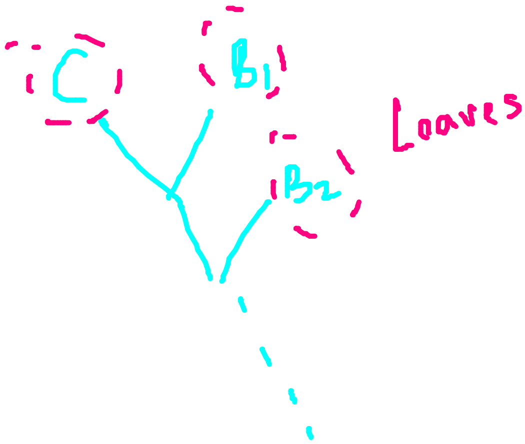
$C \succeq_B D$ iff $C \succeq (D \cup B)$

LG Given

5 Procedural viewpoint

$C \geq_B D$ iff $\{C\} \cup B \vdash_{Res} D$

C occurs at most once as a leaf.



Verify: If $C = Q(x) \leftarrow P(x)$
$D = Q(a)$
$B = \{P(a)\}$

Think of resolution tree as an inverted tree

(6) LGG does not exist always for $\geq_B$.

$C_1 = Q(x) \leftarrow P(x, f(x))$

B not needed

$B\checkmark$

$C_2 : Q(x) \leftarrow P(x, f(x)), P(x, f^2(x))$

$\geq$

$\vdots$

$C_i : Q(x) \leftarrow P(x, f(x)), P(x, f^2(x)) \ldots P(x, f^i(x))$

$D(x)$

LGRS of D_1, D_2 does not exist

$D_1 : Q(a)$ $\quad$ $D_2 : Q(b)$

$B = \{ P(a, y), P(b, y) \}$

$\forall i;\ C_i \geq_B D_1 \quad C_i \geq_B D_2$

3. We can restrict $\mathcal{B}$ to guarantee existence & computability of LGRS

1. Language can be horn
2. $\mathcal{B} \subseteq \mathcal{C}$ is a finite set of ground literals

$$\Downarrow$$

$\forall\ S \subseteq C \quad (S \subseteq \mathcal{H})$

LGRS(S) exists.

Proof:-

If (i) D is a tautology or $B \cap D = \emptyset$

$B \models D$?

We can find an int I making every literal in B true (model) & a var assignment so that D is true

For this simplified case, $C \succ_B D \quad \forall C$

$(\because B \not\models (D \leftarrow C))$

Remove from S → all tautologies
→ all D s.t. $D \cap B = \emptyset$ } S'

If $S' = \{ \}$, any tautology is an LGRS of S

Proof: (only when B is ground)

If $B \cap D = \phi$ then $C \succeq_B D$ iff $C \succeq (D \vee \bar{B})$

RSub Sub.

$\Rightarrow$ Suppose $C \succeq_B D$ ---> $B \models C\theta \rightarrow D$ for some θ.

Suppose $C\theta \not\subseteq D \cup \bar{B}$. Then $\exists$ $L \in C\theta$

s.t $L \notin D$ & $L \notin \bar{B}$

Since $B \cap D = \phi$, $\exists$ I s.t I makes every lit in B true & var assignment s.t L ($\because C\theta$) is true under I & U

while no lit in D is true $\Rightarrow$ $C\theta \rightarrow D$ is false under I ! $\therefore$ $B \not\models C\theta \rightarrow D$ $\Rightarrow$ $C\theta \subseteq D \vee \bar{B}$

$\Leftarrow$ Now, if $C \geq (D \cup \bar{B}) \Rightarrow \exists \theta$ s.t

$C\theta \subseteq D \cup \bar{B}$

Let M be a model of B & V be a var assignment s.t $C\theta$ is true under M & V. ($\Rightarrow$ every lit in $\bar{B}$ is false under M)

$L \in D \cup \bar{B}$

To prove: D is also true under M & V.

Now atleast one $L \in C\theta$ is true under M & V.

$\Rightarrow L \in D \Rightarrow D$ is true

$\underbrace{B}_{M} \models (\underbrace{\theta}_{V \cup M} \to D)$

Let $S' = \{D_1 \ldots D_n\}$

By construction, each D_i is neither a tautology Nor has $D_i \cap B = \phi$

We are now interested in the least element C s.t $C \geq_B D_i \;\; \forall i$

$\Updownarrow$

By "sublemma"

We are interested in the least element C s.t $C \subseteq \bar{B} \vee D_i \;\; \forall i$ } see what you have got

$$LGRS(S) = LGRS(S') = LGS\{(\bar{B} \vee D_1), (\bar{B} \vee D_2), \ldots (\bar{B} \vee D_i)\}$$

Q: If S is only horn clauses?

If B = set of ground Atoms ....

then each $\bar{B} \vee D_i$ is horn if D_i is horn

$\Downarrow$

$$LGRS(S) = \text{horn} = LGS\{(\bar{B}_1 \vee D_1), (\bar{B} \vee D_2) \ldots\}$$

Golem ... uses this idea.

(8) Since Ssub is strictly stronger than Sub, non existence of finite chains of covers carries over.

Relative subsumption & ILP.

We are interested in theory/hypothesis H.

$$H \succeq_B e$$

$$
\begin{array}{lr}
e: & gfather(henry, john) \leftarrow \\
B: & father(henry, jane) \leftarrow \\
 & father(henry, joe) \leftarrow \\
 & parent(jane, john) \leftarrow \\
 & parent(joe, robert) \leftarrow \\
C: & gfather(X,Y) \leftarrow father(X,Z), parent(Z,Y)
\end{array}
$$

But $C \not\succeq e$

For this B, C, e with $\theta = \{X/henry, Y/john,\ Z/jane\}$, $B \cup \{C\theta\} \models e$

Putting together many eqn stmts (for B's gnd lits)

$\equiv C \succeq_B e \equiv B \models (e \leftarrow C\theta). \equiv B \models \overline{C\theta} \leftarrow \bar{e}$

deduction

$\equiv \underbrace{B \cup \{\bar{e}\}}_{B \wedge \bar{e}} \models \overline{C}\theta \equiv C \subseteq \overline{B} \vee e$

Since B is only ground lits

Let $a_1 \wedge a_2 \ldots \wedge a_m$ be ground literals true in all models of $(\overline{B \vee e}) = (B \wedge \bar{e})$

ie $B \wedge \bar{e} \models a_1 \wedge a_2 \ldots \wedge a_m$

$\Rightarrow a_i$, for $i = 1$ to m $\in MM(G(B \wedge \bar{e}))$ } By a prev thm (T_2)

Let $\bot(B, e) = \overline{MM(G(B \wedge \bar{e}))}$

by negating both sides

$\overline{a_1 \wedge a_2 \ldots \wedge a_m} = \bot(B, e) \models \bar{B} \vee e$

note $C \succeq \bar{B} \vee e$

If $C \succeq \bot(B, e)$

$\Downarrow$

$C \models \bot(B, e) \Rightarrow C \models \bar{B} \vee e$

C

$\bot(B, e)$

$\bar{B} \vee e$

Reason for "Bottom clause"

In some sense, model lattice is inversion of implication order

Σ = defn

$\{$compl. set$\}$

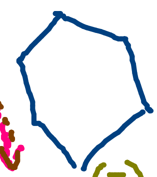
$\Sigma = B \wedge \overline{E}$

set of models

$MM(\Sigma)$

Model lattice

models



C

C'

$M(C)$
$\cap$
$M(C')$

$\overline{MM}(\Sigma) = \bot(B, e)$

Implication order

An example of finding $\perp$ is:

Goal: To come up with D_i s.t $D_i \succeq_B e_i$

Recipe: Find lowest D_i s.t $D_i \succeq \perp(B,e_i) \models \overline{B} \vee e_i$

$\mathcal{B}$:

gfather(X,Y) $\leftarrow$ father(X,Z), parent(Z,Y)
father(henry,jane) $\leftarrow$
mother(jane,john) $\leftarrow$
mother(jane,alice) $\leftarrow$

e_i:

gfather(henry,john) $\leftarrow$

Conjunction of ground atoms provable from $B \cup \overline{e_i}$:

a_1 ¬parent(jane,john) $\wedge$
a_2 father(henry,jane) $\wedge$
a_3 mother(jane,john) $\wedge$
a_4 mother(jane,alice) $\wedge$
a_5 ¬gfather(henry,john)

Minimal model of $\mathcal{MM}(\Sigma_i)$

$\Sigma_i = \{B\}_i \cup \{\overline{e}_i\}$

$B \cup \{\overline{e}_i\} = B \wedge \overline{e}_i \models a_1 \wedge a_2 \ldots \wedge a_n$

$\perp(B, e_i)$:

gfather(henry,john) $\vee$ parent(jane,john) $\leftarrow$
father(henry,jane),
mother(jane,john),
mother(jane,alice)

$\perp(B,e_i) \models \overline{B} \vee e_i$

D_i:

parent(X,Y) $\leftarrow$ mother(X,Y)

Verify: $D_i \succeq \perp(B,e_i)$

① Dropping lit ② Add a var

Mode declarations

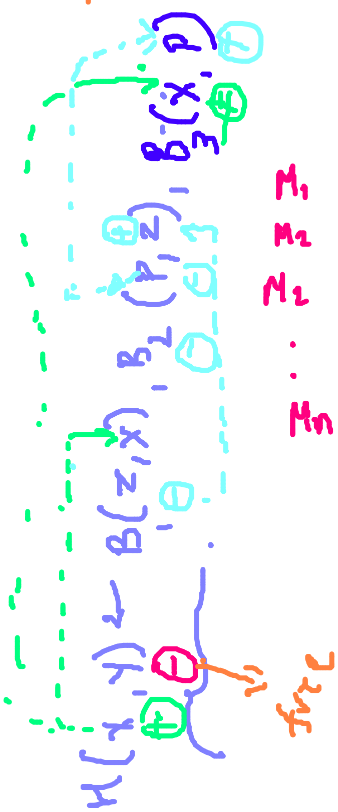
$\mathcal{D}_i$

$\vee$

$\bot(B, e_i)$

Problem: $\bot(B, e_i)$ may be infinite!

Mode decl: Constrained subset of definite clauses to construct finite, most specific clauses



M_1 modeh(*,gfather(+person,-person))

M_2 modeh(*,parent(+person,-person))

M_2 modeb(*,father(+person,-person))

: modeb(*,parent(+person,-person))

M_n modeb(*,mother(+person,-person))

$L(M)$

clause C is in $L(M)$ iff : C: $h \leftarrow b_1 \ldots b_n$

① h is the atom of a modeh with "+" & "-" replaced by vars & "#" replaced by gnd terms

② $\forall b_i$, b_i must be in modeb in $L(M)$ with "+", "-" & "#" replaced as in ①

③ Every var of "+" in b_i is either of type "+" in h or "-" in a b_j for $1 \leq j < i$ ordering

An addition framework for restricting language of $\mathcal{L}$

capture "distance" of var from head

Definition 3 *Let C be a definite clause, v be a variable in an atom in C, and U_v all other variables in body atoms of C that contain v*

$$d(v) = \begin{cases} 0 & \textit{if } v \textit{ in head of } C \\ (\max_{u \in U_v} d(u)) + 1 & \textit{otherwise} \end{cases}$$

For example, if $C : gfather(X, Y) \leftarrow father(X, Z), parent(Z, Y)$

Then $d(X) = d(Y) = 0$, $d(Z) = 1$. Putting together the definitions of mode language and depth, we next define depth bounded definite mode language.

$U_X := \{Z\}$

$U_Y : \{Z\}$

$U_Z := \{X, Y\}$

$d(Z) = \max\{v \in U_Z\} d(v) + 1 = 1$

Definition 4 *Let C be a definite clause with an ordering over literals. Let M be a set of mode declarations. C is in the depth-bounded definite mode language $\mathcal{L}_d(M)$ iff all variables in C have depth at most d*

To minimize dependence & avoid free vars in body

1. Mode defn language brings in variables
   But vars can introduce long chains
2. So depth boundedness

Verify that D :: $paren(X, Y) \leftarrow mother(X, Y)$ is in $L_2(M)$

Properties:

① $\exists$ a $\bot_d(B, e_i)$ s.t $\bot_d(B, e_i) \succeq \bot(B, e_i)$

& $\bot_d(B, e_i)$ is finite

② If $C \succeq \bot_d(B, e_i)$ then $C \succeq \bot(B, e_i)$

③ $\bot(B, e_i)$ may not be Horn.

④ $\bot(B, e_i)$ may not be finite in general

⑤ $rlgg_B(e_i, e_j) = lgg(\bot(B, e_i), \bot(B, e_j))$

$\perp(B,\, e_i)$:

> gfather(henry,john) $\vee$ parent(jane,john) $\leftarrow$
>
> father(henry,jane),
>
> mother(jane,john),
>
> mother(jane,alice)

modes:

> *modeh(*,parent(+person,-person))*
>
> *modeb(*,mother(+person,-person))*
>
> *modeb(*,father(+person,-person))*

$\perp_0(B, e_i)$**:**

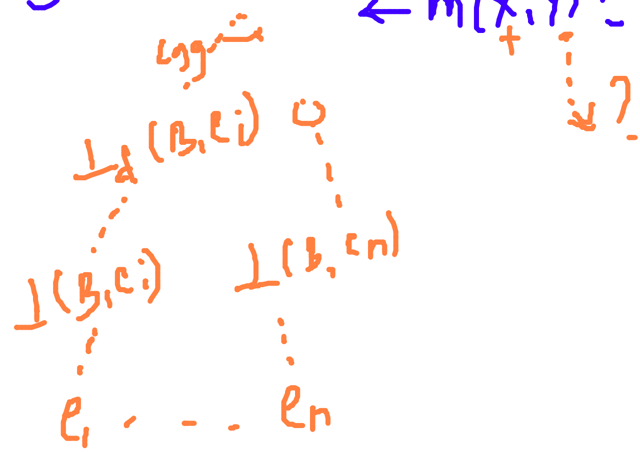
> parent(X,Y) $\leftarrow$

$\perp_1(B, e_i)$**:**

> parent(X,Y) $\leftarrow$
>
> mother(X,Y),
>
> mother(X,Z)

~ verify.

Q: Why $\perp_0(B,e_i) \neq p(\overset{+}{X},\overset{-}{Y}) \leftarrow m(\overset{+}{X},\overset{-}{Y})$?



Relative implication ($\models_B$)

C, D = clauses
B = bkgd.

We say $C \models_B D$ if

$C \cup B \models D$ ← difference?

Recall: "$C \geq_B D$ if $\exists\, \theta$ st $B \cup \{C\theta\} \models D$"

Diff.

Rel. imp. $\Rightarrow$ Rel sub (More flexibility)

$$\begin{array}{ccc} C & & B \\ & \searrow \; \swarrow \; \text{Res.} & \\ & \vdots & \\ & D & \end{array} \qquad\qquad \begin{array}{ccc} C & & \\ \theta \downarrow & & \\ C\theta & & B \\ & \searrow \; \swarrow & \\ & \vdots & \end{array}$$

Eg: $B = \{p(a)\}$, $C = P(f(x)) \leftarrow P(x)$, $D = P(f^2(a))$

Then $C \models_B D$