Incorporating Background knowledge

Plotkin's relative subsumption $\succeq_B$

C, D = clauses.   B = set of clauses

Note: $\forall$ does not distribute over $\vee$

$C \succeq_B D$ if $\exists \theta$ s.t. $B \vdash \forall(\underbrace{(C\theta}_{\text{\& may not be a clause}} \to D)$

C & D may have vars in common

pure subsumption

$C \succeq D$ if $\exists \theta$ s.t. $C\theta \subseteq D$

For prev ex: $B_3 \succeq_{(B_1, H_B)} D_2$

$C\,P(x) \leftarrow S(x), F(x), P(x)$

$C\,P(x) \leftarrow C(x), F(x)$

$S(x) \leftarrow C(x)$

$P(x) \leftarrow C(x)$

Note that in general:

Position of $\forall$ is imp: Deduction thm holds for wff

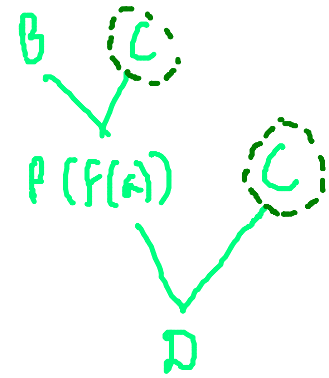
$$B \models \forall(C \to D) \not\equiv \underbrace{\{C\} \cup B \models D}_{\text{Relative implication}} \equiv B \models \forall(C) \to \forall(D)$$

$$\Rightarrow B \models \forall(C \to D)$$

Eg: If $B = \{P(a)\}$, $C = P(f(x)) \leftarrow P(x)$

$D = P(f^2(a))$

Then, $\{C\} \cup B \models D$ (ie $C \models_B D$)



But $C \not\models_B D$ since C has to be used $>$ once for derivation of D.

Recall deduction thm. $\forall \alpha, \beta$ wff & Σ a set of wff, $\Sigma \cup \{\alpha\} \models \beta \equiv \Sigma \models [\beta \leftarrow \alpha]$

Properties of $\succeq_B$

B is fixed

① Reflexive & transitive $\Rightarrow$ Induces a quasi order & ∴ also a partial order

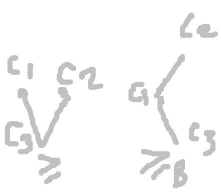
Each B induces its own p.o

$\{q(a)\}$ $\{q(a), p(b)\}$

∴ Diff p.o!

② Strictly stronger than subsumption (connects more clauses)

ⓐ ie if $C \succeq D$, then $C \succeq_B D$ for any B



$\exists \theta$ s.t. $C\theta \subseteq D \Rightarrow \forall(C\theta \models D) \Rightarrow \forall(C\theta \rightarrow D)$ is a tautology.

By ded'n thm

∴ For any B, $B \models \forall(C\theta \rightarrow D)$

(b) But Not vice versa: $C \succeq_B D \not\Rightarrow C \succeq_\theta D$

(1) P, Q, R be props } propositions ~ preds with 0 arity

$C = P \quad D = Q \quad B = \{Q \leftarrow P\}$

Then $B \models (C \rightarrow D)$?

(2) More general example

$B = \phi, \quad D \equiv T$ (tautology), & Given any C

then $\forall C, \exists \theta = \epsilon$ s.t. $B \models \forall (C\theta \rightarrow D)$

But does $C \succeq_\theta D$?

$Q \qquad P \leftarrow P$

(3) If C & D are non-tautologous clauses & B is a finite set of ground literals with $B \cap D = \phi$, then

$C \succeq_B D$ iff $C \succeq_\theta (D \cup \bar{B})$

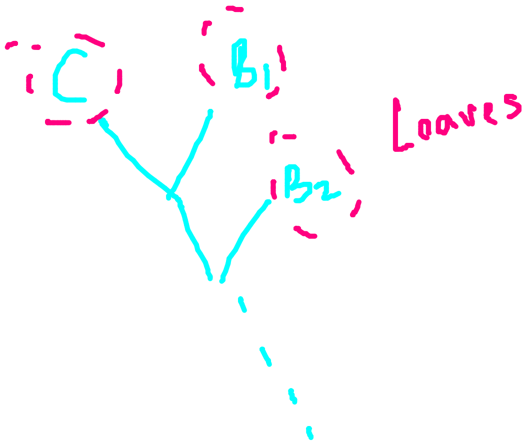
LG Gwen

5 Procedural viewpoint

$C \succcurlyeq_B D$ iff $B \models \forall(C \to D)$ iff $\{C\} \cup B \vdash_{Res} D$

C occurs at most once as a leaf.

Verify: If $C = Q(x) \leftarrow P(x)$
$D = Q(a)$
$B = \{P(a)\}$



Think of resolution tree as an inverted tree

⑥ LGG does not exist always for $\succeq_B$.

$C_1 = Q(x) \leftarrow P(x, f(x))$

B not needed ; B $\vee$

$C_2 = Q(x) \leftarrow P(x, f(x)), P(x, f^2(x))$

$\vee$

$\vdots$

$C_i = Q(x) \leftarrow P(x, f(x)), P(x, f^2(x)) \ldots P(x, f^i(x))$

$D\ \boxed{x}$

LG&S of D_1, D_2 does not exist

$D_1 = Q(a)$ $\qquad$ $D_2 = Q(b)$

$B = \{ P(a, y), P(b, y) \}$

$\forall i, \quad C_i \succeq_B D_1 \qquad C_i \succeq_B D_2$

(3) We can restrict $\mathcal{B}$ to guarantee existence & computability of LGRS

(1) Language can be horn

(2) $\mathcal{B} \subseteq \mathcal{C}$ is a finite set of ground literals

$\Downarrow$

$\forall\ S \subseteq C \quad (S \subseteq \mathcal{H})$

LGRS(S) exists.

Proof:-

If (i) D is a tautology or $B \cap D = \emptyset$

$B \models D$?

We can find an int I making every literal in B true (model) & a var assignment so that D is true

For this simplified case, $C \succ_B D \quad \forall C$

$(\because B \not\models \forall(D \leftarrow C))$

Remove from S → all tautologies; → all D s.t. $D \cap B = \emptyset$ $\Big\}$ S'

If $S' = \{\ \}$, any tautology is an LGS of S

Proof: (only when B is ground)

If $B \cap D = \phi$ then $C \succeq_B D$ iff $C \succeq (D \vee \bar{B})$

rsub sub.

$\Rightarrow$ Suppose $C \succeq_B D$ $\dashrightarrow$ $B \models \forall(C\theta \rightarrow D)$ for some θ.

Suppose $C\theta \not\subseteq D \cup \bar{B}$. Then $\exists\ L \in C\theta$

s.t. $L \notin D$ & $L \notin \bar{B}$

Since $B \cap D = \phi$, $\exists\ I$ s.t. I makes every lit in B true & var assignment

s.t. $L\ (\because C\theta)$ is true under I & V

while no lit in D is true $\Rightarrow C\theta \rightarrow D$ is false under I ! $\therefore B \not\models \forall(C\theta \rightarrow D) \Rightarrow C\theta \subseteq D \vee \bar{B}$

$\Leftarrow$ Now if $C \geq (D \cup \bar{B}) \Rightarrow \exists \theta$ s.t

$C\theta \subseteq D \cup \bar{B}$

Let M be a model of B & V be a var assignment s.t $C\theta$ is true under M & V. ($\Rightarrow$ every lit in $\bar{B}$ is false under M)

$L \in D \cup \bar{B}$

To prove:- D is also true under M & V.

Now atleast one $L \in C\theta$ is true under M∪V.

$\Rightarrow L \in D \Rightarrow D$ is true under M & V.

$B \models (\theta \rightarrow D)$

$\underbrace{B}_{M}$ $\underbrace{\theta}_{V \cup M}$

Let $S' = \{D_1 \ldots D_n\}$

By construction, each D_i is neither a tautology Nor has $D_i \cap B = \phi$

We are now interested in the least element C s.t $C \geq_B D_i \ \forall i$

$\Updownarrow$

By "sublemma"

We are interested in the least element C s.t $C \subseteq \bar{B} \vee D_i \ \forall i$ } see what you have got

$$LGRS(S) = LGRS(S') = LGS\{(\bar{B} \vee D_1), (\bar{B} \vee D_2), \ldots (\bar{B} \vee D_i)\}$$

Q: If S is only horn clauses?

If B = set of ground Atoms ....

then each $\bar{B} \vee D_i$ is horn if D_i is horn

$$\Downarrow$$

$$LGRS(S) = \text{horn} = LGS\left\{ (\bar{B}_1 \vee D_1),\ (\bar{B} \vee D_2) \ \ldots \right\}$$

Golem ... uses this idea.

8 Since Rsub is strictly stronger than Sub, non existence of finite chains of covers carries over.

Relative subsumption & ILP. : Let us further assume ground e;s

We are interested in theory/hypothesis $\mathcal{H}$.

$$\mathcal{H} \succeq_B e$$

$$
\begin{array}{ll}
e: & gfather(henry, john) \leftarrow \\
B: & father(henry, jane) \leftarrow \\
 & father(henry, joe) \leftarrow \\
 & parent(jane, john) \leftarrow \\
 & parent(joe, robert) \leftarrow \\
C: & gfather(X,Y) \leftarrow father(X,Z), parent(Z,Y)
\end{array}
$$

But

$c \not\succeq e$

For this B, C, e with $\theta = \{X/henry, Y/john, Z/jane\}$, $B \cup \{C\theta\} \models e$

Putting together many eqn stmts (for B's grnd lits)

$\equiv C \succeq_B e \equiv B \models (e \leftarrow C\theta) . \equiv B \models \overline{C\theta} \leftarrow \overline{e}$

$\equiv B \cup \{\bar{e}\} \models \overline{C\theta} \equiv C \subseteq \overline{B} \vee e$

$B \wedge \bar{e}$

Since B is only ground lits

Let $a_1 \wedge a_2 \ldots \wedge a_m$ be ground literals true in all models of $\left(\overline{\overline{B} \vee e}\right) = (B \wedge \overline{e})$

ie $B \wedge \bar{e} \models a_1 \wedge a_2 \ldots \wedge a_m$

$\Rightarrow a_i$, for $i = 1$ to m $\in MM(G(B \wedge \bar{e}))$ } By a prev thm (I_Σ)

Let $\bot(B,e) = \overline{MM(G(B \wedge \bar{e}))}$

by negating both sides

$\overline{a_1 \wedge a_2 \ldots \wedge a_m} = \bot(B,e) \models \overline{B} \vee e$

Note $C \succeq \overline{B} \vee e$

If $C \succeq \bot(B,e)$

$\Downarrow$

$C \models \bot(B,e) \Rightarrow C \models \overline{B} \vee e$

C

$|$

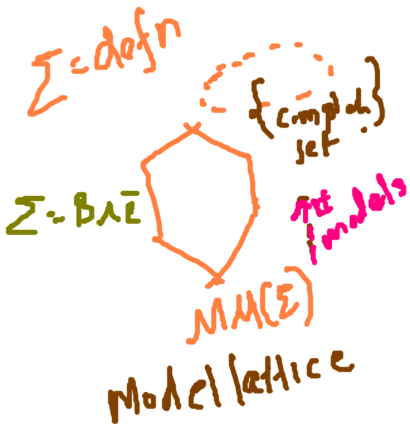
$\bot(B,e)$

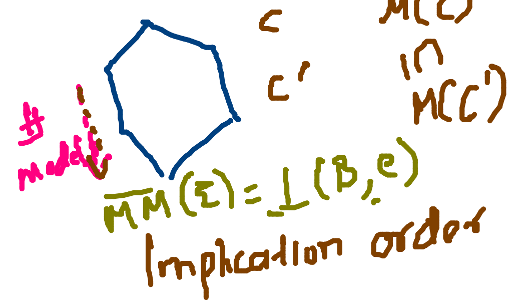
$|$

$\overline{B} \vee e$

Reason for "Bottom clause"

In some sense, model lattice is inversion of implication order





An example of finding $\perp$ is:

Goal: To come up with D_i s.t $D_i \geq_B e_i$

Recipe: Find lowest D_i s.t $D_i \geq \perp(B,e_i) \models \bar{B} \vee e_i$

$\mathcal{B}$:

gfather(X,Y) ← father(X,Z), parent(Z,Y)
father(henry,jane) ←
mother(jane,john) ←
mother(jane,alice) ←

e_i:

gfather(henry,john) ←

Conjunction of ground atoms provable from $B \cup \overline{e_i}$:

a_1 ¬parent(jane,john) ∧
a_2 father(henry,jane) ∧
a_3 mother(jane,john) ∧
a_4 mother(jane,alice) ∧
a_5 ¬gfather(henry,john)

Minimal model of $\Sigma_i = \{B\}_i \cup \{\bar{e}_i\}$

$\mathcal{M}\mathcal{M}(\Sigma_i)$

$B \cup \{\bar{e}_i\} = B \wedge \bar{e}_i \models a_1 \wedge a_2 \ldots \wedge a_n$

$\perp(B, e_i)$:

gfather(henry,john) ∨ parent(jane,john) ←
father(henry,jane),
mother(jane,john),
mother(jane,alice)

$\perp(B,e_i) \models \bar{B} \vee e_i$

D_i:

parent(X,Y) ← mother(X,Y)

Verify: $D_i \geq \perp(B,e_i)$

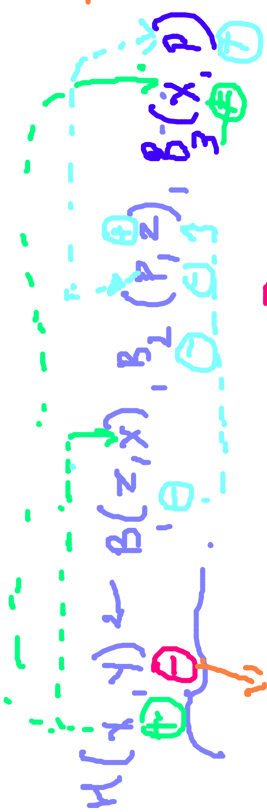
① Dropping lit ② Add a var

Mode declarations

D_i
$\sqsupseteq$
$\bot(B, e_i)$

Problem: $\bot(B, e_i)$ may be infinite!

Mode decl: Constrained subset of definite clauses to construct finite, most specific clauses



$gf(X,Y) \leftarrow f(X,Z),\ p(Z,Y)$

bnd free

M_1	modeh(*,gfather(+person,-person))
M_2	modeh(*,parent(+person,-person))
M_2	modeb(*,father(+person,-person))
:	modeb(*,parent(+person,-person))
M_n	modeb(*,mother(+person,-person))

$L(M)$

free

clause C is in $L(M)$
iff : C: $h \leftarrow b_1 \ldots b_n$

① h is the atom of a modeh with "+" & "-" replaced by vars & "#" replaced by gnd terms

② $\forall b_i$, b_i must be in modeb in $L(M)$ with "+", "-" & "#" replaced as in ①

③ Every var of "+" in b_i is either of type "+" in h or "-" in a b_j for $1 \le j < i$ (ordering)

An addition framework for restricting language of $\mathcal{L}$

captures "distance" of var from head

Definition 3 *Let C be a definite clause, v be a variable in an atom in C, and U_v all other variables in body atoms of C that contain v*

$$d(v) = \begin{cases} 0 & \text{if } v \text{ in head of } C \\ (\max_{u \in U_v} d(u)) + 1 & \text{otherwise} \end{cases}$$

For example, if $C : gfather(X, Y) \leftarrow father(X, Z), parent(Z, Y)$

Then $d(X) = d(Y) = 0$, $d(Z) = 1$. Putting together the definitions of mode language and depth, we next define depth bounded definite mode language.

$U_X := \{Z\}$
$U_Y : \{Z\}$
$U_Z := \{X, Y\}$

$d(Z) = \max\{v \in U_Z\} d(v) + 1 = 1$

Definition 4 *Let C be a definite clause with an ordering over literals. Let M be a set of mode declarations. C is in the depth-bounded definite mode language $\mathcal{L}_d(M)$ iff all variables in C have depth at most d*

To minimize dependence & avoid free vars in body

① Mode defn language brings in variables
But vars can introduce long chains

② So depth boundedness

Verify that D :: $parent(X, Y) \leftarrow mother(X, Y)$
is in $\mathcal{L}_2(M)$

Properties:

① $\exists$ a $\bot_d(B, e_i)$ s.t $\bot_d(B, e_i) \succeq \bot(B, e_i)$

& $\bot_d(B, e_i)$ is finite

② If $C \succeq \bot_d(B, e_i)$ then $C \succeq \bot(B, e_i)$

③ $\bot(B, e_i)$ may not be Horn.

④ $\bot(B, e_i)$ may not be finite in general

⑤ $rlgg_B(e_i, e_j) = lgg(\bot(B, e_i), \bot(B, e_j))$

$\perp(B,\, e_i)$:

gfather(henry,john) $\vee$ parent(jane,john) $\leftarrow$
    father(henry,jane),
    mother(jane,john),
    mother(jane,alice)

modes:

modeh(,parent(+person,-person))*
modeb(,mother(+person,-person))*
modeb(,father(+person,-person))*

$\perp_0(B, e_i)$:

parent(X,Y) $\leftarrow$

$\perp_1(B, e_i)$:

parent(X,Y) $\leftarrow$
    mother(X,Y),
    mother(X,Z)

verify.

Q: Why $\perp_0(B,e_i) \nvdash p(X^+,Y^-) \leftarrow m(X^+,Y^-)$?

lgg ?

$\perp_d(B,e_i)$?

$\perp(B,e_i)$ $\perp(B,e_n)$

e_1 . . . e_n

Relative implication ($\models_B$)

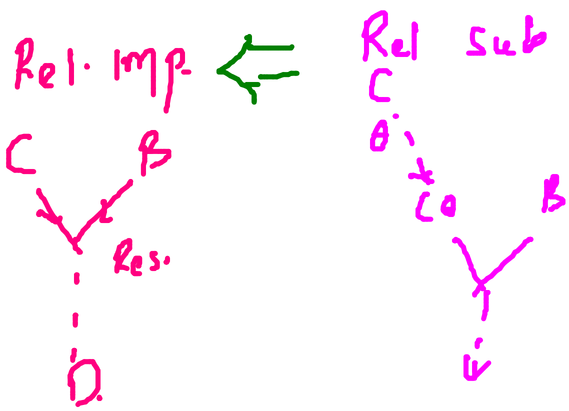
C, D = clauses
B = bkgd.

We say $C \models_B D$ if

$C \cup B \models D$ ← difference?

"Recall: $C \geq_B D$ if
$\exists \theta$ st $B \cup \{C\theta\} \models D$"

Diff.



Eg: $B = \{p(a)\}$, $C = P(f(x)) \leftarrow P(x)$ $\quad D = P(f^2(a))$

Then $C \models_B D$, But $C \not\geq_B D$

Claim: If $C \succcurlyeq_{\mathcal{B}} D$ then $C \models_{\mathcal{B}} D$ $(\{C\} \cup \mathcal{B} \models D)$

If $C \succcurlyeq_{\mathcal{B}} D$ then $\mathcal{B} \models \forall (C\theta \to D)$ for some θ

We want to show that if

$\mathcal{B} \models \forall (C\theta \to D)$ then $\{C\} \cup \mathcal{B} \models D$

ie $\exists E$ s.t

$\{C\} \cup \mathcal{B} \vdash E$ &

$E \succcurlyeq D$.

Since $C \succcurlyeq_{\mathcal{B}} D \Rightarrow \exists$ a derivation s.t C appears at most once as a leaf $\Rightarrow \exists$ a derivation of D from $\mathcal{B} \cup \{C\}$

Relative Subs.

$\forall(C\theta \to D) \equiv \forall(\neg(L_1\theta \vee L_2\theta \cdots L_n\theta) \vee D)$

$= \forall(\neg L_1\theta \vee D) \wedge \forall(\neg L_2\theta \vee D)$

$\cdots \wedge \forall(\neg L_n\theta \vee D)$

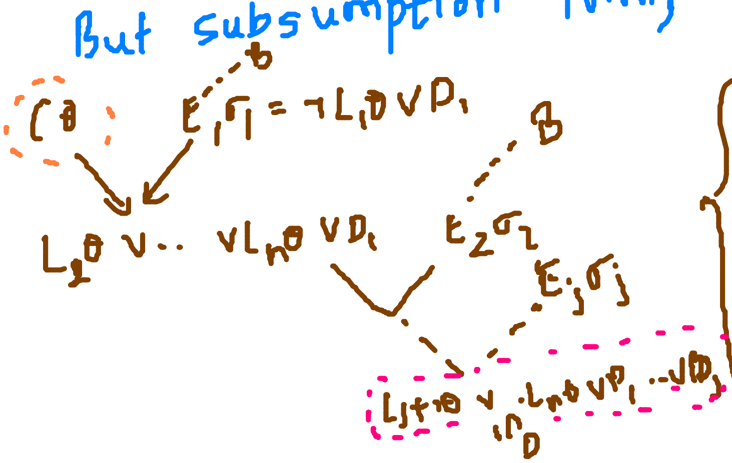
$L_1 \vee L_2 \cdots \vee L_n$

If $C \succeq_B D$ then $B \models \forall(C\theta \to D)$

$\equiv \forall i = 1..n, \; B \models \forall(\neg L_i\theta \vee D)$

Trivially: Note that if for any i, $\neg L_i\theta \notin E_i\sigma_i$, then $B \models D$

But subsumption thm, $\exists E_i$ s.t. $B \vdash_R E_i$ & $E_i \succeq \neg L_i\theta \vee D$



B

$\downarrow$

E_i

$\sqcap \sigma_i$

$\neg L_i\theta \vee D$

2 cases

$E_i\sigma_i = \neg L_i\theta$ (form for $i = j+1, n$)

$E_i\sigma_i = D_i \vee \neg L_i\theta$, $D_i \subseteq D$ (form for $i = 1$ to j)

As an aside:- $C \models_B D$ iff $\exists$ a deriv of E from $\{C\} \cup B$ & $E \succeq D$

(subsumption thm)

Negative result: LGRI does not always exist

... carries over from LGI

for S, LGI exists if $\exists$ at least one fn-free non-tautolog clause

Eg:- if $S = \{D_1, D_2\}$

$D_1 = P(a)$, $D_2 = P(b)$

$B = \{(P(a) \vee \neg Q(x)), (P(b) \vee \neg Q(x))\}$

$LGRI(D_1, D_2) = E$ st

$C_1 \quad C$

$B \cup D_1 \models E$

$C_2: B \cup D_2 \models E$

$C_{1,2,3}$

& $\nexists$ no E' s.t. $B \cup E' \models E$

& $B \cup D_1 \models E'$, $B \cup D_2 \models E'$

LGI

$\nexists$ no C

$C_1 \quad C_2$

Suppose $\mathcal{D}$ is $LGRI_B(D_1, D_2)$

① If $\mathcal{D}$ contains $P(a)$, then any int that makes only $P(a)$ true & all other ground lits false would be a model of $B \cup \{D\}$ but not of D_2

$\Rightarrow D \not\models_B D_2$ [ie $B \cup \{D\} \not\models D_2$]

$B(x')\ C(x)$

x'/d

$P(a) \vee P(x)$

x/a

$P(a)$

② If $\mathcal{D}$ contains $P(b)$, similarly

$D \not\models_B D_1$

③ If d is a constant not appearing in $\mathcal{D}$. Say

$C: P(x) \vee Q(d) \Rightarrow C \models_B \{D_1\} \cup \{D_2\} \equiv C \cup B \models \{D_1\}$, $C \cup B \models \{D_2\}$

$\Downarrow$ ✗

$C \models \{D_1\} \cup \{D_2\}$ By defn of LGRI, $C \models_B D$

$\Rightarrow$ By subsumption thm, $\exists$ E s.t

$$B \cup \{C\} \vdash E \succeq D$$

What can E possibly be

$B = \{(P(a) \vee \neg Q(x)), (P(b) \vee \neg Q(x))\}$

$C = P(x) \vee Q(d)$

$\vdash$

E is either

B_1: $B_1 \not\succeq D$ By ①

B_2: $B_2 \not\succeq D$ By ②

C: $C \not\succeq D$ (by ass d was a constant not in D) By construction of C

$P(a) \vee P(x)$: $\not\succeq D$ since $P(a) \notin D$ By ①

$P(b) \vee P(x)$: $\not\succeq D$ $P(b) \notin D$ By ②

$\Rightarrow \nexists$ no LGRI D.

Positive result.

If B is a finite set of fn-free ground literals then

$$C \models_B D \quad \text{iff} \quad C \models (D \vee \bar{B})$$

(Recall that under similar conditions, (fn free not req'd)

$$C \geq_B D \quad \text{iff} \quad C \geq (D \vee \bar{B})$$
)

Proof: If $C \models_B D$ i.e. $\{C\} \cup B \models D$, Let M be a model of C. Need to show that M is also a model of $D \vee \bar{B}$

① If $M \notin \mathcal{M}(B)$, since B is a set of ground lits, $M \in \mathcal{M}(\bar{B})$ (not otherwise)

$\Rightarrow M \in \mathcal{M}(\bar{B} \vee D)$

② If $M \in \mathcal{M}(\mathcal{B})$, since $\{C\} \cup \mathcal{B} \models D \Rightarrow \mathcal{M}$ is a model of $D \vee \bar{\mathcal{B}}$

(only if part proved)

If part

If $C \models (D \vee \bar{\mathcal{B}})$ then $C \models_{\mathcal{B}} D$

① If M is a model of $\{C\} \cup \mathcal{B} \Rightarrow M$ is a model of C as well as $\mathcal{B}$

$\Rightarrow M$ is a model of $\mathcal{B}$ but not $\bar{\mathcal{B}}$

$\Rightarrow M$ must be a model of D

To find $LGRI_{(\mathcal{B})}$ of $S = \{D_1 \ldots D_n\}$ where $\mathcal{B}$ is a finite set of fn free ground lits, compute $LGI(\{(D_1 \vee \bar{\mathcal{B}}), (D_2 \vee \bar{\mathcal{B}}) \ldots\ldots (D_n \vee \bar{\mathcal{B}})\})$

Buntine's Generalized Subsumption (For definite clause)

$C \geq_B D$ (g-subsume) if all H-models M of B, and for every ground atom A s.t. D covers A under M, we must have that C also covers A under M

$C = C^+ \leftarrow C^-$

Definite clause C is said to "cover" atom A under M if $\exists$ a gnd substitution (θ being ground) st M is a θ & $C^+\theta = A$

$B_1 \quad Pet(x) \leftarrow Cat(x)$

$B_2 \quad Pet(x) \leftarrow Dog(x)$

$B_3 \quad Small(x) \leftarrow Cat(x)$

$C = CP(x) \leftarrow S(x), P(x)$

$D = CP(x) \leftarrow C(x)$

$A = CP(t) : \theta = \{x/t\}$

require C should cover A under M.

M

(1) C is more general than D in any situation (Model) consistent with what we know thru B

↳ C can be used to prove atleast as many results as D.

If $\overbrace{\text{small pets are cuddly pets}}^{C}$, then $\overbrace{\text{cats are cuddly pets}}^{D}$, since we already know that $\underbrace{\text{cats are small pets}}_{B}$

① Subsumption $\Rightarrow$ g-subsumption $(C : C^+ \leftarrow C^-,\ D : D^+ \leftarrow D^-)$

If $C \geqslant D \Rightarrow C\theta \subseteq D \Rightarrow \underline{C^+\theta = D^+}$ & $C^-\theta \subseteq D^-$ for some θ

If D covers A under some M

$\Rightarrow D^+\gamma = A$ & $D^+\gamma$ & $D^-\gamma$ are true under M.

But :- $D^+\gamma = C^+\theta\gamma = A$ is true under M

Also :- $\underline{D^-\gamma} \supseteq C^-\theta\gamma$ is true under M

is a conjunction of lits

$\Rightarrow C$ covers A under M.

$\Rightarrow$ if $C \geqslant D$, then $C \geqslant_B D$

Converse: If $B = \{q(a)\}$, $C = Q(a) \leftarrow P(a)$ & $D = Q(a)$

then $C \geqslant_B D$ but $C \not\geqslant D$.

② $\geq_B$ is a quasi order that induces a partial order.

③ Procedural viewpoint:-

Further restriction of relative subsumption [No proof]

C

C as head, B as input | SLB

E

$\exists\theta \mid \cap$

D

Strengthwise

$\models_B$

$\downarrow \quad \succ_B$

$\downarrow \quad \geq_B$

④ ∴ It follows

If C, D & B are def progs

$\Rightarrow$ If $C \geq_B D$ then $C \succ_B D$

Since SLD deduction is a deduction

An LGGS of a finite set $\mathcal{S}$ (of definite program clauses which all have the same predicate symbol in their respective heads) always exists either if

(a) All clauses in $\mathcal{S}$ are atoms, and the background knowledge $\mathcal{B}$ implies only a finite number of ground atoms (i.e., $M_{\mathcal{B}}$ is finite) → Minimal models

(b) $\mathcal{S}$ and $\mathcal{B}$ are all function-free (see [Bun88]).

(c) $\mathcal{B}$ is ground. This case differs from the first, because $\mathcal{B}$ may imply only a finite number of ground examples, and still be non-ground itself. For example, $\mathcal{B} = \{P(a), (Q(x) \leftarrow P(x))\}$.

$$|||$$

$$M_B = \{P(a), Q(a)\}$$

(c) is a special case of (a)

Common to the 3 cases :- If $S = \{D_1, .. D_n\}$ & $\forall\ 1 \le i \le n$, if σ_i is a skolemn subs for D_i w.r.t $B \cup S$ & M_i is MM of $B \cup \bar{D_i \sigma_i}$ & M_i is finite then

$$LGGS_{\geq_B}(D_1, .. D_n) = LGS(\{D_1^+\sigma_1 \vee \bar{M_1}\}, \ldots \{D_n^+\sigma_n \vee \underset{\text{bot}}{\bar{M_n}}\})$$