Literal :- Negated or non-negated prop symbol

Clause :- Disjunction of literals.

$A \equiv A \wedge A$

$A \equiv \neg\neg A$

$A \text{ iff } B \equiv B \text{ iff } A$

$(A \text{ or } B) \text{ or } C \equiv A \text{ or } (B \text{ or } C)$

$A \leftarrow B \equiv A \vee \neg B$

$(A \leftarrow B) \leftarrow C \equiv A \leftarrow (B \wedge C)$

$A \leftarrow (B \leftarrow C) \equiv (A \leftarrow B) \wedge (A \vee C)$

$A \text{ iff } B \equiv (A \wedge B) \vee (\neg A \wedge \neg B)$

Any formula not in clausal form can be transformed to an equiv form in clausal form

$(\neg B \text{ iff } C) \equiv$

$$\neg(B \text{ iff } C) \equiv (B \vee C) \wedge \neg(B \wedge C)$$

$$\equiv (B \vee C) \wedge (\neg B \vee \neg C)$$

Conjunctive Normal form

$$\underbrace{(\neg B \text{ iff } C)} \equiv \{ \underbrace{(B \vee C)}_{\text{clause 1}}, \underbrace{(\neg B \vee \neg C)}_{\text{clause 2.}} \}$$

CNF . . . set of clauses

En

Conditional ≡ Contrapositive

$\alpha \leftarrow \beta \equiv \neg\beta \leftarrow \neg\alpha$

Props:-

(1) F is in CNF

G is in DNF

$\neg F$ is in DNF

$\neg G$ is in CNF

(2) Every formula F

can be written as

$\equiv F_1$ in CNF $\equiv F_2$ in DNF

Γ

Q: Given F & its truth table, can you construct DNF

A	B	C	Form
F	T	F	T
T	T	F	T

① Look all rows wher Form = T.

② Conjunction :- $\neg A \wedge B \wedge \neg C$

③ Disjunction: $(\neg A \wedge B \wedge \neg C) \vee (A \wedge B \wedge \neg C)$

1) Replace all $A \leftarrow B$ with $A \vee \neg B$

2) $A \leftrightarrow B$ with $(A \vee \neg B) \wedge (\neg A \vee B)$

3) Eliminate double negs

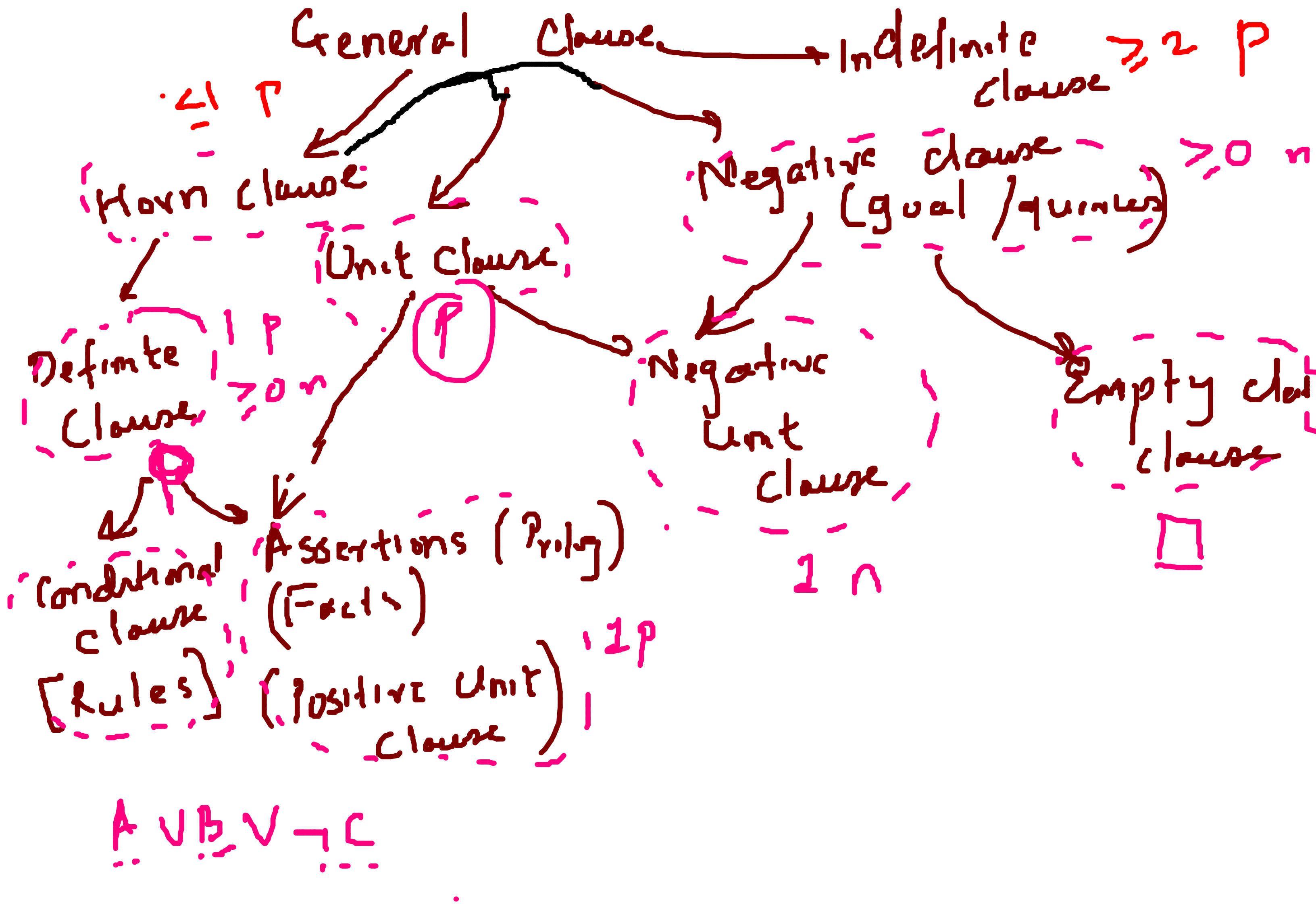
$\neg\neg A \longrightarrow A$

4) Distribute the disjunct $\vee$

$A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$

CNF

Imp:- Clausal form Logic is basis for ILP, SRL


General Clause
Indefinite clause
≥ 2 P
Horn clause
≤ 1 P
Negative clause (goal / queries)
≥ 0 n
Unit Clause
P
Definite Clause
1 P
≥ 0 n
Negative Unit clause
1 n
Empty clause
□
Conditional clause
[Rules]
Assertions (Prolog)
(Facts)
(Positive Unit Clause)
1 P
A ∨ B ∨ ¬C

Modes tollens

$\{\neg A, A \leftarrow B\} \vdash \neg B$

Inference

$\Sigma \overset{?}{\models} \alpha$

Modes ponens

$\{B, A \leftarrow B\} \vdash A$

Derivation

$\Sigma \vdash \alpha$

using well understood rules

Rules of proof/inference → Mechanize logical implication

+ Wff

Prop calculus

Deduction thm

$\Sigma \models \alpha$ iff $\Sigma - \{\beta_i\} \models (\alpha \leftarrow \beta_i)$

$\Sigma = \{\beta_1 \ldots \beta_n\}$

(1) $\alpha \leftarrow \beta$: Sentence, true / false based on I

material impl. $\alpha = F$, $\beta = F$

(2) $\Sigma \models \alpha$: logical impl

$\beta \models \alpha$ ... Every int of β is int of α

In scientific community

Σ :- Axioms

Model for Σ :- one that
sets all stmts
in Σ to true

$\Sigma \models \alpha$

...

Every
model of Σ must be
a model of α.

Derivation

$\Sigma \vdash \alpha$

High level view of inf

If $\Sigma \models \alpha$

can a mechanical proc. show this?

(1) Mechanical: Truth table enum

$\times$ unintelligent

(2) Mechanical system:

Inference system

Σ : Axioms

α :- Target/theorem

R : Rules of inference

$R + \underbrace{\text{propositions}}_{\text{prop calculus}}$

$\Sigma \vdash_R \alpha$ sometimes skipped

$\Sigma + R$ = inference system

$\Sigma \cup \alpha$:- theory

if $\Sigma \vdash_R \alpha$ then $\Sigma \models \alpha$

SOUNDNESS

&

COMPLETENESS

if $\Sigma \models \alpha$ then $\Sigma \vdash_R \alpha$

Making sense of R...

$\Sigma \models \alpha$

$\Sigma \models \alpha$ ...

$\models$: $\vDash$ $\qquad$ $\models$: $\in$

Axiom: String 'a'

Rules of inf :-

(i) From any thm x, ending with a, infer "x ba"

(2) From any thm X, infer the string obtained by replacing one substring bab of X with c

$$a \vdash_{(i)} aba \vdash_{(i)} ababa \vdash_{(i)} a\underbrace{bab}aba$$

$$\vdash_{(2)} acaba \vdash_{(i)} acab\underbrace{aba}a \quad \ldots$$

$$\{\{a\} \cup \{(ax)^n a \mid x \in \{b, c\}\ n \geq 1\}\}$$

Rules of inf for logic

Modus ponens (sound)

$\{\beta, \alpha \leftarrow \beta\} \vdash \alpha$

Modus tollens

$\{\neg\alpha, \alpha \leftarrow \beta\} \vdash \neg\beta$

Sound ?

Q:

Show:- $\{\beta, \alpha \leftarrow \beta\} \models \alpha$.

To prove that $\mathcal{R}$ is sound,

show that

whenever $\Sigma \vdash_{R} \alpha$

then it must be that

$$\Sigma \models \alpha$$

$$\{\beta, \alpha \leftarrow \beta\} \models \alpha \;?$$

$$M(I): \begin{array}{l} \rightarrow \beta \\ \rightarrow \alpha \end{array} \Bigg\}$$

Show that, any prob solvable using ponens is solvable using tollens

$A \to A^{x}$

$A^{x} = \neg A \quad C^{x} = \neg C$

$B^{x} = \neg B$

$\underbrace{A \leftarrow B}$ $\equiv \neg B \leftarrow \neg A \equiv B^{x} \leftarrow A^{x}$

Equivalence of conditional

& contrapositive

Proof Procedure:-

A strategy to ① Select (~~Comp~~ Selection)

& ② Apply rules from R (Computation)

Resolution:- (Generalization of modus ponens..)

β_1 :- Fred is an ape ← Fred is human

β_2 :- Fred is human ← Fred walks upright, Fred has a large brain

α :- Fred is an ape ← Fred walks upright, Fred ha[illegible] brain

$$\{\underbrace{(P \leftarrow Q_1, \ldots Q_i, \ldots Q_n)}_{\Sigma}, (Q_i \leftarrow R_1 \ldots R_m)\}$$

$$\vdash P \leftarrow Q_1, \ldots Q_{i-1}, R_1 \ldots R_m, Q_{i+1} \ldots Q_n$$

$$\{(P \vee \neg Q_1 \vee \ldots \vee \neg Q_i \vee \ldots \neg Q_n) \; (Q_i \vee \neg R_1 \ldots$$

negative   +ve

complementary literals

$$\vdash (P \vee \neg Q_1 \ldots \vee \neg Q_{i-1} \vee \neg R_1 \ldots \vee \neg R_m \vee \neg Q \ldots$$

Resolvent

Claim: Resolution is Sound.

C_1, C_2: clauses

$L \in C_1, \quad \neg L \in C_2$

$R = \{(C_1 - \{L\}) \cup \{C_2 - \{L\}\}\}$

Clause: - Disjunction of literals

+/- prop

Clause represented as a set

Q: Does $\{C_1, C_2\} \models R$?

$M_{C_1'} = M_{C_1} = M_{C_2}$

Let $C_1 = C \vee L$

$C_2 = D \vee \neg L$

If L = True in M

D must be True

$\Rightarrow$ $R = C \vee D$ must be True

Similarly :- If L = false in M

C must be True

If $\{C_1, C_2\} \vdash R$

then

$\{C_1, C_2\} \models R$

Suppose: $\{C_1, C_2\} \models \cancel{R}\, K$

Should: $\{C_1, C_2\} \vdash_{\text{(resolution)}} \cancel{R}\, K$?

Recall: L in C_1 & $\neg L$ in C_2

$\{A, B\} \models A \leftarrow B$

$\{A, B\} \vdash (A \leftarrow B) \wedge (B \leftarrow A)$

A iff B

Note: MP | MT special cases of resolution

Resolution proof procedure.

1) Let S be a set of clauses

α be a prop formula.

Resolution : Not complete

But it is sound.

Resolution is refutation complete

If Σ is inconsistent, then using resolution repeatedly, you CAN derive $\square \equiv$ False

(empty clause)

$\{\ \}$: Empty set of clauses $\equiv$ True

$\Sigma = \{ A, B, \neg(A \leftarrow B) \}$: Inconsistent (no model)

$\{A, B, \neg(A \leftarrow B)\}$

$= \{A, B, \neg A \wedge B\}$

$= \{A, B, \neg A, B\}$

$$\begin{tikzcd}
A \arrow[dr] & & \neg A \arrow[dl] \\
& \square &
\end{tikzcd}$$

? course(CS709, Person?)

$\boxdot$

$\underline{A \leftarrow B}$: goal),

Ref comp:

If $\Sigma \models \alpha$ (By deduction thm. $\Sigma \wedge \neg\alpha \models \square$)

then using resolution repeatedly, you can show:- $\Sigma \wedge \neg\alpha \vdash \square$.

Resolution: complete wrt inconsistent axioms.