$\Sigma \vdash_{\Sigma} R_1 \vdash_{\Sigma \cup \{R_1\}} R_2 \vdash_{\Sigma \cup \{R_1, R_2\}} \cdots$

Linear Res. (is refutation complete).

$\Sigma \vdash_{\Sigma} R_1 \vdash_{\Sigma} R_2 \vdash_{\Sigma} R_3 \vdash_{\Sigma} \cdots$

Input resolution (Not refutation complete)

Statement:
If Σ is unsat & $C \in \Sigma$ s.t. $\Sigma \setminus \{C\}$ is sat, $\exists$ a lin deriv of $\square$ starting at C

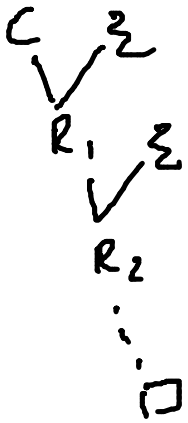
Induction on $\|\Sigma\|$ = # of unique atoms in Σ

Base case $\|\Sigma\| = 0$: $\Sigma = \{\square\}$

$C = \square$ $\Sigma \setminus \{\square\} = \{\}$ is a tautology

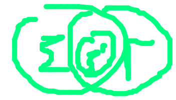
$\{\square\}$ not sat

$C = P$?



Induction: Assume for $n \leq m$ & $\|\Sigma\| = m+1$

2 cases



(a) C = L (a literal)

Σ —(1) Delete all clauses with L → Γ → Σ'

(2) From clauses containing ¬L, remove "¬L"

Γ: neither L nor ¬L uses Γ

an unsat subset of Γ s.t. every prop subset is sat. el for Σ?

If M model for Γ, is MU (yes)

contradiction

∴ $\Sigma \not\models \square$ & $\Gamma \models \square$

Show Σ' contain D' s.t. D' was obtained from $D \in \Sigma$ by removing ¬L?

(yes)

If $\Sigma' \subseteq \Sigma$ (we know that $L \notin \Sigma'$)

$\Sigma' \subseteq \Sigma \setminus \{C\}$ ... Σ' sat.

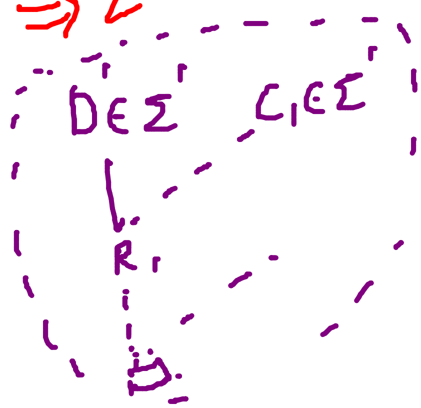
(L)

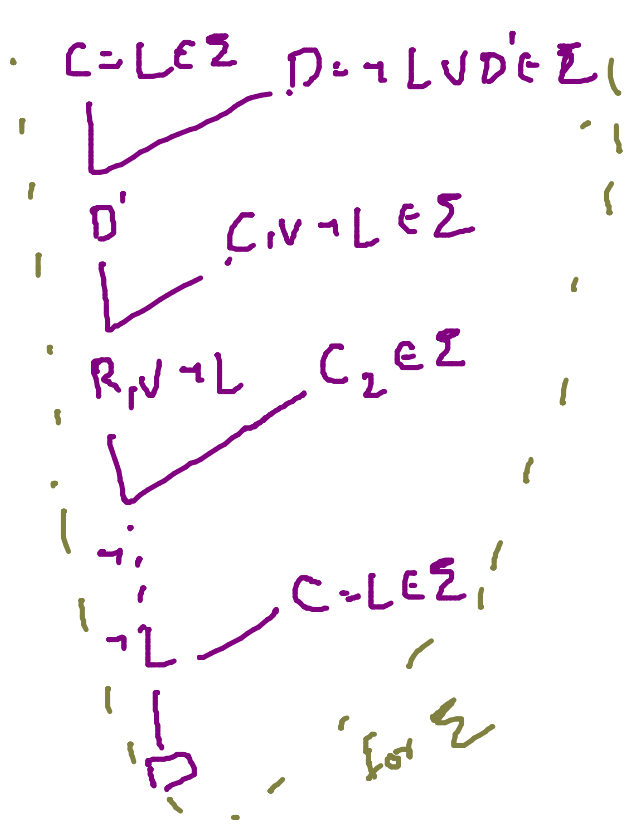
⇒ $\Sigma' \setminus \{D'\}$ is sat

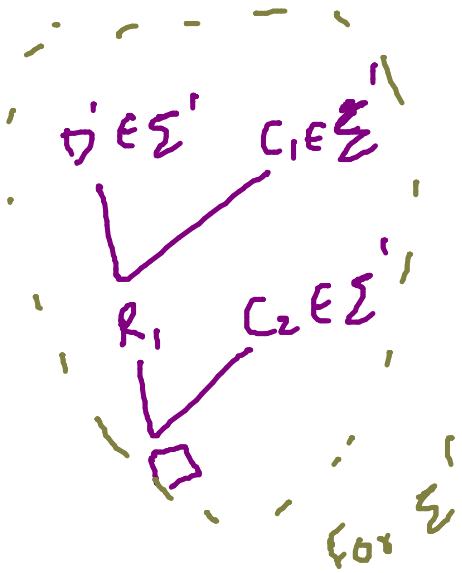
But by construction Σ' is unsat

By induction assumption on $\|\Sigma'\| \leq m$

$D' \vdash_{LR}^{\Sigma'}$ ...




C = L ∈ Σ
D = ¬L ∨ D' ∈ Σ
D'
C₁ ∨ ¬L ∈ Σ
R₁ ∨ ¬L
C₂ ∈ Σ
¬L
C = L ∈ Σ
□
for Σ

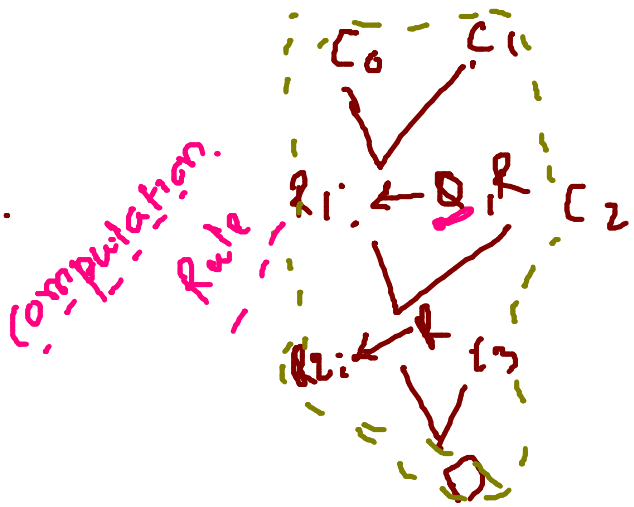

D' ∈ Σ'
C₁ ∈ Σ'
R₁
C₂ ∈ Σ'
□
for Σ'

SLD Resolution ⊇ Input resolution

Refutation comp for Horn clauses

$\Sigma = \{(\neg P), (P \leftarrow Q, R), \ldots, (Q), (R)\}$

$C_0 \quad C_1 \quad C_2 \quad C_3$



Let

head ← Body

Lit Lits.

$H \leftarrow B_1, \quad H_2 \leftarrow B_2 \ldots$

$R_i \xrightarrow[\Sigma]{} R_{i+1}$

R_{i+1} is obtained by resolving the lit in head of $C_i \in \Sigma$ with lit in body of R_i