Finding models for Σ:

Comes close to induction

A subset of prop symbols (atoms)

ILP :- Model inference

$\Sigma \models \alpha$ $\quad$ ($\Sigma \cup \{\neg\alpha\}$ is unsat)

co-NP complete prob

... sat NP comp · We will do

... unsat: co-NP comp · what we have done

Resolution:- Inverse resol, Inverse sub

ILP

Sat problem: Given Σ, find M_Σ.

(1) ILP :- Structure space of M_Σ

(2) $\underbrace{\text{Constraint}}_{\Sigma}$ Satisfaction Problem:-

→ DPLL (Davis Putnam Logemann, Loveland)
(Most efficient)

→ GSat, Walksat MaxWalkSat

Σ : clause set $\mathcal{V}$: variables

Simplify (Σ, v) :
① Remove C from Σ if $v \in C$
② Remove $\neg v$ from all $C \in \Sigma$
return modified Σ.

DPLL(Σ) {

① If $\Sigma = \phi$, return 'sat'

② If $\Box \in \Sigma$ return 'unsat'

③ Unit Prop rule:- If Σ contains $\underbrace{C = \{c\}}_{\text{unit clause}}$
assign c=TRUE, $\Sigma' =$ simplify(Σ, c)
call DPLL(Σ')

else

④ Splitting Rule:- Select from $\mathcal{V}$, a v which is not yet assigned T/F value. Assign v to some T/F value

$\Sigma'' =$ simplify(Σ, v), retval = DPLL(Σ'')

ⓐ If retval = "unsat", $\Sigma''' =$ simplify $(\Sigma, \neg v)$
set v = F/T DPLL(Σ''')

ⓑ If retval = "sat", return "sat".

Convergence?
Complete?
(Sound) correct?

}

Example

$\Sigma = \{\{a, b, \neg c\}, \{\neg a, \neg b\}, \{c\}, \{a, \neg b\}\}$

③ $\Sigma' = \{\{a, b\}, \{\neg a, \neg b\}, \{a, \neg b\}\}$

$DPLL(\Sigma')$

$M = \{a, c\}$

④ $a := v := T$

$\Sigma'' = \{\{\neg b\}\}$

③ $b = false$

$\Sigma''' = \{\}$

— sat.

$\{\{a, \neg b\}, \{c, \neg d\}, \{b, \neg e\}, \{d\}\}$

(worst case: exponential in # lit)

Smallest = $\{d, c\}$

DPLL = ? $\{a, b, d, c\}$

what if Σ contains only horn clauses?

$\Sigma = \{\{a, \neg b, \neg c, \neg d\}, \{b, \neg d\}, \{c, \neg d\}, \{d\}\}$

Efficiency?

Is step (4) reqd.? (4) is not reqd / superfluous

$\{\{a\}, \{\underset{F}{\underline{b}} \leftarrow \underset{F}{\underline{c}}\}, \{a \leftarrow c\}\}$
T T

Step (4): If no unit lit, set everything remaining to false / True.

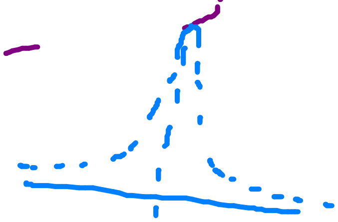
Phase Transitions

DPLL: worst case exp.

CNF:- $\underbrace{+/-/0}_{1/3}$ of lit in clause

DPLL:- Quad time

Randomly Formulae have generally high prob of being satisfiable.



Identify hard to solve prob instances

phase transition conjecture

All NP-C have atleast 1 order param & hard to solve probs, lie around a critical value of the param.

initially confirmed for Graph c & Ham path.

→ SAT

$$\frac{\#\text{ vars } (\Sigma)}{\#\text{ of clauses } (\Sigma)} : \text{order param}$$

3-SAT

clause : var = 4.3

$\{\{a, b, \neg c, d, f, g, l\}, \{g, \neg h, \neg l, c, a\}\}$

$\frac{8}{2} = 4$ $\left(\frac{2}{8}\right)$ More degree of freedom

low ⇒ sat

$\{\{a, b\}, \{\neg a, c\}, \{c, \neg b, a\}, \{a, c, \neg b\}\}$

$\frac{3}{4}$ $\left(\frac{4}{3}\right)$

$\{\{a\}, \{\neg b\}, \{\neg a\}, \{b\}, \{\neg a, b\}, \{a, \neg b\} \ldots\}$

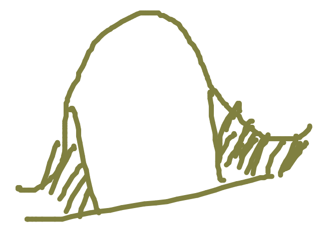
$9/2$:- high :- unsat

Approx, Local search :- Walksat, Gsat, . . .

Exploring local nbrhood.

try enhancing . . . till you dont get better

Better configs, thru local modifn.



Quantify?

# of satisfied clauses

INPUT: A set of clauses Σ, MAX-FLIPS, and MAX-TRIES.
OUTPUT: A satisfying truth assignment of Σ, if found.

for $i = 1$ to MAX-TRIES **do**
  T = a randomly-generated truth assignment.
  for j:=1 to MAX-FLIPS **do**
    if T satisfies Σ **then**
      return T
    end if
    v = a propositional variable such that a change in its truth assignment gives the largest increase in the number of clauses of Σ that are satisfied by T.
    T = T with the truth assignment of v reversed.
  end for
end for
return "Unsatisfiable".

random restart

$v \in C$ C is unsat

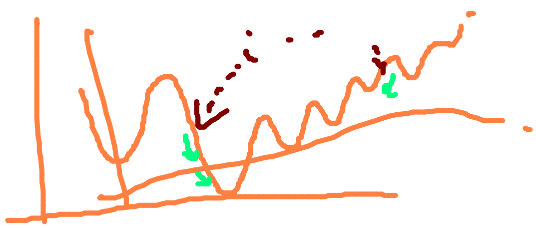
T #sat(T)

#sat(T')

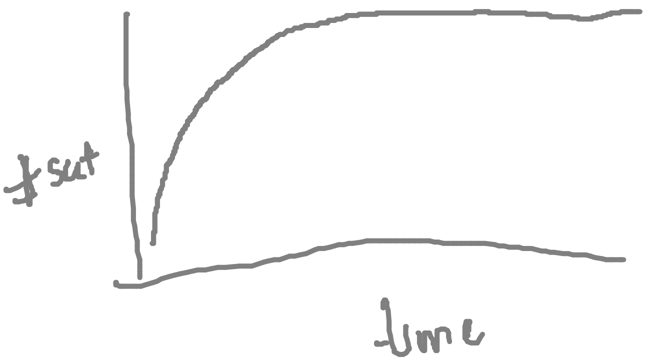
#sat(T') > #sat(T)

Gsat

Performs well for under-constrained & over-constrained probs.



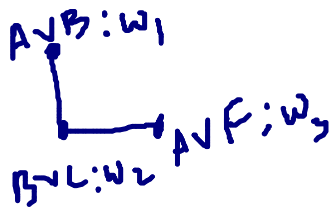
WALKSAT: - v should not unsatisfy a previously sat clause



Variant of Walksat: MaxWalkSat

weights to clauses.

sat



wtsat( )

Model structure

Lattice of Models

Σ : set of definite clauses

$\mathcal{M}(\Sigma)$:- set of all models

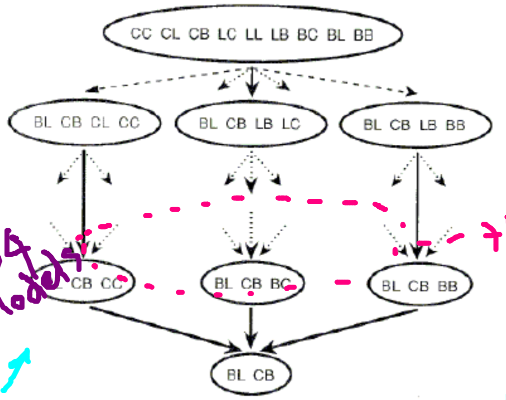
$M_1 \leq M_2$ iff $M_1 \subseteq M_2$

$M_1, M_2 \in \mathcal{M}(\Sigma)$

$M_1 \sqcap M_2$ as $M_1 \cap M_2$

$M_1 \sqcup M_2$ as $M_1 \cup M_2$

Claim: $\langle \leq, \mathcal{M}(\Sigma) \rangle$ is a p.o. It is also a complete lattice with $\sqcap$ = glb $\sqcup$ = lub



64 Models

→ Fail safe way of getting model for df Σ.

$2^{B(\Sigma)}$

64
512 : Ints
64 : Models
448 : Counter Models

DPLL?

Least glb : MM(Σ)

$\{CC, CL, CB, BL, CL, LL, BC, BB\} = \underbrace{B(\Sigma)}_{\text{base}}$ $\{BL, CB, CL, CC\}$.

Model lattice

Int lattice

$\Sigma = \{\{CC \leftarrow CL\}, \{CB \leftarrow BL\}, \{CL \leftarrow LL\}, \{BL\}\}$.

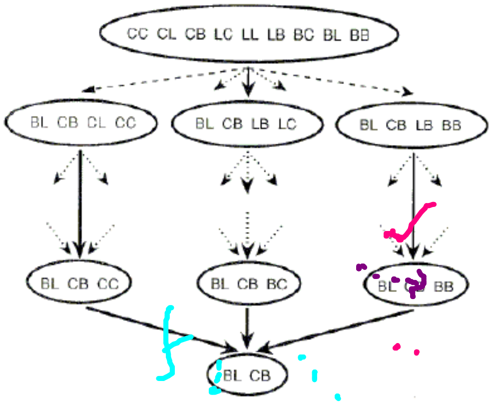
Thm

Σ Σ'

$MM(\Sigma) = \{BL, CB\}$

Function

$2^8 = 512$: $2^{B(\Sigma)}$: Lattice



Fine grained steps with lattice of models.

f: monotonic

continuous

downward closed!

Reflexive

GSAT: Steps in lattice of interps.