Logic

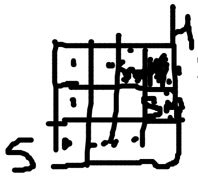
- Study of arguments & correct reasoning

1. 0 order Logic
2. 1st order logic [Predicate logic]

Richness of knowledge representation

Godel's incompleteness thm.

AI :-



Hider/seeker problem

1. Meemory of seeker
2. Hints abt hider
3. Strategy
4. Representation } Need for making m/cs mechanically take actions.

Declarative Language :-

(1) RDBMS
(2) PROLOG

↑ efficiency ↓ Richness

(1) System believes $\underbrace{\text{stmt } S}_{\text{syntax}}$ iff it $\underbrace{\text{considers } S \text{ to be True}}_{\text{semantics}}$

(2) Kb must be precise enough so :- we should know

[Syntax] (1) which symbols represent $\underbrace{\text{legitimate}}_{\text{syntax}}$ sentences

[Model theory] (2) what it means for the sentence to be 'True'

[Proof theory] (3) When a sentence follows from another set of sentences

0-Order Logic (Propositional)

~~Who goes there?~~

The earth is flat. ✓ (assertions)

X may be elected PM.

X elected PM : 0.9

SLP, BLP, RMN

Syntax:
Propositions :- Caps letters P, Q, R . . .
connectives :- and, or, if

Syntactic sugar : (,)

Simple Statement :- Prop P, Q . . .

Compound stmt :- $P \wedge Q$ (and) $\quad P \vee Q$ (or) $\quad \neg P$

$P \leftarrow Q$ (if)

① P :- Path A as rabbit's choice

② Q :- Path B as rabbit's choice

Disjunctive syllogism :- (by Chrysippus)

A dog is chasing a rabbit. The dog arrives at a fork, sniffs at one path & dashes down the other

$(P \vee Q) \wedge (\neg P)$

F

① $P \vee Q$ (T) semantics ② $\neg P$ (T) ③ $\therefore Q$ proof theory

Syntax: Vocab

Prop symbols: P, Q, R.

Logical conn: $\neg, \wedge, \vee, \leftarrow$

Brackets: (,)

wff

1. A prop symbol is a wff : P Q R
2. If α is wff, $\neg\alpha$ is also
3. If α & β are wff, $(\alpha \wedge \beta)$, $(\alpha \vee \beta)$ . .
$(\alpha \leftarrow \beta)$

$(P \leftarrow (Q \wedge R))$

$P \leftarrow (Q \wedge R)$ informally acceptable

$(P \vee Q \wedge R)$

$((P \vee Q) \wedge R)$

$(P \vee (Q \wedge R))$

Normal Forms

Every wff $\equiv$ A CNF .... $F = \bigwedge_{i=1}^{n} \left(\bigvee_{j=1}^{m} L_{ij} \right)$ Clause..

A DNF .... $F = \bigvee_{i=1}^{n} \left(\bigwedge_{j=1}^{m} L_{ij} \right)$

SEMANTICS :-

1. Interpretations (I) .. Assignment of T/F values to Prop symbols
2. Model (M)
3. Logical consequence ($\models$)

($\not\Rightarrow$)

	P,	Q.	R
Diff world .. I_1	T	T	F
.. I_2	T	F	F

For N prop symbols

2^N

Semantics of connectives

α	$\neg\alpha$
F	T
T	F

α	β	$\alpha \wedge \beta$
F	F	F
F	T	F
T	F	F
T	T	T

α	β	$\alpha \leftarrow \beta$
T	F	T
F	T	F
T	T	T
F	F	T

Truth value of an wff

1. Find truthvalues of prop using the int. I
2. Iteratively keep determining truth vals

$P, Q, R = \{T, T, F\}$

$T \quad (P \leftarrow \underbrace{(Q \wedge R)}_{F})$

$I = \{P, Q\}$

$I = \{Q, R\}$

MODEL :-

I is a model for α if I makes α true

$$\Sigma = \{\alpha_1, \alpha_2 \ldots \alpha_n\}$$

I is a model of Σ iff I is a model for for each α_i;

$$\alpha_1 \wedge \alpha_2 \ldots \wedge \alpha_n$$

1. wff for which every I is a model :- Valid or tautology

$$P \vee \neg P$$

2. wff for which no int I is a model :- inconsistent/unsatisfiable

(3) if for α, I exists st I is a model
α is satisfiable

Ex: $\neg(A \text{ iff } (B \vee \neg C))$:- $I = \{B\}$ $I = \{B, C\}$
$I = \{\}$

Find a model

$P \text{ iff } Q$ means $(P \leftarrow Q) \wedge (Q \leftarrow P)$

$((A \leftarrow B) \leftarrow C) \leftrightarrow (A \leftarrow (B \wedge C))$: Valid

$((A \leftarrow B) \leftarrow C) \equiv (A \leftarrow (B \wedge C))$

Logical consequence

$(P \vee Q) : T$

$\neg P : T$

$\Sigma \vDash \alpha$

Axioms — theorem

$\Sigma = \{ (P \vee Q), \neg P \} \vDash ?$ Q

$I = \{Q\}$

$I = \{Q\}, \{P, Q\}$

$\Sigma \vDash \alpha$ iff every model of Σ is also a model of α

(entails)

$(\beta_1 \wedge \beta_2 \dots \wedge \beta_n) \vDash \alpha$

$(\beta_1 \wedge \beta_2 \dots \wedge \beta_n) \vDash (\alpha_1 \wedge \alpha_2 \dots \alpha_n)$

Deduction theorem

$\Sigma \models \alpha$ if & only if $\Sigma - \{\beta_i\} \models (\alpha \leftarrow \beta_i)$

$\Sigma = \{\beta_1, \beta_2 \ldots \beta_n \ldots\}$

$\{\underbrace{\{P \vee Q\}, \{\neg P\}}_{\Sigma}\}$

$\Sigma \models Q$

Proof:- (1) Consider $\Sigma \models \alpha$

$M_\Sigma \subseteq M_\alpha$

Suppose. $\Sigma - \{\beta_i\} \not\models (\alpha \leftarrow \beta_i)$

Contradiction:-

M makes $\Sigma - \{\beta_i\}$ True, β_i true

M makes $\alpha \leftarrow \beta_i$ false α false

M is model for Σ But not for α

$\Sigma \models \alpha$

$\{\beta_1 \ldots \beta_n\} \models \alpha$

$\{\beta_2 \ldots \beta_n\} \models (\alpha \leftarrow \beta_1)$

True $\{\} \models \underbrace{((((\alpha \leftarrow \beta_1) \leftarrow \beta_2) \leftarrow \beta_3 \ldots) \leftarrow \beta_n)}_{\text{VALID} \mid \text{TAUTOLOGY}}$

Any mt is a model for $\{\}$ $\emptyset$

$\alpha \leftarrow \beta \equiv \alpha \vee \neg \beta$

$\{\beta_1 \ldots \beta_n\} \models (\text{False} \leftarrow \neg\alpha)$

$\equiv \underbrace{\{\beta_1 \ldots \beta_n, \neg\alpha\}}_{\text{inconsistent}} \models \text{False}$

Recall: $\Sigma \models \alpha$ iff $M_\Sigma \subseteq M_\alpha$