

Logic

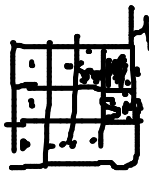
- Study of arguments & correct reasoning

- ↓
- ① 0 order Logic
 - ② 1st order logic [Predicate logic]

Richness of knowledge
representation

Godel's incompleteness thm.

AI :-



Hider/seeker
Problem

① Memory of seeker

② Hints abt hider

③ Strategy

④ Representation

} Need for making m/rs
mechanically take actions.

Declarative Language :-

① როგორც

② Prolog

↑ Efficiency

↓ Richness

- (1) System believes stmt S iff it considers S to be True
 syntax semantics

- ② Kb must be precise enough so - we should know
[Syntax] (1) which symbols represent legitimate sentences
syntax

- [Model theory] (2) what it means for the sentence to be 'true'

- [Proof them] ③ When a sentence follows from another set of sentences

0-Order Logic (propositional)

Who goes there?

The earth is flat. ✓ (assertions)

X may be elected PM

X elected PM : 0.9

SLP, BLP, RMN

Syntax

Propositions:- Caps letters

connectives:- and, or, if

P, Q, R...

syntactic sugar: '(', ')'

Simple Statement :- Prop P & Q . . .

Compound stmt :- $P \wedge Q$ $P \vee Q$ $\neg P$
(and) (or)

$P \leftarrow Q$
(if)

- ① P :- path A as rabbit's choice
- ② Q :- path B as rabbit's choice

Disjunctive syllogism :- (by chrysippus)

A dog is chasing a rabbit. The dog arrives at a fork, sniffs at one path & dashes down the other



① $P \vee Q$ ② $\neg P$ ③ $\therefore Q$
(T) premises (T) proof theory

$(P \vee Q) \wedge (\neg P)$
F

Syntax. Vocab

Prop symbols: P, Q, R .

Logical conn: $\neg, \wedge, \vee, \leftarrow$

Brackets: $(,)$

wff

① A prop symbol is a wff: P, Q, R

② If α is wff, $\neg \alpha$ is also

③ If α & β are wff, $(\alpha \wedge \beta), (\alpha \vee \beta),$
 $(\alpha \leftarrow \beta)$

$(P \leftarrow (Q \wedge R))$

$P \leftarrow (Q \wedge R)$
informally acceptable $(P \vee Q) \wedge R$
 $(P \vee (Q \wedge R))$

Normal Forms

Every rff $\equiv$ $\begin{cases} \text{A CNF} \dots F = \bigwedge_{i=1}^n \left(\bigvee_{j=1}^m L_{ij} \right) & \text{(Clause)} \\ \text{A DNF} \dots F = \bigvee_{i=1}^n \left(\bigwedge_{j=1}^m L_{ij} \right) \end{cases}$

SEMANTICS :-

- ① Interpretations ($\mathcal{I}$) -- Assignment of T/F values to Prop symbols
- ② Model ($\mathcal{M}$)
- ③ Logical consequence ($\models$)

Diff world

	$\mathcal{I}_1$	$\mathcal{I}_2$	P	Q	R
...	T	T	T	T	F
...	T	F	T	F	F

For N prop symbols
 2^N

Semantics of connectives

α	$\neg \alpha$
F	T
T	F

α	β	$\alpha \wedge \beta$
F	F	F
F	T	F
T	F	F
T	T	T

α	β	$\alpha \leftarrow \beta$
T	F	T
F	T	F
T	T	T
F	F	T

Truth value of an wff

① Find truth values of prop using the int. I

② Iteratively keep determining truth value

$$T \left\{ \begin{array}{l} (P \leftarrow (Q \wedge R)) \\ \underbrace{F}_{i=\{Q, R\}} \end{array} \right. \quad \begin{array}{l} P, Q, R \in \{T, F\} \\ I = \{P, Q\} \end{array}$$

MODEL ..

I is a model for α if I makes α true

$$\Sigma = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$$

I is a model of Σ iff I is a model for each α_i

$$\alpha_1 \wedge \alpha_2 \dots \wedge \alpha_n$$

① wff for which every I is a model :- valid or tautology

$$P \vee \neg P$$

② wff for which no I is a model :- inconsistent/unsatisfiable

(3) if for α , $\mathcal{I}$ exists st $\mathcal{I}$ is a model
 α is satisfiable

Ex: $\neg(A \text{ iff } (B \vee \neg C)) \vdash \mathcal{I} = \{B\} \quad \mathcal{I} = \{\bar{B}, C\}$
 $\mathcal{I} = \{\}$

Find a model

$P \text{ iff } Q$

means

$$(P \leftarrow Q) \wedge (Q \leftarrow P)$$

$$((A \leftarrow B) \leftarrow C) \wedge \neg(A \leftarrow (B \wedge C)) \vdash \text{Valid}$$

$$(A \leftarrow B) \leftarrow C \equiv (A \leftarrow (B \wedge C))$$

Logical Consequence

$$(P \vee Q) : T$$

$$\neg P : T$$

$$\Sigma = \{ (P \vee Q), \neg P \} \models ?$$

$$I = \{0\}$$

$$\Sigma \models \alpha$$

(entails)

$$(P_1 \wedge P_2 \dots \wedge P_n) \models \alpha$$

$$(P_1 \wedge P_2 \dots \wedge P_n) \models (A_1 \wedge A_2 \dots A_n)$$

$$\Sigma \models \alpha$$

Axioms

theorem

$$I = \{0\}, \{1, 0\}$$

iff every model of Σ is also a model of α

Deduction theorem

$$\Sigma \vdash \alpha \quad \text{if \& only if} \quad \Sigma - \{\beta_i\} \vdash (\alpha \leftarrow \beta_i)$$
$$\Sigma = \{\beta_1, \beta_2, \dots, \beta_n, \dots\}$$

Proof:- (i) Consider $\Sigma \vdash \alpha$

$$M_\Sigma \subseteq M_\alpha$$

Suppose $\Sigma - \{\beta_i\} \not\vdash (\alpha \leftarrow \beta_i)$

Contradiction

M makes $\Sigma - \{\beta_i\}$ True, β_i true
 M makes $\alpha \leftarrow \beta_i$ false α false

M is model for Σ But not for α

$$\{\{p \vee q\}, \{\neg p\}\}$$
$$\Sigma \vdash q$$

$$\Sigma \models \alpha$$

$$\{\beta_1 \dots \beta_n\} \models \alpha$$

$$\{\beta_2 \dots \beta_n\} \models (\alpha \leftarrow \beta_1)$$

$$\vdash_{\text{true}} \{ \} \models \underbrace{(((\alpha \leftarrow \beta_1) \leftarrow \beta_2) \leftarrow \beta_3 \dots) \leftarrow \beta_n)}_{\text{VALID TAUTOLOGY}}$$

Any mt is a model for $\{ \}$ $\checkmark$

$$\begin{aligned} & \{\beta_1 \dots \beta_n\} \models (\text{False} \leftarrow \neg \alpha) \\ \equiv & \underbrace{\{\beta_1 \dots \beta_n, \neg \alpha\}}_{\text{inconsistent}} \models \text{False} \end{aligned}$$

$$\alpha \leftarrow \beta \equiv \alpha \vee \neg \beta$$

Recall:

$$\Sigma \models \alpha \text{ iff } \mathcal{M}_\Sigma \subseteq \mathcal{M}_\alpha$$