

CS717: Midsem

25 Marks, Closed book

1. Does $\Sigma \models \alpha$ hold for the α below, when $\Sigma = \{\}$? Please justify your answer.

- $\alpha = p \leftarrow (((p \vee (p \leftarrow q) \leftarrow q)) \vee \neg(q \leftarrow p)))$

3 Marks

2. Assuming that T_Σ is continuous on the lattice $\langle 2^{\mathcal{B}(\Sigma)}, \subseteq \rangle$, prove that T_Σ must also be monotonic.

3 Marks

3. Application of T_Σ to some definite clause set Σ containing just one clause C , starting with $I_1 = \mathcal{B}(\Sigma)$ yields $I_1 = \{A\} = I_k$ for all $k > 1$.

- There are infinitely many possible choices for C which would yield this outcome. What are they?

3 Marks

- Now prove the following claim: *If the method of applying T_Σ begins with a non-minimal model and converges, then the model which it converges upon is not necessarily the minimal model.*

3 Marks

4. An adequate set C of connectives for propositional logic is a set such that for every formula of propositional logic there is an equivalent formula with only connectives from that set. For example, the set $\{\neg, \vee\}$ is adequate for propositional logic, because any occurrence of \wedge and \leftarrow can be removed by using the equivalences $\xi \leftarrow \phi \equiv \neg\phi \vee \xi$ and $\xi \wedge \phi \equiv \neg(\neg\phi \vee \neg\xi)$. Show that if $C \subseteq \{\neg, \wedge, \vee, \leftarrow\}$ is adequate for propositional logic, then $\neg \in C$.

3 Marks

5. Prove that

- (a) $\alpha \equiv \beta$ is equivalent to saying that $\alpha \leftrightarrow \beta$ is a tautology.

2.5 Marks

- (b) α is valid if and only if $\neg\alpha$ is unsatisfiable.

2.5 Marks

6. A man is rescued from a fire by a noble soul but the rescuer wants to remain anonymous. One of the following three must have been the rescuer: Arjun, Madhav, and Raghav. Arjun says “I didn’t do it. The victim was an old acquaintance of Madhav’s. But Raghav hated him.” Madhav states “I didn’t do it. I didn’t know the guy. Besides I was out of town all the week.” Raghav says “I didn’t do it. I saw both Arjun and Madhav downtown with the victim that day; one of them must have saved him.” Assume that the noble rescuer might not be telling the truth, whereas the other two are telling the truth.

Formalize/represent in propositional logic and use propositional **resolution** to determine who the noble rescuer was.

3 Marks

7. Use the predicates

- (a) $A(x, y)$: x admires y
- (b) $C(x, z)$: x attended z
- (c) $M(y)$: y is a Carnatic music singer
- (d) $S(x)$: x is a student of Carnatic music
- (e) $T(z)$: z is a Carnatic music concert

to translate the following into predicate logic:

- (a) Every student of Carnatic music admires some Carnatic singer

1 Mark

- (b) No student of Carnatic music attended every Carnatic music concert

1 Mark