CS717: Endsem

20 Marks, Open book

1. We have seen that if ϕ is a well formed existentially quantified formula (wff) and ϕ^S is the formula ϕ in Skolem Normal form, then $\phi \models \phi^S$ whereas, $\phi^S \not\models \phi$. If ϕ has finite domain (that is, the domain has a finite number of constants), is it possible to replace ϕ with some equivalent formula ϕ^E such that

   - $\phi \equiv \phi^E$ and
   - ϕ^E is free from existential quantifiers

   If yes, what can ϕ^E be? Note that such a ϕ^E can be useful in many situations.

   2 Marks

2. (a) Define

   $$C_n = \{P(x_i, x_j) \mid i \neq j \text{ } and \text{ } 1 \leq i, j \leq n\}, \quad n \geq 2$$

   First prove that each C_n for $n \geq 2$ is reduced.
   2 Marks

   (b) Let $C = \{P(x_1, x_1)\}$. Now prove that each C_n properly subsumes C_{n+1} and C. That is $C_2 \succ C_3 \succ \ldots C_n \succ \ldots C$.
   2 Marks

   (c) Finally, making use of (2a) and (2b) above, prove that if $\mathcal{C}$ is a clausal language containing a binary predicate P, then $C = \{P(x_1, x_1)\}$ has no upward cover in $\mathcal{C}$.
   3 Marks

3. Let $C = Q(x) \leftarrow P(x)$, $D = Q(a)$. Identify $\mathcal{B}$ under which $C \succeq_{\mathcal{B}} D$. For which $\mathcal{B}$ does $C \geq_{\mathcal{B}} D$?

   1 Mark

4. Let $\mathcal{B}$ be a Bayesian Logic Program. You are asked to compute $\Pr(q_1, q_2, \ldots, q_n \mid e_1, e_2, \ldots, e_m)$, where $q_1, q_2, \ldots, q_n, e_1, e_2, \ldots, e_m \in \mathcal{H}(\mathcal{B})$ (the Herbrand base of $\mathcal{B}$). Let $\mathcal{M}(\mathcal{B})$ be the minimal (least Herbrand) model of $\mathcal{B}$. Answer the following. Roughly and briefly explain your answer.

(a) What is $\Pr\left(q_1, q_2, \ldots, q_i, \ldots, q_n \mid e_1, e_2, \ldots, e_m\right)$ if $q_i \notin \mathcal{M}(\mathcal{B})$.
1.5 Marks

(b) What is $\Pr\left(q_1, q_2, \ldots, q_n \mid e_1, e_2, \ldots, e_i, \ldots, e_m\right)$ if $e_i \notin \mathcal{M}(\mathcal{B})$.
1.5 Marks

5. Let $\mathcal{B} = \{Polygon(X) \leftarrow Rectangle(X), Rectangle(X) \leftarrow Square(X)\}$. Let $C = Pos(X) \leftarrow Red(X), Square(X)$. Compute $\perp_{(C,\mathcal{B})}$.

   2 Marks

6. Let $\mathcal{B} = \{P(t,a), M(t), F(a), P(j,p), M(j), M(p)\}$, $C_1 = Fa(t,a) \leftarrow$ and $C_2 = Fa(j,p) \leftarrow$. Compute

   (a) $rlgg_{\mathcal{B}}(C_1, C_2)$
   2.5 Marks

   (b) $lgg\left(\perp_{(C_1,\mathcal{B})}, \perp_{(C_2,\mathcal{B})}\right)$
   2.5 Marks

   Verify that they are the same.