Homework Exercise 2

1. Construct a model-theoretic argument which establishes that $\{A, B\} \models C$ if and only if $\{A\} \models (C \leftarrow B)$

   (**1 mark**)

2. Confirm

   (a) $\neg(B \ iff \ C) \equiv (B \vee C) \wedge \neg(B \wedge C)$

   (b) $(A \vee B) \ iff \ \neg A \equiv \neg A \wedge B$

   (c) $(A \leftarrow B) \leftarrow C \equiv A \leftarrow (B \wedge C)$ (**1 Mark**)

3. Derive

   (a) $A \wedge B \models A \leftarrow B$

   (b) $(A \ iff \ B) \leftarrow C \models B \leftarrow (C \wedge A)$ (**1 mark**)

4. Are the following (in)consistent? Prove.

$$\begin{aligned}
&\neg A \leftarrow B \\
&B \leftarrow C \\
&A \leftarrow C \\
&C \leftarrow D \\
&D
\end{aligned}$$

   (**1 mark**)

5. We partially proved the following statement in class:

   Theorem 1 *If Σ is an unsatisfiable set of clauses, and $C \in \Sigma$ such that $\Sigma \setminus \{C\}$ is satisfiable, then there is a linear refutation of Σ with C as top clause.*

   In the proof by induction, there were two cases to be considered. Prove the induction step for the second case when $C = L \vee C'$, where C' is a non-empty clause.

   (**3 Marks**)

6. Making use of the model intersection property, prove that the intersection I^* of all models of Σ is the minimal model $MM(\Sigma)$.

   (**3 Marks**)