Tutorial 5

Friday 19$^{\text{th}}$ February, 2016; 07:46

Problem 1. Consider n p-dimensional vectors $\phi(x_1)$, ..., $\phi(x_n)$. The Gram matrix of the collection is the $m \times m$ matrix $\mathcal{K}$ with elements $\mathcal{K}_{ij} = \phi^T(x_i)\phi(x_j)$.
Prove that a Gram Matrix is always positive semi-definite.

Problem 2. Kernels: Show that following kernels are positive semi-definite
Hint: The property proved in problem 1 might be useful in some of these proofs

1. $K(x_1, x_2) = cos(x_1 - x_2)$
2. $K(x_1, x_2) = K_1(x_1, x_2) + K_2(x_1, x_2)$, where K_1 and K_2 are positive semi-definite kernels.
3. $K(x_1, x_2) = (< x_1, x_2 > +c)^d$, where $< x_1, x_2 >$ is inner product of vectors x_1 and x_2 and $d = 2$.
   Can this be generalized to polynomial of higher order (d $>$ 2)?
4. $K(x_1, x_2) = \exp(< x_1, x_2 >)$.
   Does this prove that RBF (gaussian) Kernel discussed in class ($K(x_1, x_2) = \exp(-\frac{||x_1 - x_2||^2}{2\sigma^2})$) is positive semi-definite?

Problem 3. SMO (Sequential Minimal Optimization) algorithm. As described in the class , the SMO algorithm selects two β parameters, β_i and β_j and optimizes the objective value jointly for both these β's. Derive the closed form expression for β_i^{new} and β_j^{new}

Problem 4. Remember the Lasso regression discussed in the class. Can you rewrite the optimization objective in a kernelized form?