# Lecture 19 contd: Neural Network Training using Backpropagation, Convolutional And Recurrent Neural Networks

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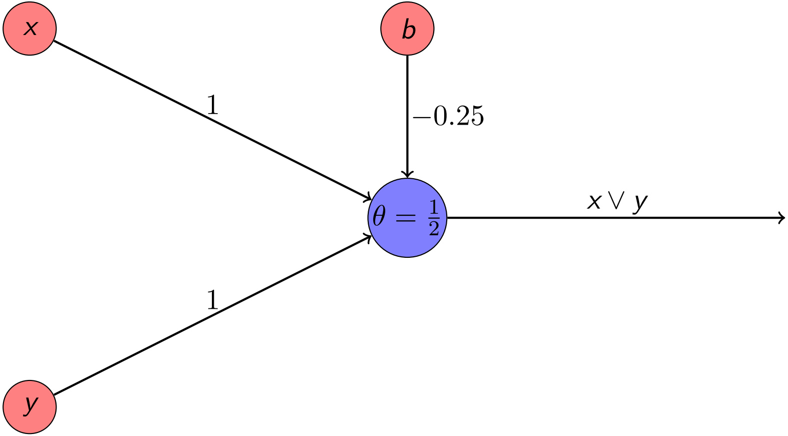
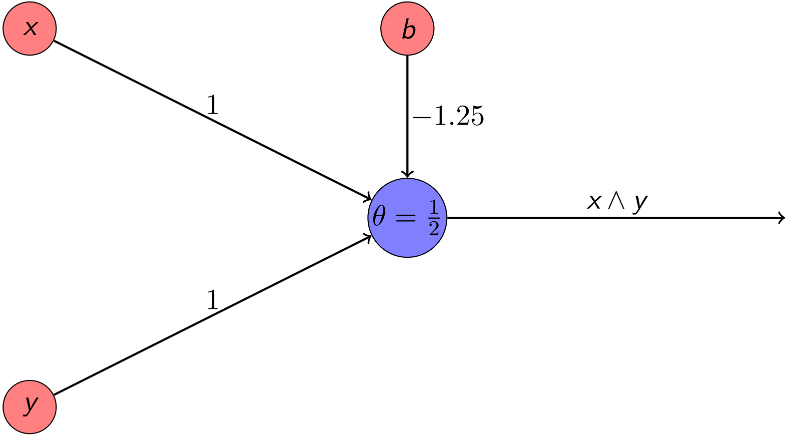
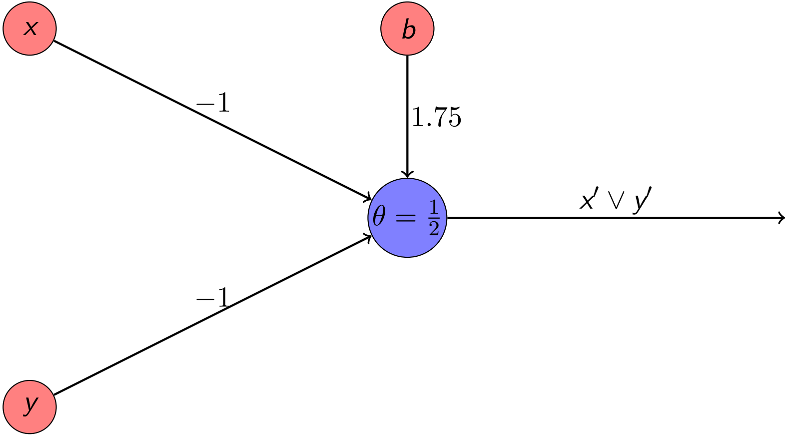
# OR using perceptron



# AND using perceptron

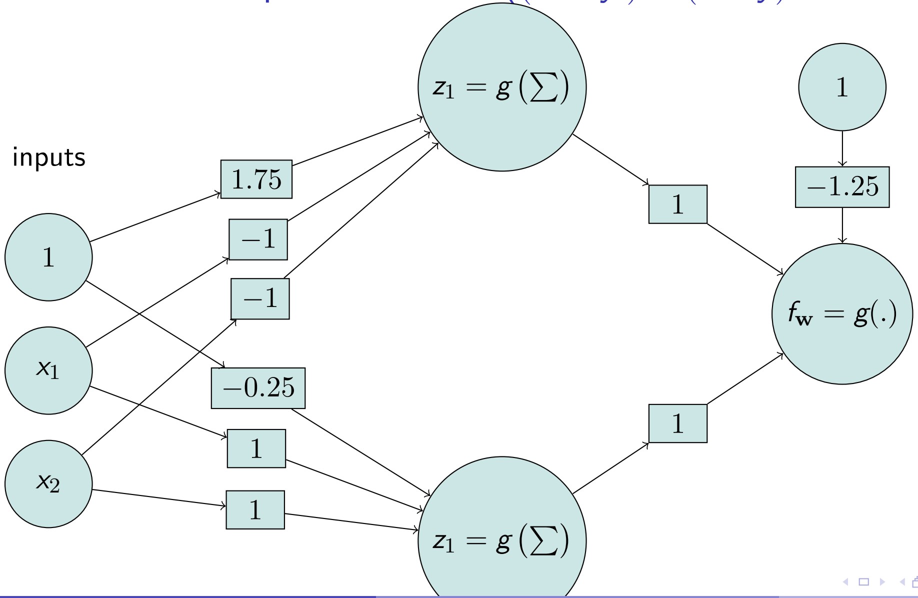


# x'Vy' using perceptron

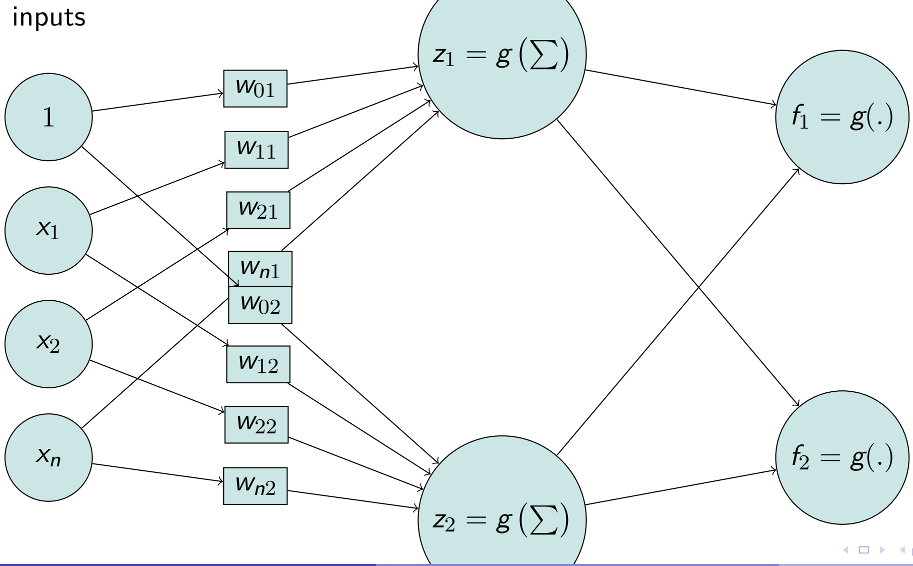


# How about XOR $((x' \vee y') \wedge (x \vee y))$?

# Cascade of Perceptrons for XOR($(x' \vee y') \wedge (x \vee y)$ with $\theta = 0$ )



# Feed-forward Neural Nets



# Training a Neural Network

STEP 0: Pick a network architecture

- Number of input units: Dimension of features $\phi\left(\mathbf{x}^{(i)}\right)$.
- Number of output units: Number of classes.
- Reasonable default: 1 hidden layer, or if $>1$ hidden layer, have same number of hidden units in every layer.
- Number of hidden units in each layer a constant factor (3 or 4) of dimension of $x$.
- We will use
  - the smooth sigmoidal function $g(s) = \frac{1}{1+e^{-s}}$: **We have now learnt how to train a single node sigmoidal (LR) neural network**
  - instead of the non-smooth step function $g(s) = 1$ if $s \in [\theta, \infty)$ and $g(s) = 0$ otherwise.

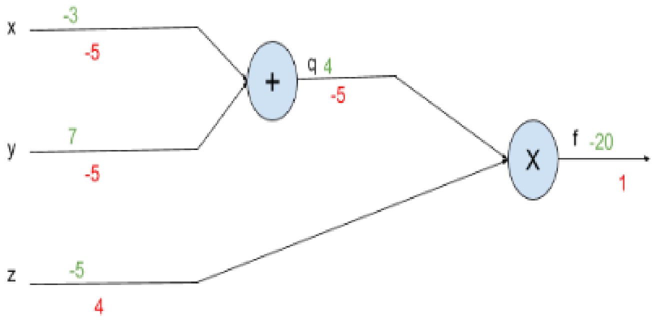
# **High Level Overview** of Backpropagation Algorithm for Training NN

1. Randomly initialize weights $w_{ij}^{l}$ for $l = 1, \ldots, L$, $i = 1, \ldots, s_l$, $j = 1, \ldots, s_{l+1}$.
2. Implement **forward propagation** to get $f_w(\mathbf{x})$ for any $x \in \mathcal{D}$.
3. Execute **backpropagation**
   1. by computing partial derivatives $\frac{\partial}{\partial w_{ij}^{(l)}} E(w)$ for $l = 1, \ldots, L$, $i = 1, \ldots, s_l$, $j = 1, \ldots, s_{l+1}$.
   2. and using gradient descent to try to minimize (non-convex) $E(w)$ as a function of parameters $\mathbf{w}$.
4. Verify that the cost function $E(w)$ has indeed reduced, else resort to some random perturbation of weights $\mathbf{w}$.

# Intuition for Backpropagation

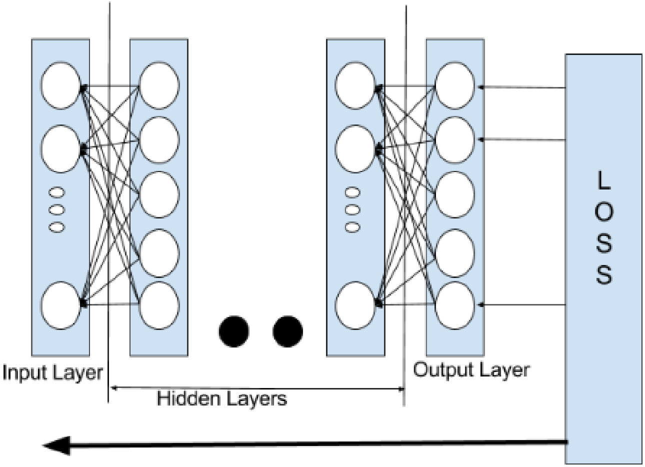
$f(x, y, z) = (w_x x + w_y y)w_q + w_z z,\ q = w_x x + w_y y$

At input (-3,7,-5):-
(gradient of f w.r.t. v shown in red where $v \in \{w_x, w_y, w_z, w_q\}$)



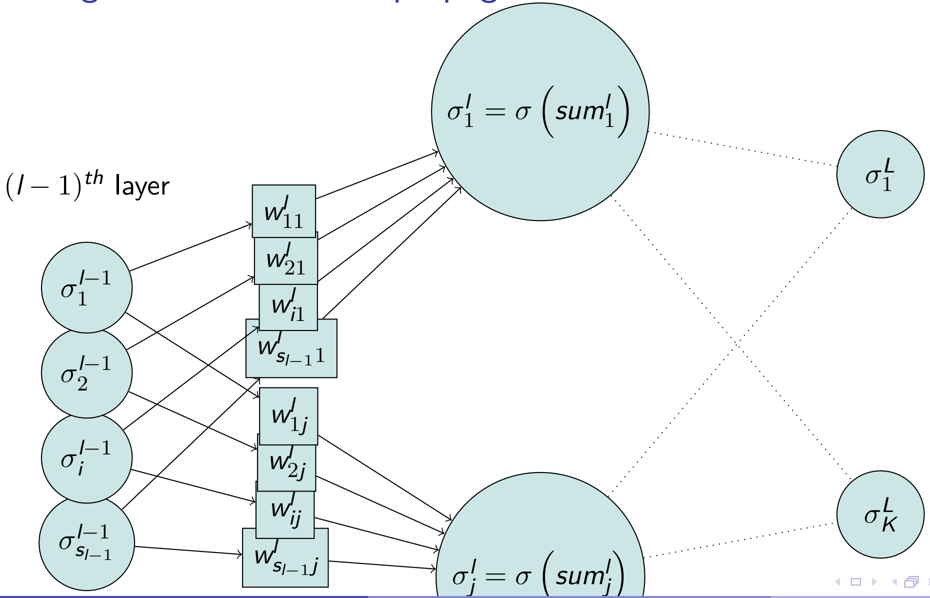

Should we decrease $w_y$ or should we increase $w_x$ to decrease f?

# Intuition for Backpropagation




Iteratively update the weights along the direction where Loss decreases.

# Setting Notation for Backpropagation



# Gradient Computation

- The Neural Network objective to be minimized:

$$E(\mathbf{w}) = -\frac{1}{m}\left[\sum_{i=1}^{m}\sum_{k=1}^{K} y_k^{(i)} \log\left(\sigma_k^L\left(\mathbf{x}^{(i)}\right)\right) + \left(1 - y_k^{(i)}\right)\log\left(1 - \sigma_k^L\left(\mathbf{x}^{(i)}\right)\right)\right] + \frac{\lambda}{2m}\sum_{l=1}^{L}\sum_{i=1}^{s_{l-1}}\sum_{j=1}^{s_l}\left(w_{ij}^l\right)^2 \quad (1)$$

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- $sum_j^l = \sum_{k=1}^{s_{l-1}} w_{kj}^l \sigma_k^{l-1}$ and $\sigma_i^l = \frac{1}{1+e^{-sum_i^l}}$
- $\frac{\partial E}{\partial w_{ij}^l} = \frac{\partial E}{\partial \sigma_j^l}\frac{\partial \sigma_j^l}{\partial sum_j^l}\frac{\partial sum_j^l}{\partial w_{ij}^l}$
- $\frac{\partial sum_j^l}{\partial w_{ij}^l} = \frac{\partial}{\partial w_{ij}^l}\left(\sum_{k=1}^{s_{l-1}} w_{kj}^l \sigma_k^{l-1}\right) = \sigma_i^{l-1}$
- $\frac{\partial \sigma_j^l}{\partial sum_j^l} = \left(\frac{1}{1+e^{-sum_i^l}}\right)\left(1-\frac{1}{1+e^{-sum_i^l}}\right)$

- For a single example $(\mathbf{x}, y)$:

$$-\left[\sum_{k=1}^{K} y_k \log\left(\sigma_k^L(\mathbf{x})\right) + (1-y_k)\log\left(1-\sigma_k^L(\mathbf{x})\right)\right] + \frac{\lambda}{2m}\sum_{l=1}^{L}\sum_{i=1}^{s_{l-1}}\sum_{j=1}^{s_l}\left(w_{ij}^l\right)^2 \quad (2)$$

- $\frac{\partial E}{\partial \sigma_j^l} = \sum_{p=1}^{s_{l+1}} \frac{\partial E}{\partial sum_p^{l+1}} \frac{\partial sum_p^{l+1}}{\partial \sigma_j^l} = \sum_{p=1}^{s_{l+1}} \frac{\partial E}{\partial \sigma_j^{l+1}} \frac{\partial \sigma_j^{l+1}}{\partial sum_p^{l+1}} w_{jp}^{l+1}$ since $\frac{\partial sum_p^{l+1}}{\partial \sigma_j^l} = w_{jp}^{l+1}$
- $\frac{\partial E}{\partial \sigma_j^L} = -\frac{y_j}{\sigma_j^L} - \frac{1-y_j}{1-\sigma_j^L}$