Lecture 26b: Unsupervised Learning: Dimensionality Reduction, Embeddings, PCA etc

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Recall: Supervised Feature Selection based on Gain

- S is a sample of training examples, p_{C_i} is proportion of examples with class C_i in S
- Entropy measures impurity of S: $H(S) \equiv \sum_{i=1}^{K} -p_{C_i} \log_2 p_{C_i}$
- Selecting R best attributes: Let $\mathcal{R} = \emptyset$
- $Gain(S, \phi_i)$ = expected **Gain** due to choice of ϕ_i Eg: Gain based on entropy -
  $Gain(S, \phi_i) \equiv H(S) \ - \sum_{v \in Values(\phi_i)} \frac{|S_v|}{|S|} H(S_v)$
  Do:
  1. $\phi^* = \underset{\phi_i \setminus \mathcal{V}}{\text{argmax}}\, Gain(S, \phi_i)$
  2. $\mathcal{R} = \mathcal{R} \cup \{\phi^*\}$

  Until $|\mathcal{R}| = R$

Q: Other measures of **Gain**: Gini Index, Classification Error, *etc.*

Supervised Feature Subset Selection (Optional)

- One can also Optimally select subset of features using Iterative Hard Thresholding[1] for **Optimal Feature Selection**
- **Input:** Error function $\mathbf{E}(\mathbf{w})$ with gradient oracle to compute $\nabla\mathbf{E}(\mathbf{w})$ sparsity level s, step-size η:
- $\mathbf{w}^{(0)} = 0, \; t = 1$
- while not converged do
  1. $\mathbf{w}^{(t+1)} = P_s\left(\mathbf{w}^{(t)} - \eta\nabla_{\mathbf{w}}\mathbf{E}(\mathbf{w}^{(t)})\right)$ //Projection function $P_s(.)$ picks the highest weighted s features as per the update $\mathbf{w}^{(t)} - \eta\nabla_{\mathbf{w}}\mathbf{E}(\mathbf{w}^{(t)})$ and sets rest to 0
  2. $t = t + 1$
- end while
- Output: $\mathbf{w}^{(t)}$

[1] https://arxiv.org/pdf/1410.5137v2.pdf

Recap: One Hot Encoding for Characters

- With 3 characters in vocabulary, a,b and c, what would be the best encoding to inform each character occurrence to the network?
- One Hot Encoding: Give a unique key k to each character in alpha-numeric order, and encode each character with a vector of vocabulary size, with a 1 for the k^{th} element, and 0 for all other elements.

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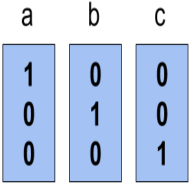

Encoding Words

How to encode the words for the task of labeling a drama reviews as "liked" or "not liked" ?

- Review 1: The drama was interesting, loved the way each scene was directed. I simply loved everything in the drama.
- Review 2: I had three boring hours. Very boring to watch.
- Review 3: I liked the role each that was assigned to each super star. Especially loved the performance of actor.
- Review 4: Though I hate all the dramas of the director, this one was an exception with lot of entertainment.

Encoding Words

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- One Hot Encoding of Words.
- Bag Of Words, similar to one hot encoding of characters
  - Use the vocabulary of highly frequent words in reviews.
  - Use the word frequency in each review instead of "1".

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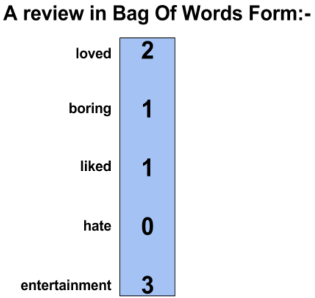

(Word) Embedding: Motivation

Limitations of Bag of Words or One Hot Encoding for words

- High Dimension: In real life scenario, the vocabulary size could be huge.
- Lacks Contextual Similarity - e.g. liked and loved are contextually similar words.

(Word) Embedding: Motivation

Dimensionality Reduction techniques.

- Bag of Frequent Words: Contextual similarity is still lacking.
- **What happens if one passes a one hot encoded word as both input and output to a NN?**

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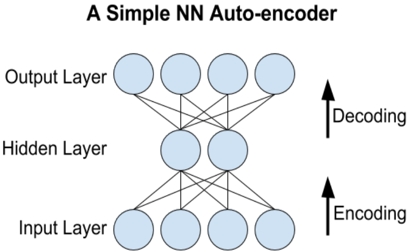
- Bag of Frequent Words: Contextual similarity is still lacking.
- **What happens if one passes a one hot encoded word as both input and output to a NN?**
- **NN Auto-encoder: Output has same form as input**. We extract the encoded vector from the hidden layer.



(Word) Embedding: Motivation

After unsupervised training with lot of online data, can a machine answer the questions like:-

- King - Man + Woman = ?
- If France:Paris, then Japan:?

(Word) Embedding: Motivation

A Hypothetical Word Vector Representation

	King	Queen	Woman	Princess
Royalty	0.98	0.98	0.01	0.93
Masculinity	0.98	0.04	0.02	0.02
Femininity	0.05	0.92	0.99	0.95
Age	0.7	0.6	0.5	0.2

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- King - Man + Woman = ?
- If King:Man then Queen:?

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- What would be the vector for Man? [0.01, 0.98, 0.05, 0.6]'
- King - Man + Woman = ? Queen(as vector subtraction and addition give nearly same result as the vector for Queen)
- If King:Man then Queen:? Woman(as vector differences of both pairs give nearly same results)

(Word) Embedding

- (Word) Embedding: Building a low-dimensional vector representation from corpus of text, which preserves the contextual similarity.
- In simple language, we want an efficient language of numbers which deep neural networks can understand as close as possible to the way we understand words.
- Training: Continuous Bag of Words Model.

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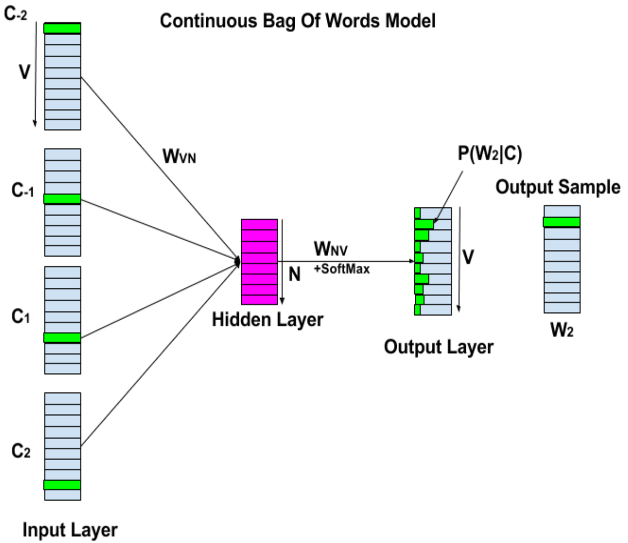
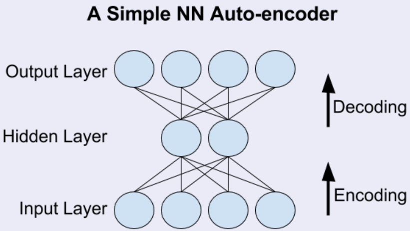
- (Word) Embedding: Building a low-dimensional vector representation from corpus of text, which preserves the contextual similarity.
- In simple language, we want an efficient language of numbers which deep neural networks can understand as close as possible to the way we understand words.
- Training: Continuous Bag of Words Model.
  - Take words in one hot encoded form. Take top V frequent words to represent each word.
  - Consider the sentence, "... I really liked the *drama*....".
  - Take a N (say 5) word window around each word and train the Neural Network with context words set $\mathcal{C}$ as input and the central word w as output.
  - For the example above use C = {"I", "really", "the" "drama"} as input and W = "liked" as output.

(Word) Embedding: Unsupervised Training



What if we want Embeddings to be Orthogonal?



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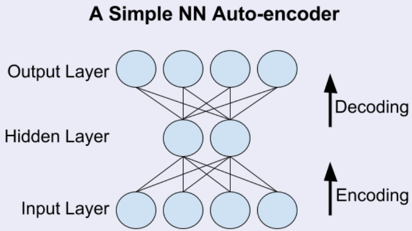

- Let $\mathbf{X}$ be a random vector and Γ its covariance matrix.
- **Principal Component Analysis:** Find a rotation of the original coordinate system and express $\mathbf{X}$ in that system so that each new coordinate expresses as much as possible of the variability in $\mathbf{X}$ as can be expressed by a linear combination of the n entries of $\mathbf{X}$. This has application in data transformation, feature discovery, feature selection and so on.

Embeddings as Generalization of PCA

- Let $\mathbf{X}$ be a random vector and Γ its covariance matrix. Let $\mathbf{e}_1, \ldots, \mathbf{e}_n$ be the n (normalized) eigenvectors of Γ.
- The n principal components of $\mathbf{X}$ are said to be $\mathbf{e}_1^T\mathbf{X}$, $\mathbf{e}_2^T\mathbf{X}$, ..., $\mathbf{e}_n^T\mathbf{X}$.

1. Let $p(X_1) = \mathcal{N}(0, 1)$ and $p(X_2) = \mathcal{N}(0, 1)$ and $cov(X_1, X_2) = \theta$. Find all the principal components of the random vector $\mathbf{X} = [X_1, X_2]^T$. [Tutorial 10]
2. Now, let $\mathbf{Y} = \mathcal{N}(\mathbf{0}, \Sigma) \in \Re^p$ where $\Sigma = \lambda^2 I_{p\times p} + \alpha^2 ones(p, p)$ for any $\lambda, \alpha \in \Re$. Here, $I_{p\times p}$ is a $p \times p$ identity matrix while $ones(p, p)$ is a $p \times p$ matrix of 1's. Find atleast one principal component of $\mathbf{Y}$. [Tutorial 10]