Learning Sets of Rules

[Read Ch. 10]
[Recommended exercises 10.1, 10.2, 10.5, 10.7, 10.8]

- Sequential covering algorithms
- FOIL
- Induction as inverse of deduction
- Inductive Logic Programming

Learning Disjunctive Sets of Rules

Method 1: Learn decision tree, convert to rules

Method 2: Sequential covering algorithm:

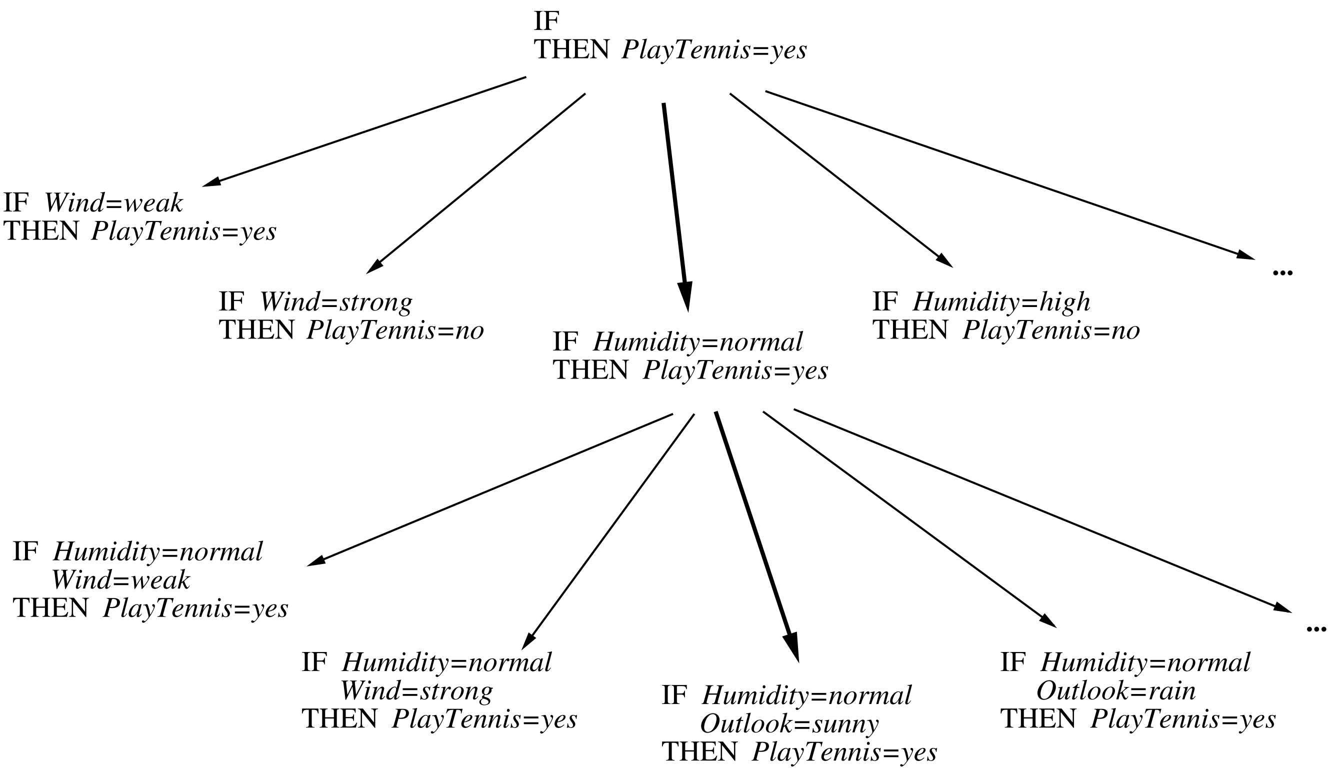
1. *Learn one rule* with high accuracy, any coverage
2. Remove positive examples covered by this rule
3. Repeat

Sequential Covering Algorithm

SEQUENTIAL-COVERING($Target\_attribute, Attributes, Examples, Threshold$)

- $Learned\_rules \leftarrow \{\}$
- $Rule \leftarrow$ LEARN-ONE-RULE($Target\_attribute, Attributes, Examples$)
- while PERFORMANCE($Rule, Examples$) $> Threshold$, do
  - $Learned\_rules \leftarrow Learned\_rules + Rule$
  - $Examples \leftarrow Examples -$ {examples correctly classified by $Rule$}
  - $Rule \leftarrow$ LEARN-ONE-RULE($Target\_attribute, Attributes, Examples$)
- $Learned\_rules \leftarrow$ sort $Learned\_rules$ accord to PERFORMANCE over $Examples$
- return $Learned\_rules$

Learn-One-Rule



Learn-One-Rule

- $Pos \leftarrow$ positive $Examples$
- $Neg \leftarrow$ negative $Examples$
- while Pos, do

  Learn a NewRule

  - $NewRule \leftarrow$ most general rule possible
  - $NewRuleNeg \leftarrow Neg$
  - while $NewRuleNeg$, do

    Add a new literal to specialize NewRule

    1. $Candidate\_literals \leftarrow$ generate candidates
    2. $Best\_literal \leftarrow \text{argmax}_{L \in Candidate\_literals}$ $Performance(SpecializeRule(NewRule, L))$
    3. add $Best\_literal$ to $NewRule$ preconditions
    4. $NewRuleNeg \leftarrow$ subset of $NewRuleNeg$ that satisfies $NewRule$ preconditions
  - $Learned\_rules \leftarrow Learned\_rules + NewRule$
  - $Pos \leftarrow Pos - \{$members of Pos covered by $NewRule\}$
- Return $Learned\_rules$

Subtleties: Learn One Rule

1. May use *beam search*
2. Easily generalizes to multi-valued target functions
3. Choose evaluation function to guide search:
   - Entropy (i.e., information gain)
   - Sample accuracy:
     $$\frac{n_c}{n}$$
     where n_c = correct rule predictions, n = all predictions
   - m estimate:
     $$\frac{n_c + mp}{n + m}$$

Variants of Rule Learning Programs

- *Sequential* or *simultaneous* covering of data?
- General $\rightarrow$ specific, or specific $\rightarrow$ general?
- Generate-and-test, or example-driven?
- Whether and how to post-prune?
- What statistical evaluation function?

Learning First Order Rules

Why do that?

- Can learn sets of rules such as

  $$Ancestor(x,y) \leftarrow Parent(x,y)$$
  $$Ancestor(x,y) \leftarrow Parent(x,z) \wedge Ancestor(z,y)$$

- General purpose programming language
  PROLOG: programs are sets of such rules

First Order Rule for Classifying Web Pages

[Slattery, 1997]

course(A) $\leftarrow$
    has-word(A, instructor),
    Not has-word(A, good),
    link-from(A, B),
    has-word(B, assign),
    Not link-from(B, C)

Train: 31/31, Test: 31/34

FOIL($Target\_predicate, Predicates, Examples$)

- $Pos \leftarrow$ positive $Examples$
- $Neg \leftarrow$ negative $Examples$
- while Pos, do

  Learn a NewRule

  - $NewRule \leftarrow$ most general rule possible
  - $NewRuleNeg \leftarrow Neg$
  - while $NewRuleNeg$, do

    Add a new literal to specialize NewRule

    1. $Candidate\_literals \leftarrow$ generate candidates
    2. $Best\_literal \leftarrow$ $\text{argmax}_{L \in Candidate\_literals} Foil\_Gain(L, NewRule)$
    3. add $Best\_literal$ to $NewRule$ preconditions
    4. $NewRuleNeg \leftarrow$ subset of $NewRuleNeg$ that satisfies $NewRule$ preconditions
  - $Learned\_rules \leftarrow Learned\_rules + NewRule$
  - $Pos \leftarrow Pos - \{$members of Pos covered by $NewRule\}$
- Return $Learned\_rules$

Specializing Rules in FOIL

Learning rule: $P(x_1, x_2, \ldots, x_k) \leftarrow L_1 \ldots L_n$
Candidate specializations add new literal of form:

- $Q(v_1, \ldots, v_r)$, where at least one of the v_i in the created literal must already exist as a variable in the rule.
- $Equal(x_j, x_k)$, where x_j and x_k are variables already present in the rule
- The negation of either of the above forms of literals

Information Gain in FOIL

$$Foil\_Gain(L,R) \equiv t\left(\log_2 \frac{p_1}{p_1+n_1} - \log_2 \frac{p_0}{p_0+n_0}\right)$$

Where

- L is the candidate literal to add to rule R
- p_0 = number of positive bindings of R
- n_0 = number of negative bindings of R
- p_1 = number of positive bindings of $R + L$
- n_1 = number of negative bindings of $R + L$
- t is the number of positive bindings of R also covered by $R + L$

Note

- $-\log_2 \frac{p_0}{p_0+n_0}$ is optimal number of bits to indicate the class of a positive binding covered by R

Induction as Inverted Deduction

Induction is finding h such that

$$(\forall \langle x_i, f(x_i)\rangle \in D) \; B \wedge h \wedge x_i \vdash f(x_i)$$

where

- x_i is ith training instance
- $f(x_i)$ is the target function value for x_i
- B is other background knowledge

So let's design inductive algorithm by inverting operators for automated deduction!

Induction as Inverted Deduction

"pairs of people, $\langle u, v\rangle$ such that child of u is v,"

$$\begin{array}{rc}
f(x_i): & Child(Bob, Sharon) \\
x_i: & Male(Bob), Female(Sharon), Father(Sharon, Bob) \\
B: & Parent(u,v) \leftarrow Father(u,v)
\end{array}$$

What satisfies $(\forall \langle x_i, f(x_i)\rangle \in D)\ B \wedge h \wedge x_i \vdash f(x_i)$?

$$h_1: \ Child(u,v) \leftarrow Father(v,u)$$
$$h_2: \ Child(u,v) \leftarrow Parent(v,u)$$

Induction is, in fact, the inverse operation of deduction, and cannot be conceived to exist without the corresponding operation, so that the question of relative importance cannot arise. Who thinks of asking whether addition or subtraction is the more important process in arithmetic? But at the same time much difference in difficulty may exist between a direct and inverse operation; ... it must be allowed that inductive investigations are of a far higher degree of difficulty and complexity than any questions of deduction....

(Jevons 1874)

Induction as Inverted Deduction

We have mechanical *deductive* operators
$F(A, B) = C$, where $A \wedge B \vdash C$

need *inductive* operators

$O(B, D) = h$ where $(\forall \langle x_i, f(x_i) \rangle \in D)\ (B \wedge h \wedge x_i) \vdash f(x_i)$

Induction as Inverted Deduction

Positives:

- Subsumes earlier idea of finding h that "fits" training data
- Domain theory B helps define meaning of "fit" the data

$$B \wedge h \wedge x_i \vdash f(x_i)$$

- Suggests algorithms that search H guided by B

Induction as Inverted Deduction

Negatives:

- Doesn't allow for noisy data. Consider

$$(\forall \langle x_i, f(x_i) \rangle \in D) \; (B \wedge h \wedge x_i) \vdash f(x_i)$$

- First order logic gives a *huge* hypothesis space H

  $\rightarrow$ overfitting...

  $\rightarrow$ intractability of calculating all acceptable h's

Deduction: Resolution Rule

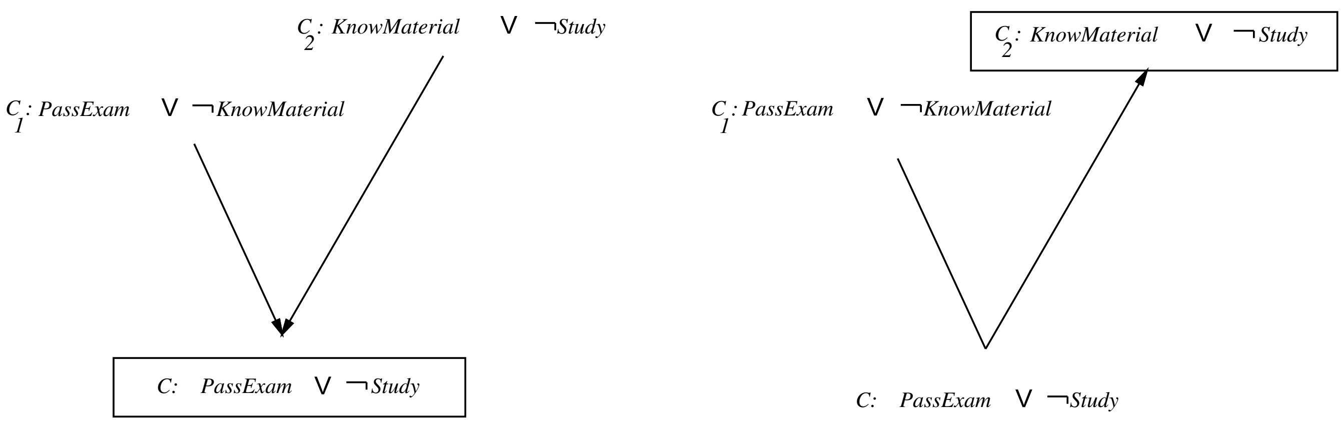
$$\frac{\begin{array}{ccc} P & \vee & L \\ \neg L & \vee & R \end{array}}{\begin{array}{ccc} P & \vee & R \end{array}}$$

1. Given initial clauses C_1 and C_2, find a literal L from clause C_1 such that $\neg L$ occurs in clause C_2
2. Form the resolvent C by including all literals from C_1 and C_2, except for L and $\neg L$. More precisely, the set of literals occurring in the conclusion C is

$$C = (C_1 - \{L\}) \cup (C_2 - \{\neg L\})$$

   where $\cup$ denotes set union, and "$-$" denotes set difference.

Inverting Resolution



Inverted Resolution (Propositional)

1. Given initial clauses C_1 and C, find a literal L that occurs in clause C_1, but not in clause C.

2. Form the second clause C_2 by including the following literals

$$C_2 = (C - (C_1 - \{L\})) \cup \{\neg L\}$$

First order resolution

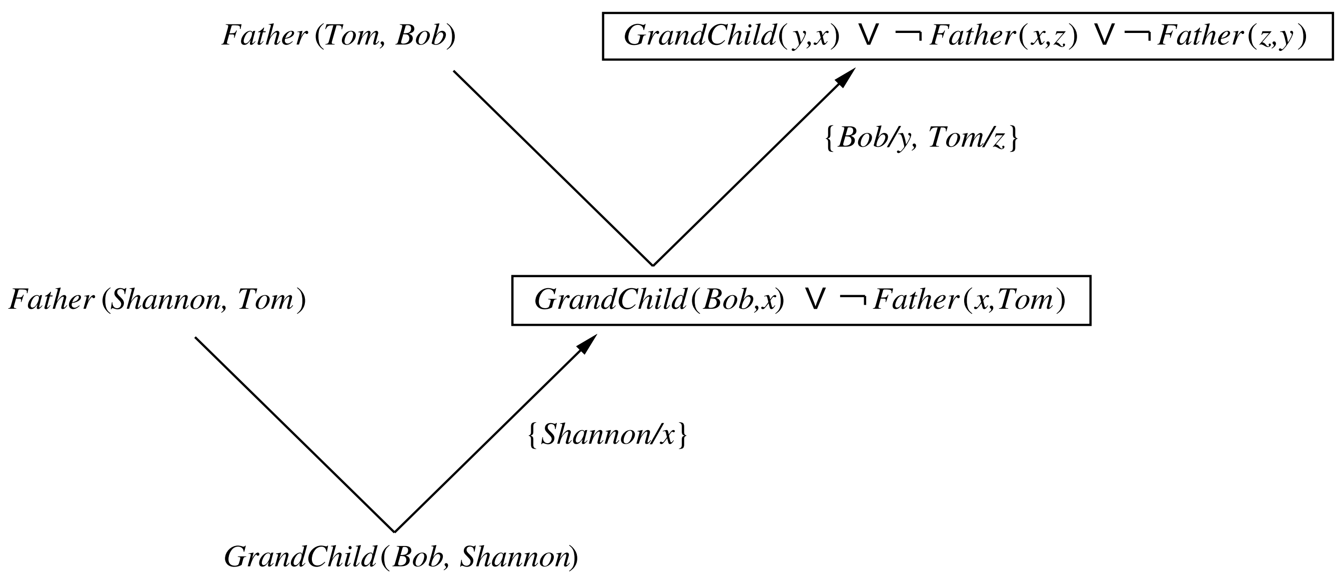
First order resolution:

1. Find a literal L_1 from clause C_1, literal L_2 from clause C_2, and substitution θ such that $L_1\theta = \neg L_2\theta$
2. Form the resolvent C by including all literals from $C_1\theta$ and $C_2\theta$, except for $L_1\theta$ and $\neg L_2\theta$. More precisely, the set of literals occurring in the conclusion C is
$$C = (C_1 - \{L_1\})\theta \cup (C_2 - \{L_2\})\theta$$

Inverting First order resolution

$$C_2 = (C - (C_1 - \{L_1\})\theta_1)\theta_2^{-1} \cup \{\neg L_1 \theta_1 \theta_2^{-1}\}$$

Cigol



Father (Tom, Bob)
GrandChild(y,x) ∨ ¬ Father(x,z) ∨ ¬ Father(z,y)
{Bob/y, Tom/z}
Father (Shannon, Tom)
GrandChild(Bob,x) ∨ ¬ Father(x,Tom)
{Shannon/x}
GrandChild(Bob, Shannon)

Progol

PROGOL: Reduce comb explosion by generating the most specific acceptable h

1. User specifies H by stating predicates, functions, and forms of arguments allowed for each
2. PROGOL uses sequential covering algorithm. For each $\langle x_i, f(x_i) \rangle$
   - Find most specific hypothesis h_i s.t. $B \wedge h_i \wedge x_i \vdash f(x_i)$
     - actually, considers only k-step entailment
3. Conduct general-to-specific search bounded by specific hypothesis h_i, choosing hypothesis with minimum description length