Mesh Simplification

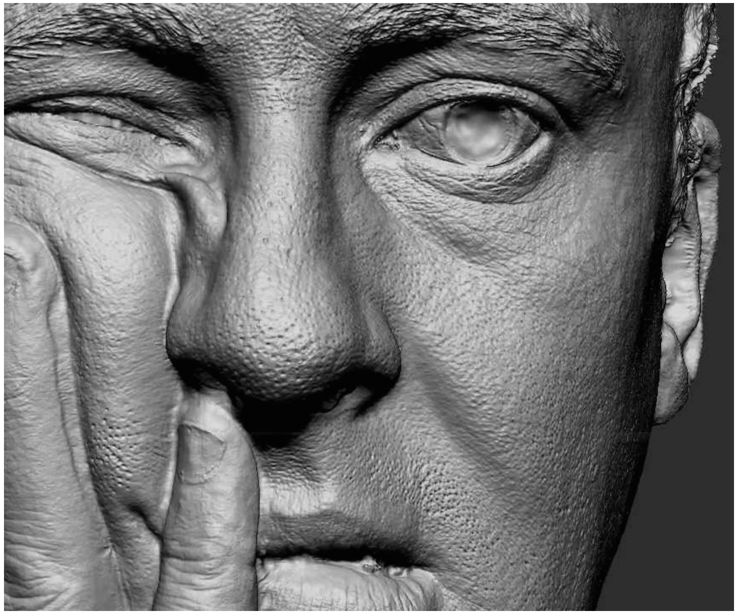
(Slides from Tom Funkhouser, Adam Finkelstein)

Siddhartha Chaudhuri

http://www.cse.iitb.ac.in/~cs749

In a nutshell

- **Problem:**
  - Meshes have too many polygons for storage, rendering, analysis etc
    - High-resolution scanning
    - Marching cubes running amok
    - Artist sculpted too many details in Zbrush



- **Solution:**
  - Simplify the mesh by reducing the poly count

Ten24

Thought for the Day #1

How can we simplify a mesh?

What is Mesh Simplification?

- **Mesh simplification** is a class of algorithms that transform a polygonal mesh into another with fewer faces/edges/vertices
- The simplification process is controlled by user-defined criteria that try to preserve properties of the original mesh as much as possible: curvature, surface metrics, edge loops etc
- Simplification reduces the complexity of a mesh

Mesh Simplification Overview

Some algorithms

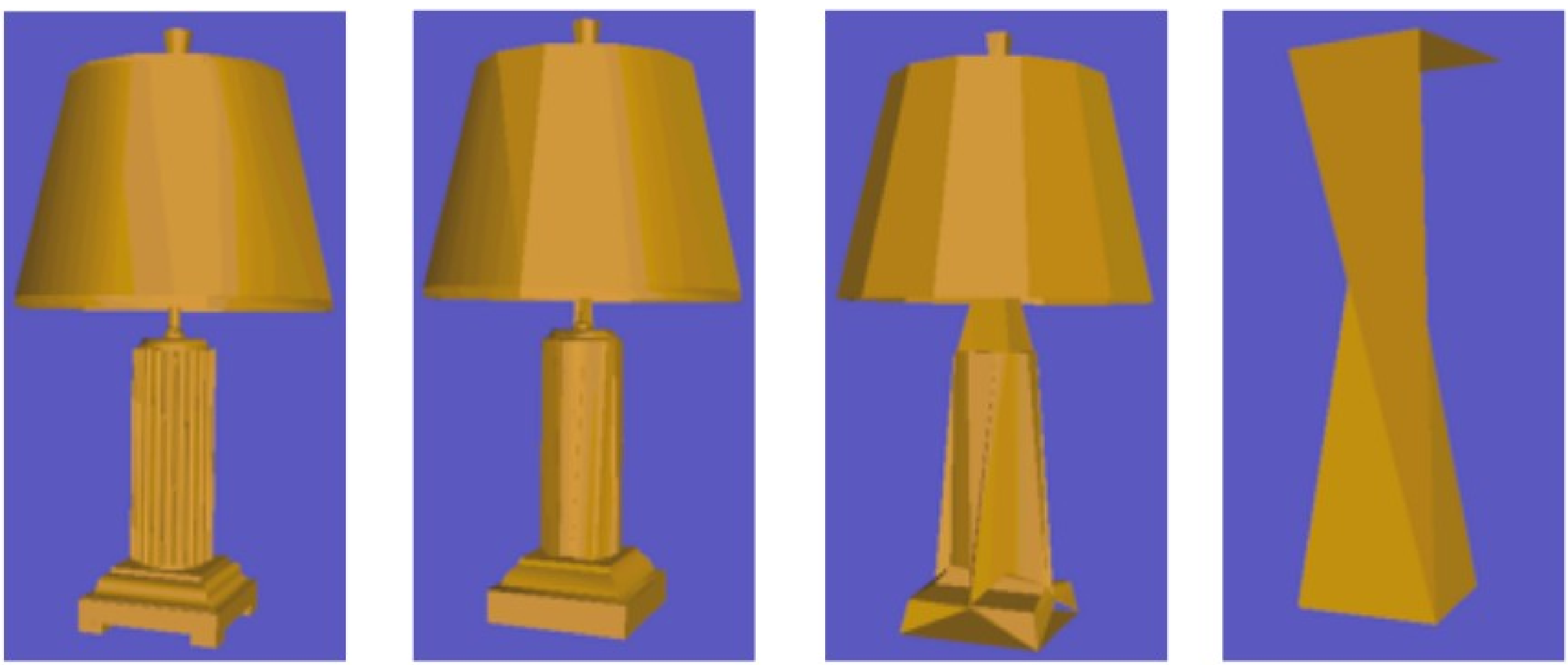
- Vertex clustering
- Mesh retiling
- Mesh optimization
- Mesh decimation

Considerations

- Speed of algorithm
- Quality of approximation
- Generality (applies to many types of meshes)
- Topology modifications
- Control of approximation quality
- Continuous levels of detail
- Smooth transitions

Vertex Clustering

- Partition vertices into clusters
- Replace all vertices in each cluster with one representative

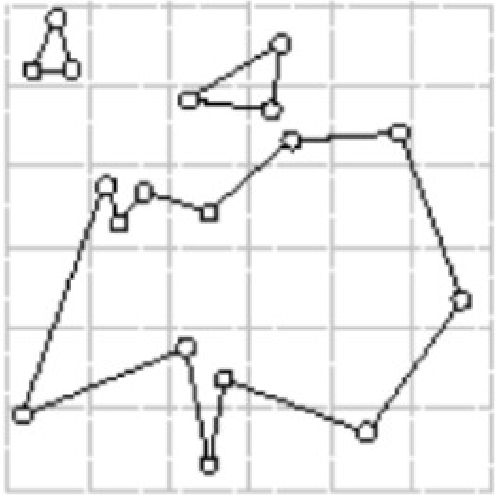


Rossignac and Borrel, “Multi-resolution 3D approximations for rendering complex scenes”, 1993

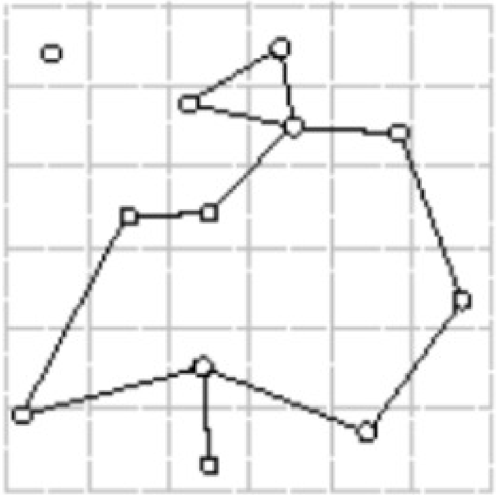
Vertex Clustering

- Algorithm [Rossignac93]:
  - Build grid containing vertices
  - Merge vertices in same grid cell
    - Select new position for representative vertex
    - Collapse degenerate edges and faces

- Pros: Fast
- Cons:
  - Collapses topology
  - Low quality
  - Hard to control



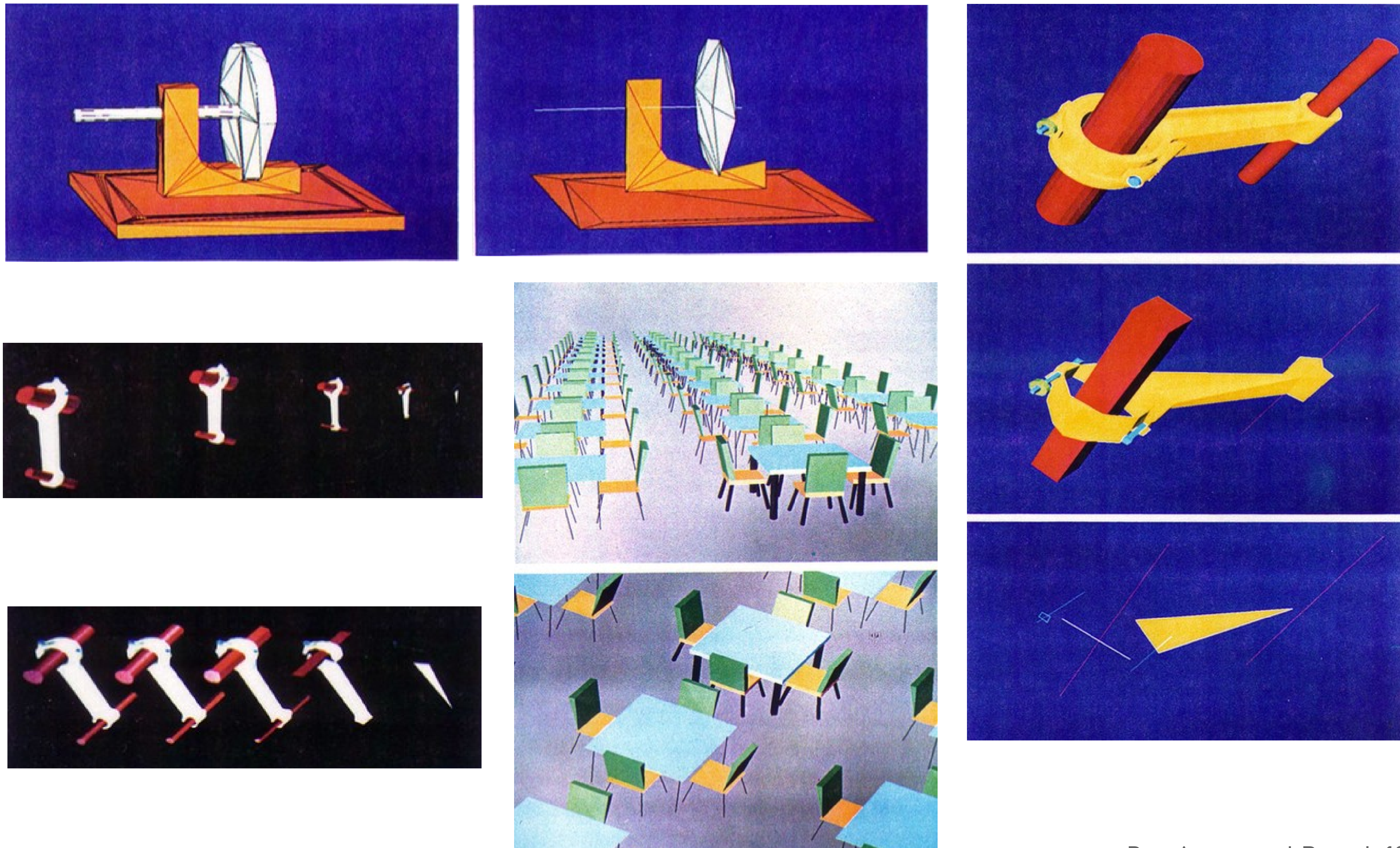
Before



After

Rossignac and Borrel, “Multi-resolution 3D approximations for rendering complex scenes”, 1993

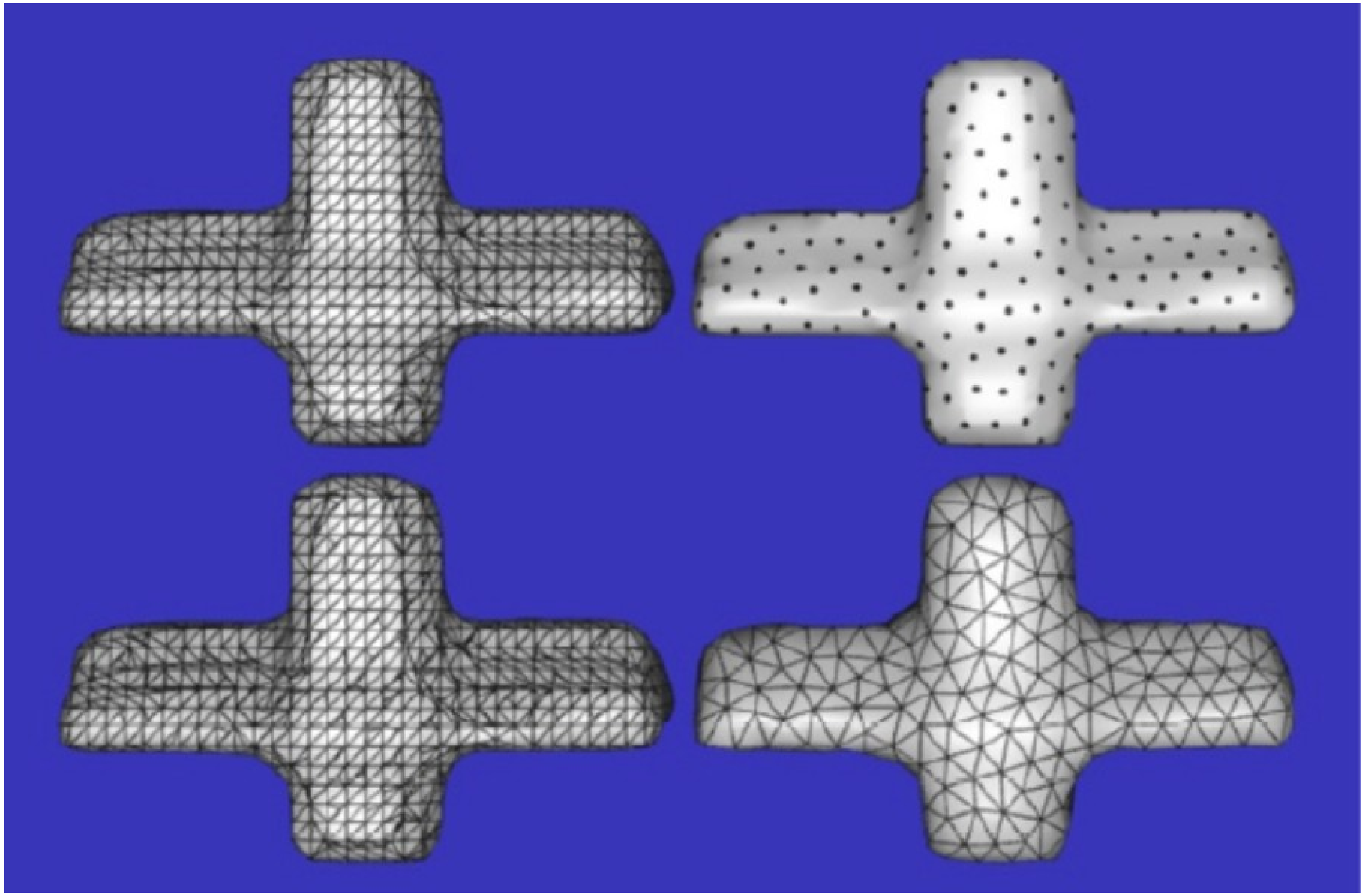
Vertex Clustering



Rossignac and Borrel, 1993

Mesh Re-tiling

- Resample mesh with "uniformly spaced" vertices

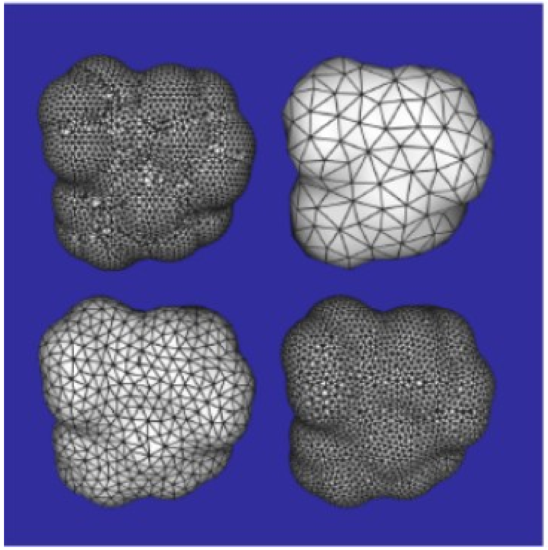


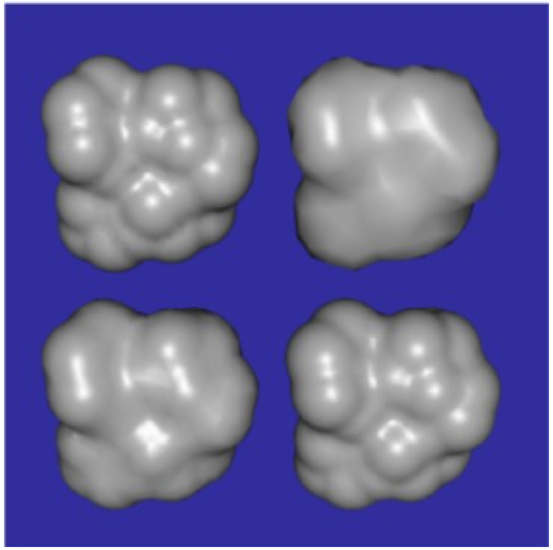
Greg Turk, "Re-tiling polygonal surfaces", 1992

Mesh Re-tiling

- Algorithm [Turk92]:
  - Generate random points on surface
  - Use diffusion/repulsion to spread them uniformly
  - Tessellate vertices (many details here!)

- Pros:
  - Respects topology
- Cons:
  - Slow
  - Blurs sharp features



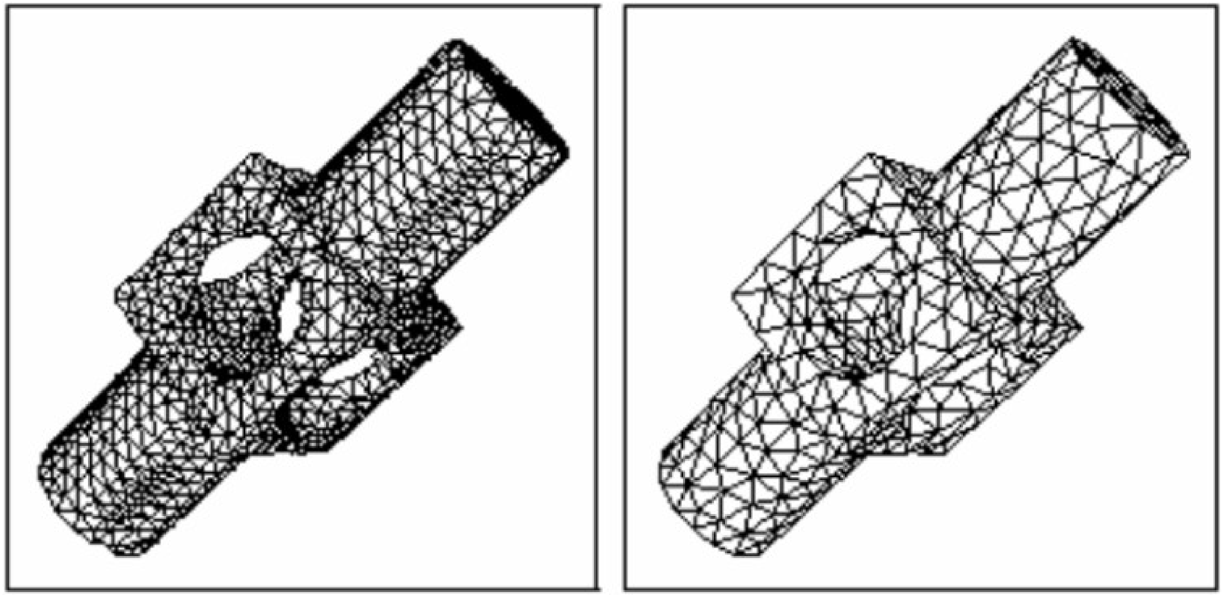


Greg Turk, “Re-tiling polygonal surfaces”, 1992

Mesh Optimization

- Apply optimization procedure to minimize an objective function $E(K, V)$

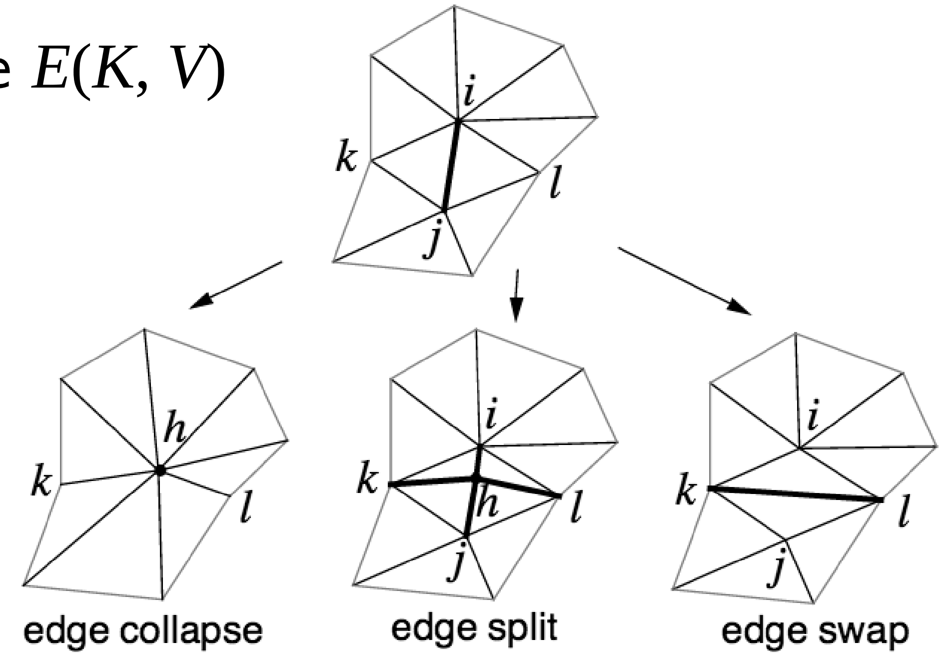
$$E(K, V) = E_{\text{dist}}(K, V) + E_{\text{rep}}(K, V) + E_{\text{spring}}(K, V)$$



Hoppe et al., “Mesh optimization”, 1993

Mesh Optimization

- Algorithm [Hoppe92]:
  - Iterate with a decreasing spring term:
    - Randomly modify topology with edge collapse, edge swap, or edge split
    - Move vertices to minimize $E(K, V)$
    - Keep change if it reduces objective function



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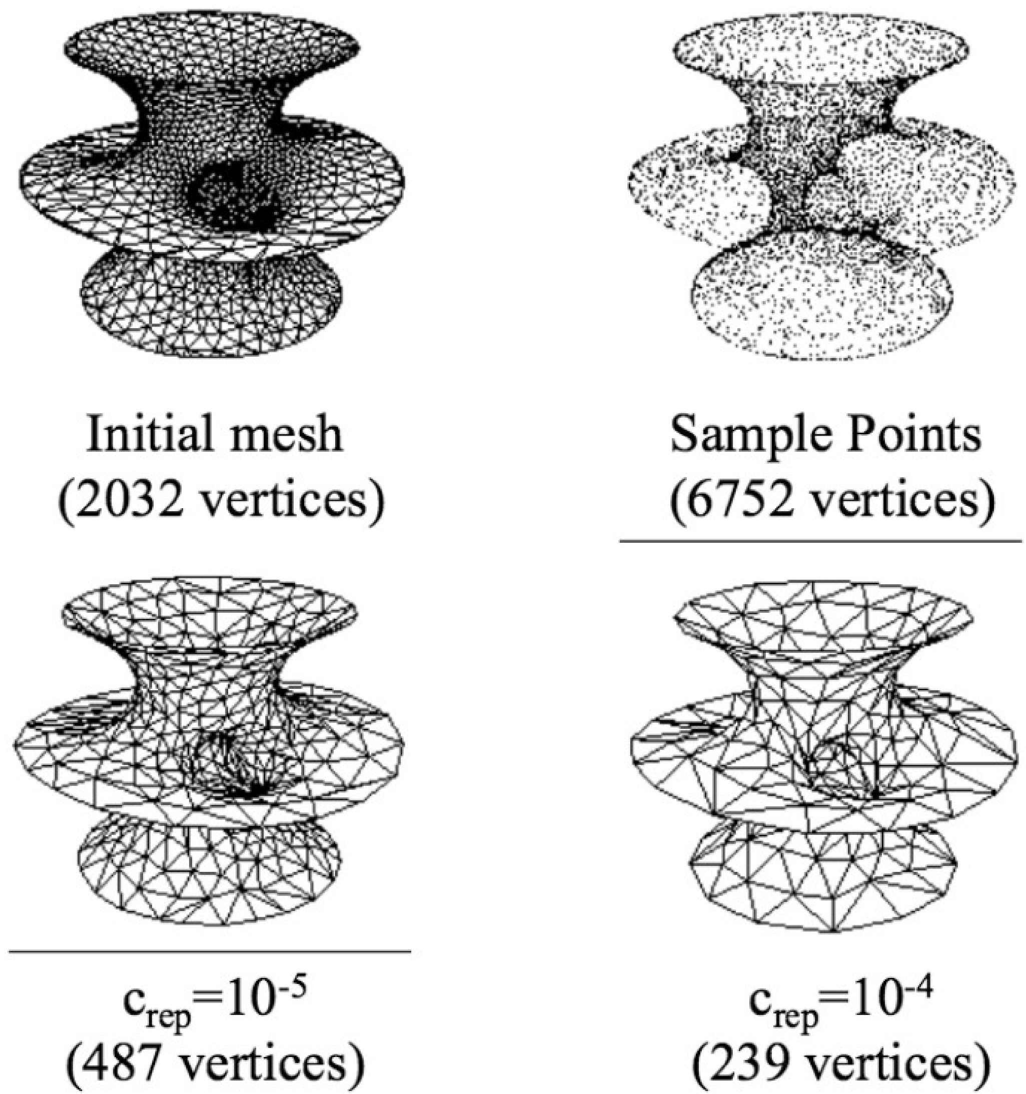
Mesh Optimization



Hoppe et al., “Mesh optimization”, 1993

Mesh Decimation

- Apply iterative, greedy algorithm to gradually reduce complexity of mesh
  - Measure error of possible decimation operations
  - Place operations in queue according to error
  - Perform operations in queue successively
  - After each operation, re-evaluate error metrics

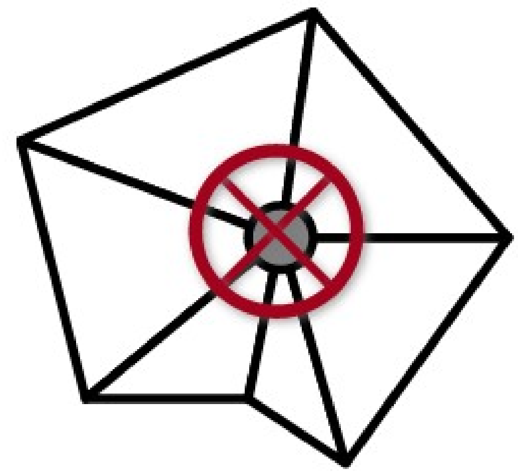
Mesh Decimation Operations

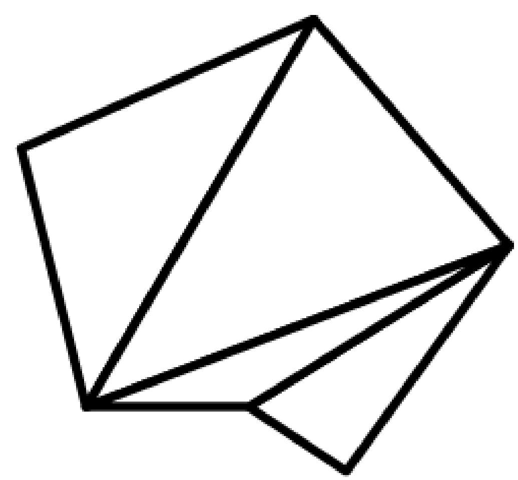
- General idea:
  - Each operation simplifies mesh by small amount
  - Apply operations successively

- Types of operations:
  - Vertex remove
  - Edge collapse
  - Vertex cluster

Vertex Remove

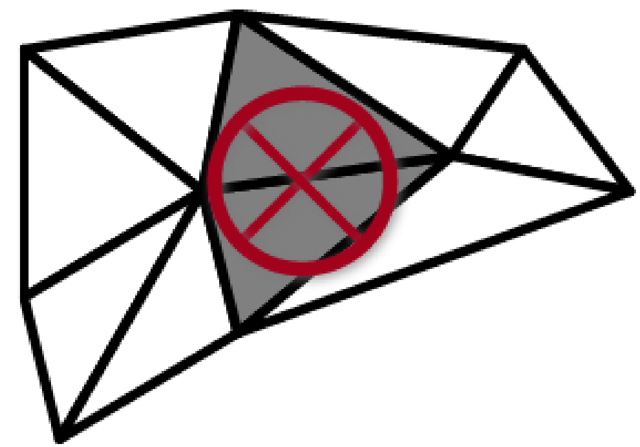
- Method:
  - Remove vertex and adjacent faces
  - Fill hole with new triangles (2 fewer triangles)
- Properties:
  - Requires manifold surface around vertex
  - Preserves local topological structure

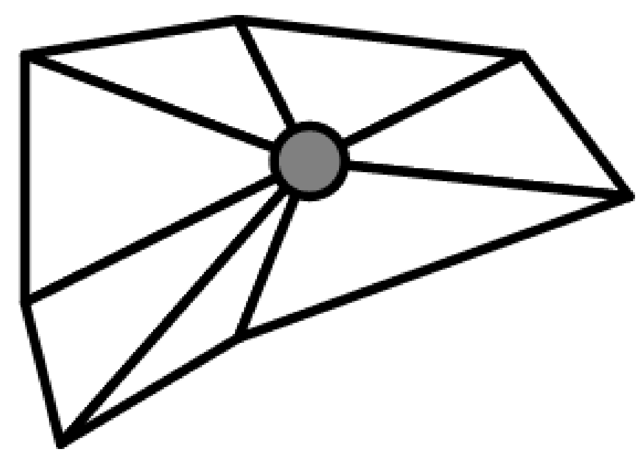




Edge Collapse

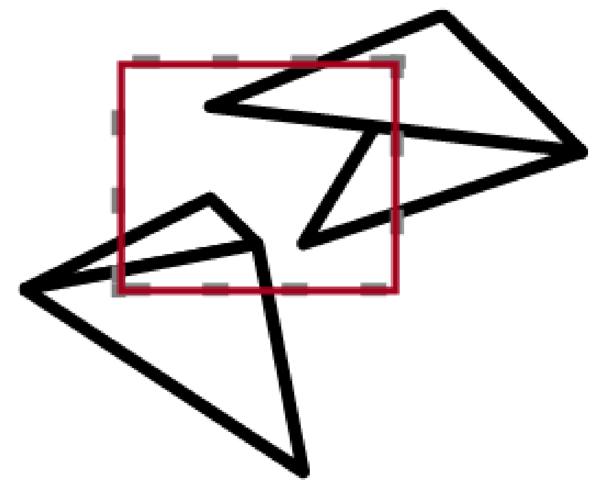
- Method:
  - Merge two vertices into one
  - Remove degenerate triangles
- Properties:
  - Requires manifold surface around vertex
  - Preserves local topological structure
  - Allows smooth transition

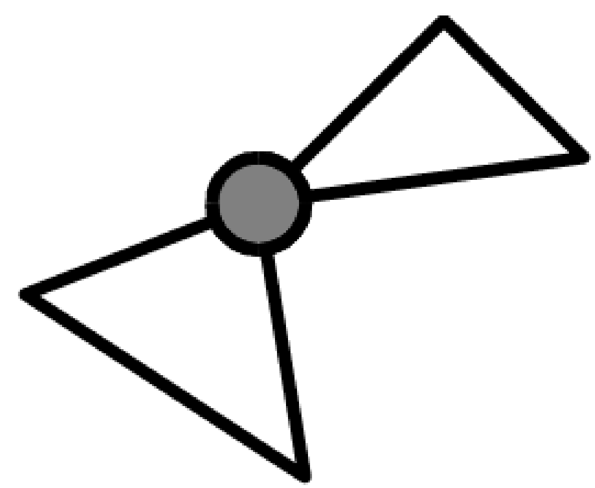




Vertex Cluster

- Method:
  - Merge vertices based on proximity
  - Triangles with repeated vertices become edges/points
- Properties:
  - General, robust
  - Topological changes possible
  - Not great quality





Operation Considerations

- Topology considerations:
    - Attention to topology promotes better appearance
    - Allowing non-manifolds increases robustness and ability to simplify
- Operation considerations:
    - Collapse-type operations allow smooth transitions
    - Vertex remove affects smaller portion of mesh than edge collapse

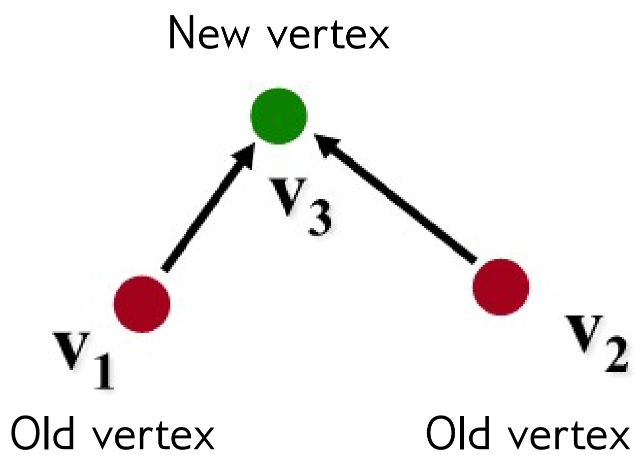
Mesh Decimation Error Metrics

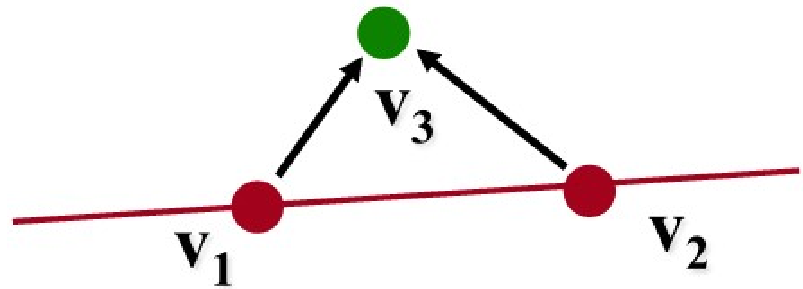
- Motivation:
  - Promote accurate 3D shape preservation
  - Preserve screen-space silhouettes and pixel coverages
- Types:
  - Vertex-Vertex distance
  - Surface-Surface distance
  - Point-Surface distance
  - Vertex-Plane distance

Vertex-Vertex Distance

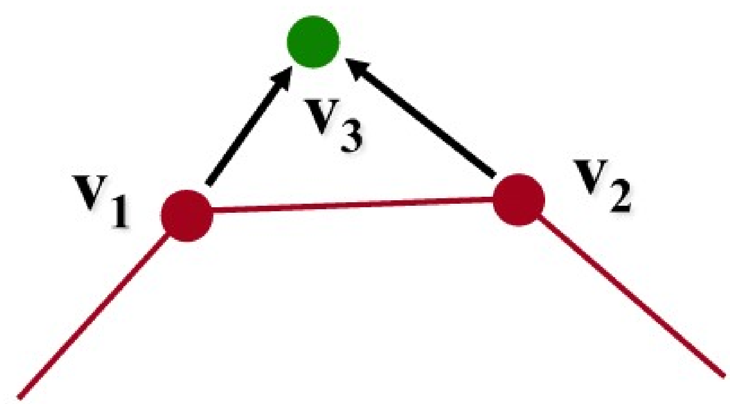
$$E = \max(\, \| v_3 - v_1 \|,\ \| v_2 - v_1 \| \,)$$

- Rossignac and Borrel 1993
- Luebke and Erikson 1997



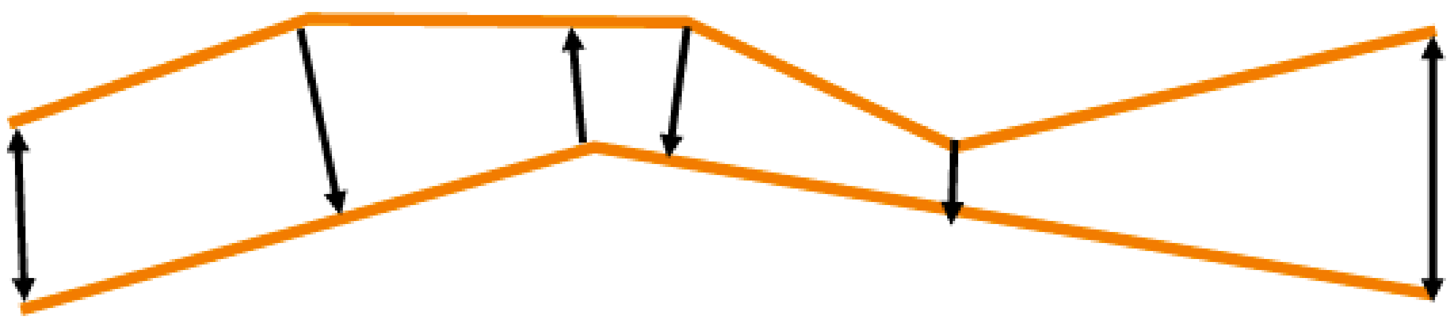


Not very discriminative, e.g. does not distinguish between these two cases



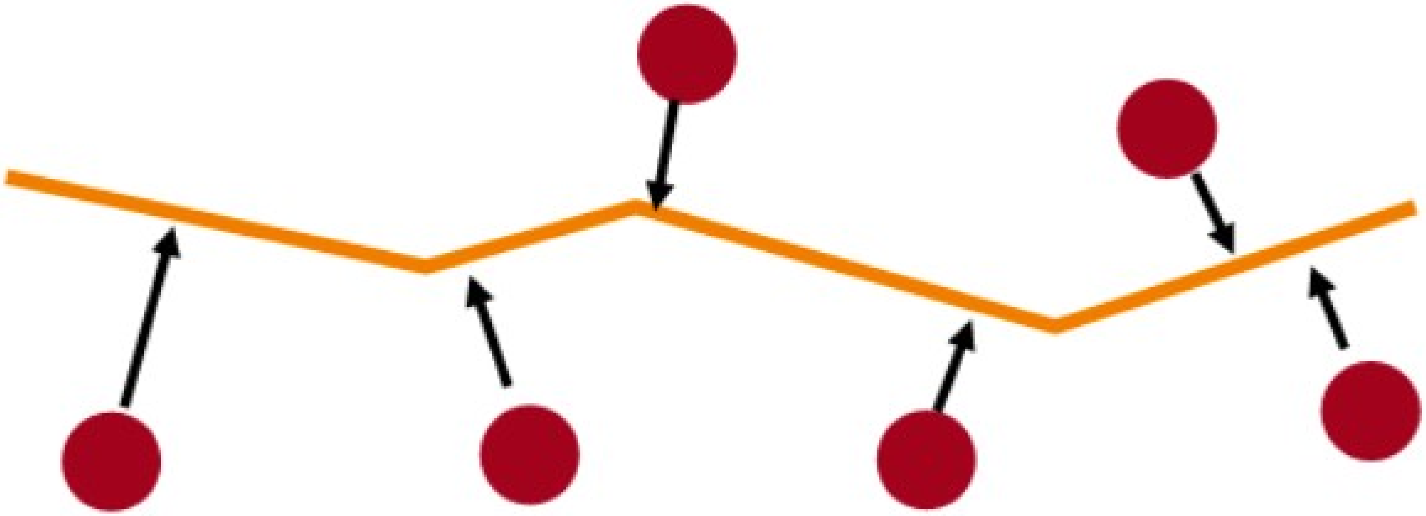
Surface-Surface Distance

- Error is maximum distance between original and simplified surface
  - Tolerance volumes [Gueziéc 1996]
  - Simplification envelopes [Cohen/Varshney 1996]
  - Hausdorff distance [Klein 1996]
  - Mapping distance [Bajaj/Shikore 1996, Cohen et al. 1997]



Point-Surface Distance

- Error is sum of squared distances from original vertices to closest points on simplified surface
  - Hoppe et al. 1992

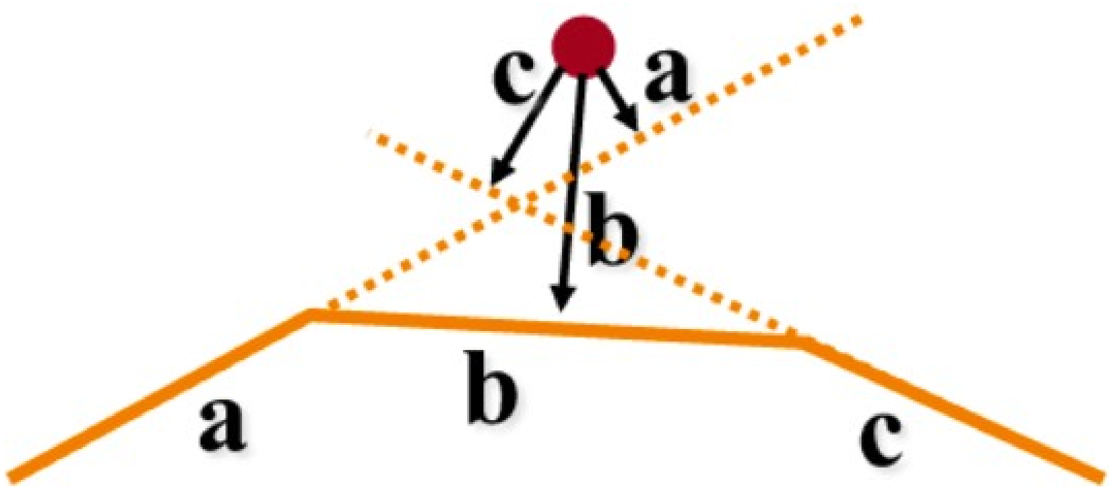


Vertex-Plane Distance

- Error is based on distances of original vertices from planes of faces in simplified surface
  - Max distance to plane
    - Maintain set of planes for each vertex [Ronfard/Rossignac 1996]
  - Sum of squared distances
    - Approximated by quadric at each vertex [Garland/Heckbert 1997]

Quadric Error Metric

- Error is sum of squared distances of original vertices from planes of faces in simplified surface
  - How to compute the error?
  - How to perform the atomic decimation operation?



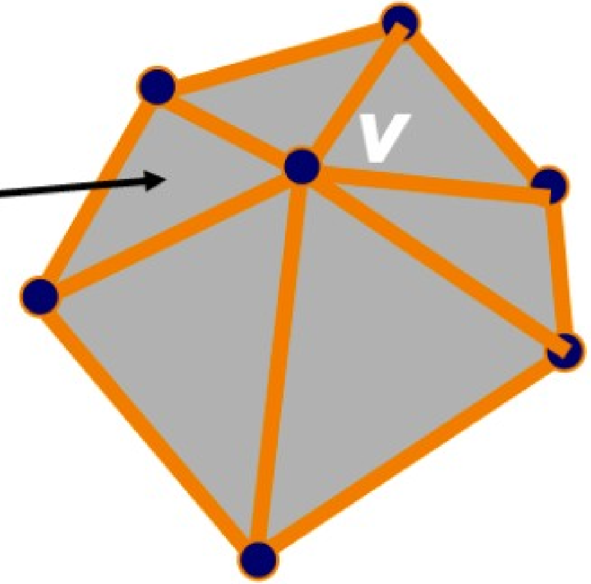
Garland and Heckbert, "Surface simplification using quadric error metrics", 1997

Quadric Error Metric

- Sum of squared distances from vertex to planes

$$\Delta_{\mathbf{v}} = \sum_{\mathbf{p}} Dist(\mathbf{v},\mathbf{p})^2$$

$$\mathbf{v} = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}, \quad \mathbf{p} = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$



$$Dist(\mathbf{v},\mathbf{p}) = ax + by + cz + d = \mathbf{p}^{\mathrm{T}}\mathbf{v}$$

Garland and Heckbert, “Surface simplification using quadric error metrics”, 1997

Quadric Error Metric

- Common mathematical trick:
  - Quadratic form = symmetric matrix Q multiplied twice by a vector $\mathbf{v}$

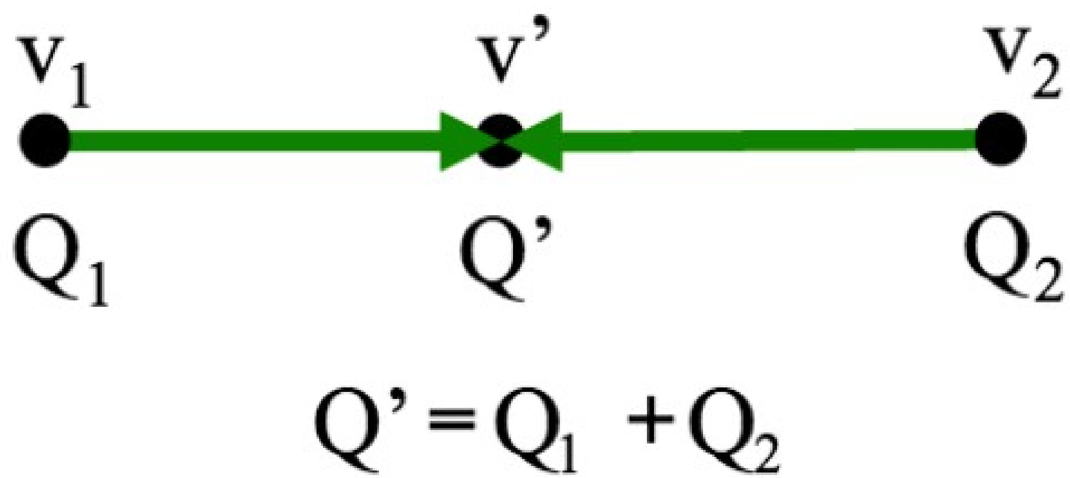
$$
\begin{aligned}
\Delta &= \sum_{\mathbf{p}} (\mathbf{p}^{\mathrm{T}}\mathbf{v})^2 \\
&= \sum_{\mathbf{p}} \mathbf{v}^{\mathrm{T}}\mathbf{p}\mathbf{p}^{\mathrm{T}}\mathbf{v} \\
&= \mathbf{v}^{\mathrm{T}}\left(\sum_{\mathbf{p}} \mathbf{p}\mathbf{p}^{\mathrm{T}}\right)\mathbf{v} \\
&= \mathbf{v}^{\mathrm{T}}\mathbf{Q}\mathbf{v}
\end{aligned}
$$

$$
Q = \begin{bmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & c^2 & cd \\ ad & bd & cd & d^2 \end{bmatrix}
$$

Garland and Heckbert, “Surface simplification using quadric error metrics”, 1997

Quadric Error Metric

- Approximate error of edge collapses
  - Each vertex $\mathbf{v}_i$ has associated quadric Q_i
  - Error of collapsing $\mathbf{v}_1$ and $\mathbf{v}_2$ to $\mathbf{v}'$ is $\mathbf{v}'^{\mathrm{T}} Q_1 \mathbf{v}' + \mathrm{v}'^{\mathrm{T}} Q_2 \mathbf{v}'$
  - Quadric for new vertex $\mathbf{v}'$ is $Q' = Q_1 + Q_2$



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Quadric Error Metric

- Find optimal location $\mathbf{v}'$ after collapse

$$\mathbf{Q}' = \begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{12} & q_{22} & q_{23} & q_{24} \\ q_{13} & q_{23} & q_{33} & q_{34} \\ q_{14} & q_{24} & q_{34} & q_{44} \end{bmatrix}$$

$$\min_{\mathbf{v}'} \mathbf{v}'^{\mathrm{T}} \mathbf{Q}' \mathbf{v}' : \quad \frac{\partial}{\partial x} = \frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0$$

Garland and Heckbert, "Surface simplification using quadric error metrics", 1997

Quadric Error Metric

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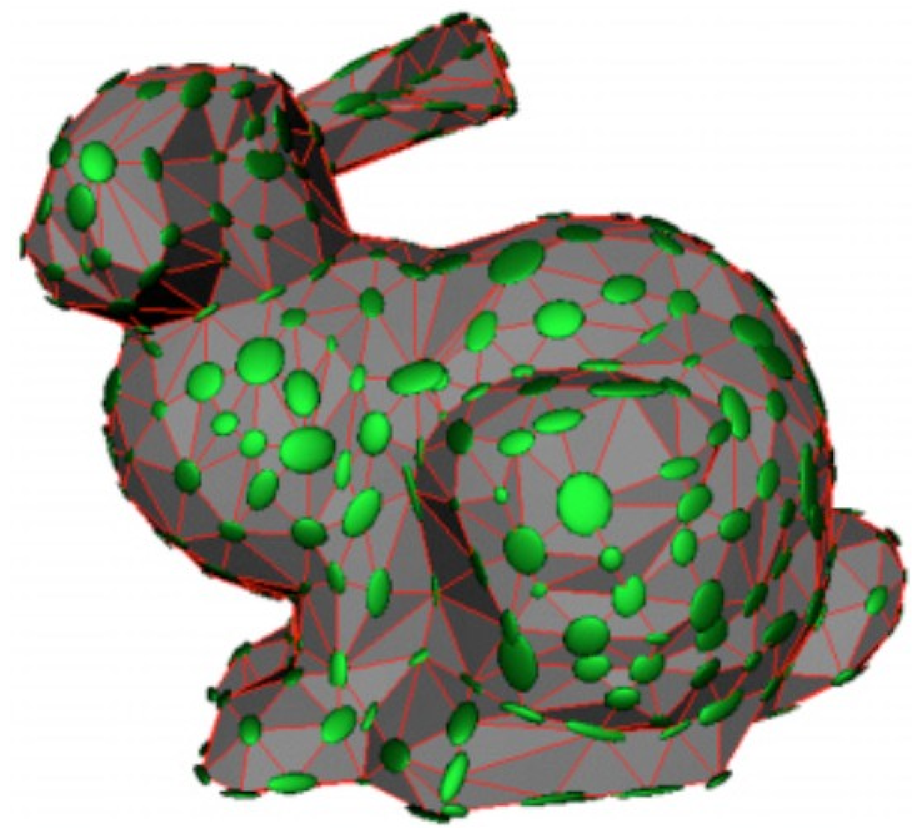
$$\begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{12} & q_{22} & q_{23} & q_{24} \\ q_{13} & q_{23} & q_{33} & q_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \mathbf{v}' = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{v}' = \begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{12} & q_{22} & q_{23} & q_{24} \\ q_{13} & q_{23} & q_{33} & q_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Garland and Heckbert, “Surface simplification using quadric error metrics”, 1997

Quadric Error Visualization

- Ellipsoids: iso-error surfaces
  - Smaller ellipsoids represent greater error for a given vertex motion
  - Lower error for motion parallel to surface
  - Lower error in flat regions than corners
  - Elongated in “cylindrical” regions near ridges



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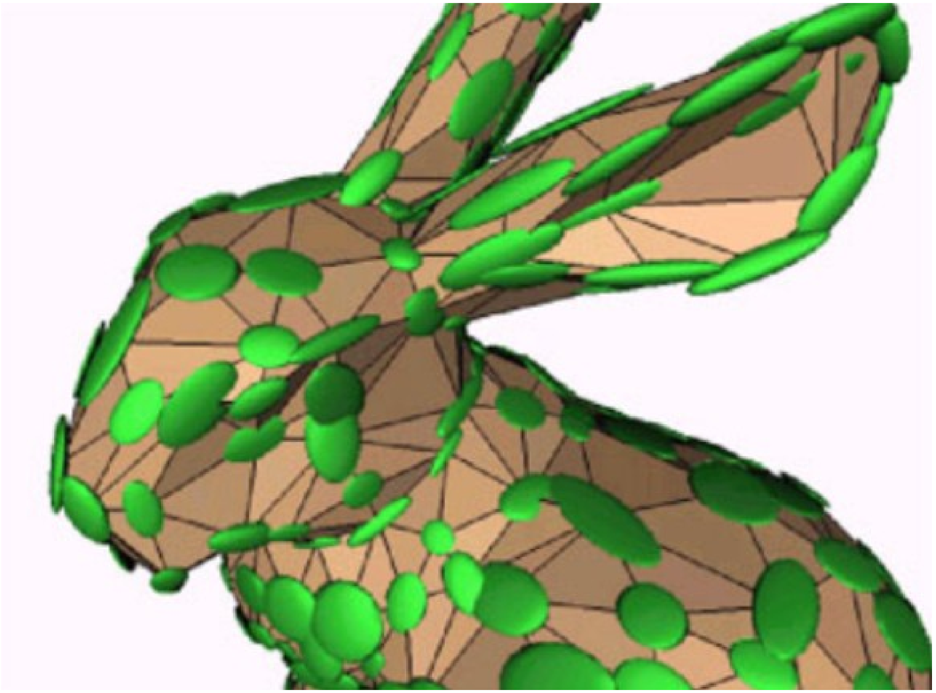
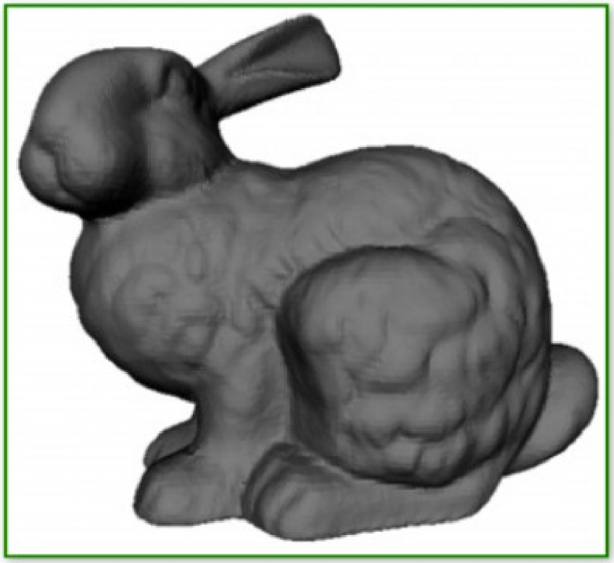
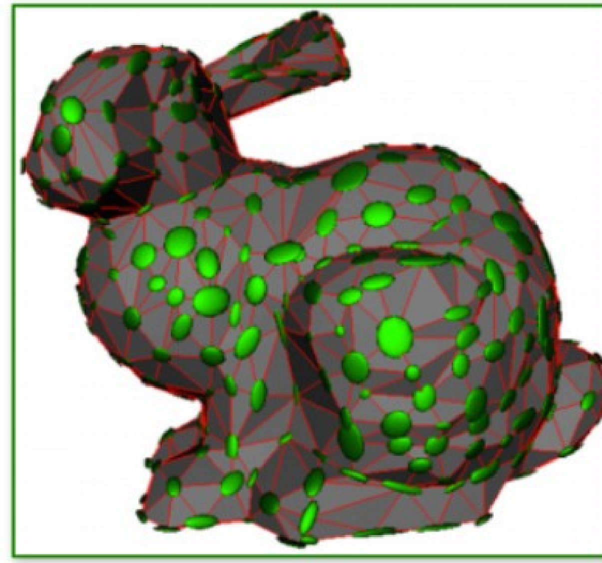


Garland and Heckbert, “Surface simplification using quadric error metrics”, 1997

Quadric Error Metric Results



Original



Quadrics

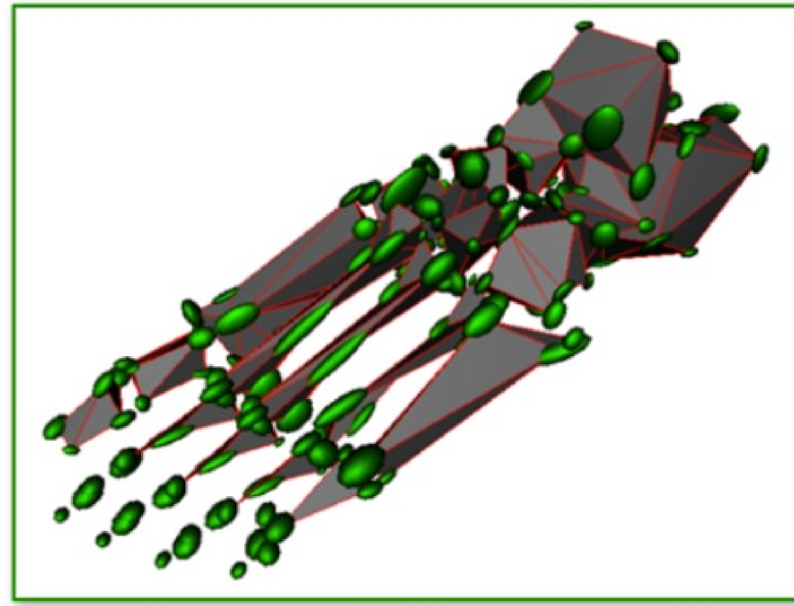
1k tris

100 tris

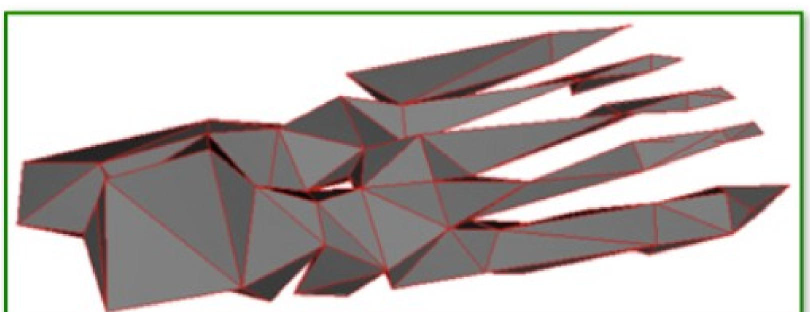
Garland and Heckbert, “Surface simplification using quadric error metrics”, 1997

Quadric Error Metric Results

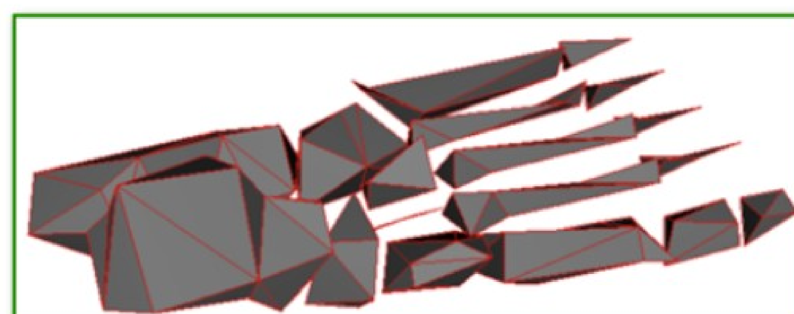
Original



Quadrics



250 tris



250 tris, edge collapses only

Garland and Heckbert, “Surface simplification using quadric error metrics”, 1997

Quadric Error Metric Details

- Boundary preservation: add planes perpendicular to boundary edges
- Prevent foldovers: check for normal flipping
- Merging nearby vertices: Create virtual edges between vertices closer than some threshold

Garland and Heckbert, “Surface simplification using quadric error metrics”, 1997