Polygon Meshes

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What is a polygon mesh?

• Like a point cloud, it is a discrete sampling of a surface

• ... but, it adds **simple polygons** (no holes or self-intersections) as linear (flat) approximations of local regions of the actual underlying surface
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The original samples become vertices of the polygons
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- Like point clouds, meshes can have different resolutions.
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• Like point clouds, meshes can have different resolutions
  • ... at different places ("adaptive meshing")
Elements of a mesh
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- Vertices
- Edges
- Faces
Elements of a mesh

- Non-Boundary Vertex
- Boundary Vertex
- Non-Boundary Edge
- Boundary Edge
- Triangular Face
- Quadrilateral (Quad) Face (should be planar!)
A mesh is a graph

This cannot be stressed strongly enough!
A mesh is an undirected graph
The vertex positions capture the **geometry** of the surface.
The mesh connectivity captures the **topology** of the surface
Mesh Geometry: Planes and Normals

• Each polygon is (assumed to be) planar
  • Triangular faces are always planar
  • Quads and higher degree faces need not be
    – Ambiguity revealed by triangulation
• Many mesh formats allow non-planar faces, but most algorithms assume planar faces. Caveat emptor.

Always planar

Same 4 non-coplanar vertices, different geometry!
Mesh Geometry: Planes and Normals

- The plane of each polygon has an associated normal vector
The plane of each polygon has an associated normal vector \( \hat{n} = \frac{\mathbf{e}_1 \times \mathbf{e}_2}{\|\mathbf{e}_1 \times \mathbf{e}_2\|} \).

**Q:** The computed normal may point into or out of the object. Which one to pick?

**A:** Either (typically outwards), but be consistent across the shape! Using our formula here, the normal is outwards if the vertices wind counter-clockwise around the face when seen from outside the shape.
Mesh Geometry: Planes and Normals

• The plane of each polygon has an associated plane equation: \( \hat{n} \cdot (p - v_0) = 0 \)
Mesh Geometry: Planes and Normals

- We can also associate vertices with normals
  - Sometimes they come with the mesh (e.g. if they were estimated when the mesh was constructed from a point cloud)
  - Sometimes we have to estimate them
Estimating vertex normals

- **Simplest**: Add up the normals of adjacent faces and unitize.
Estimating vertex normals

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- **Simple and usually a bit better**: Add up the normals of adjacent faces, weighted by face areas

Without area-weighting | With area-weighting
Estimating vertex normals

- **Simplest**: Add up the normals of adjacent faces and unitize
- **Simple and usually a bit better**: Add up the normals of adjacent faces, weighted by face areas
- **Complex**: Detect sharp edges
Mesh Topology

- **Topology** (loosely): The structure of a shape ignoring any measurements of distance, angle etc
  - i.e. the properties invariant to bending, twisting, folding, stretching... (but not tearing)
- E.g. **Genus**: The number of handles in a shape

![](genus_0.png) ![genus_1.png](genus_1.png) ![genus_2.png](genus_2.png) ![genus_3.png](genus_3.png)
Mesh Topology

• **Manifold**: A topological space that is locally Euclidean (neighborhood has the topology of the unit ball)

Some manifold shapes

- genus 0
- genus 1
- genus 2
- genus 3

Not manifold

Manifold structure of a surface is approximated by its mesh connectivity