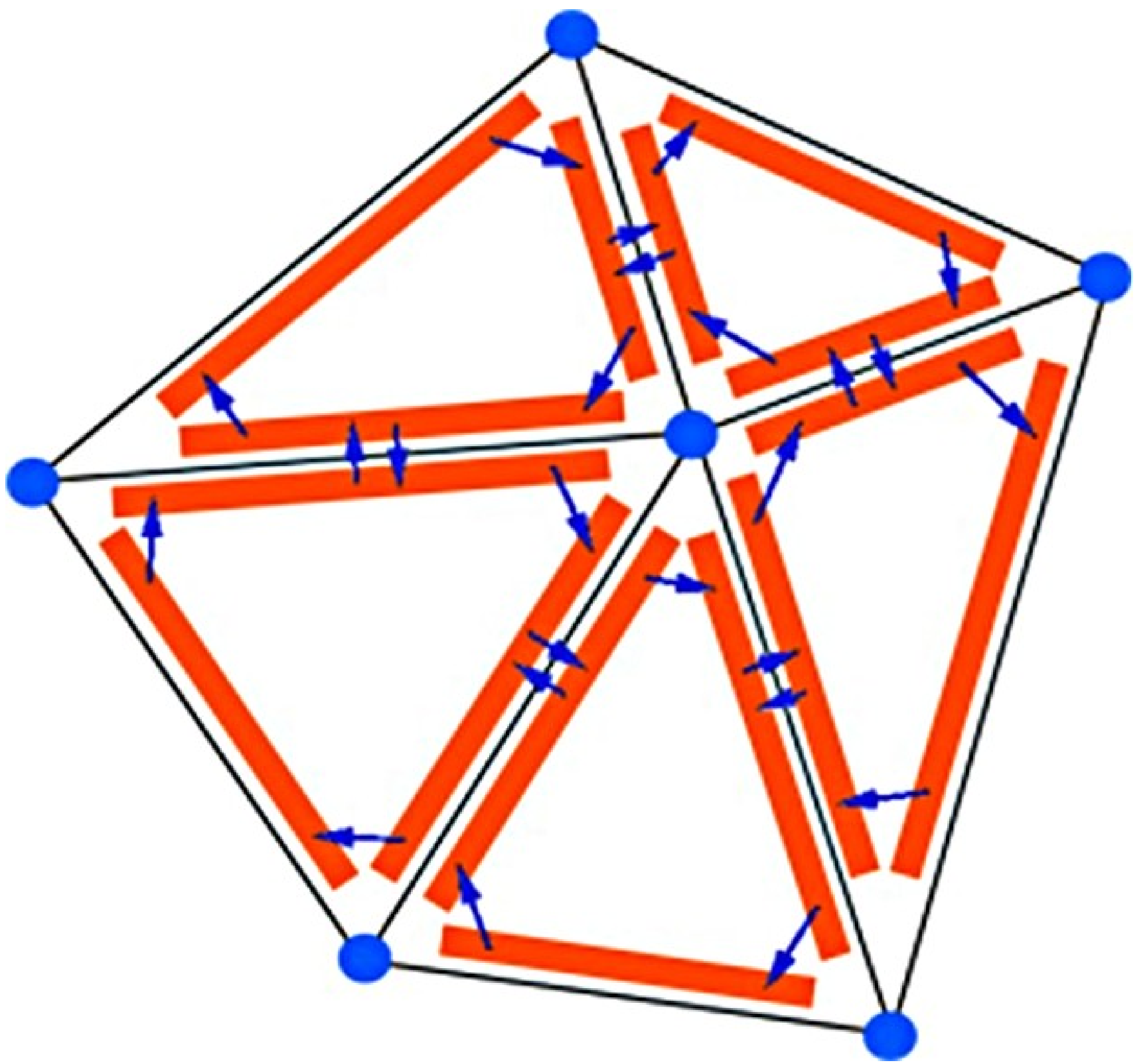
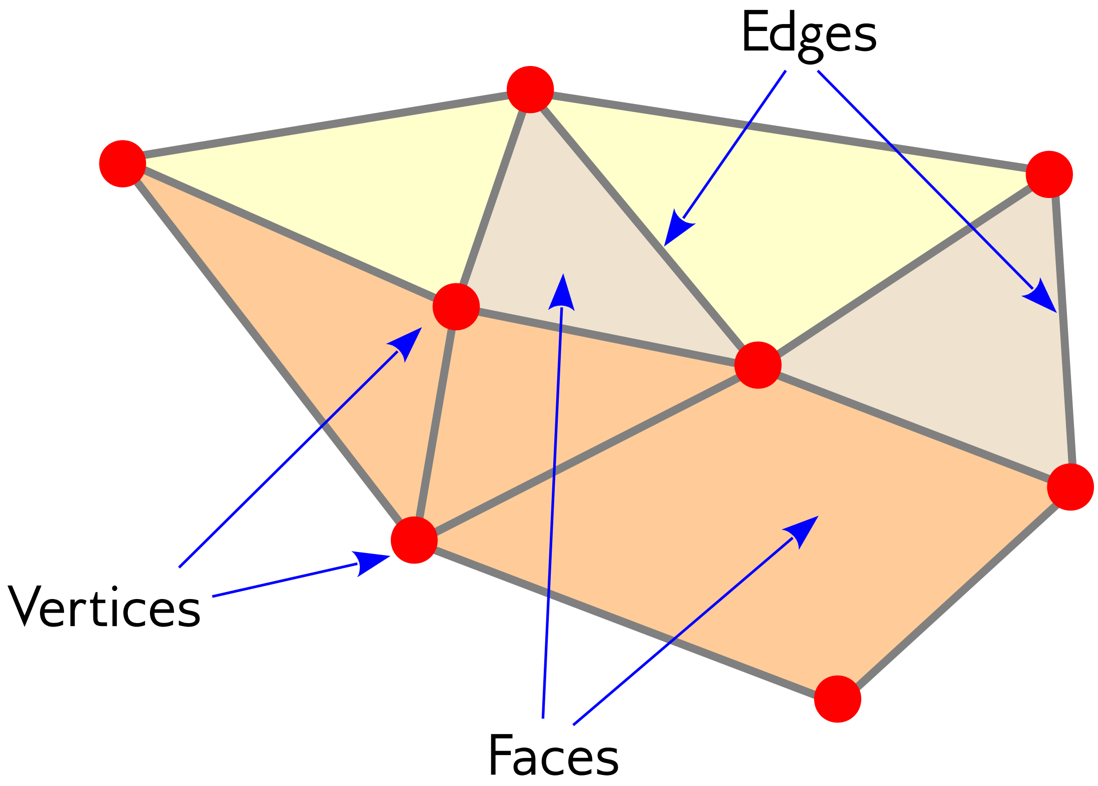


Meshes: Memory Formats

Siddhartha Chaudhuri http://www.cse.iitb.ac.in/~cs749

Recap: Polygon Meshes

- A mesh is a **discrete sampling** of a surface (vertices), plus **locally linear** approximations (simple polygons)
- A mesh is a **graph**

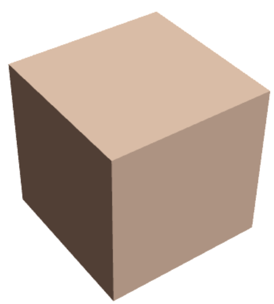


Today

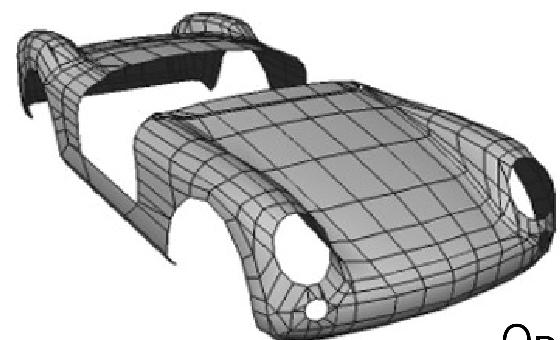
- How do we **store** a mesh?
    - In RAM
    - On disk

Closed/Open, Connected/Disconnected

- **Closed**: No boundary edges (adjacent to a single face)

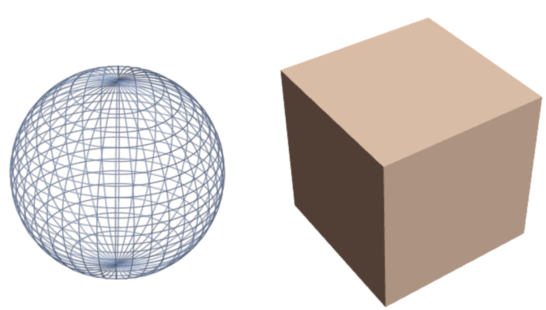


Closed



Open

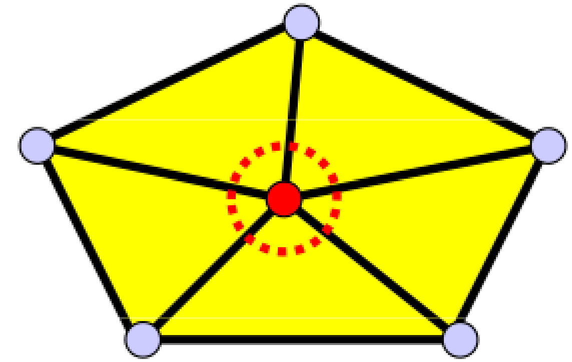
- **Connected**: Same definition (and algorithm) as for a graph



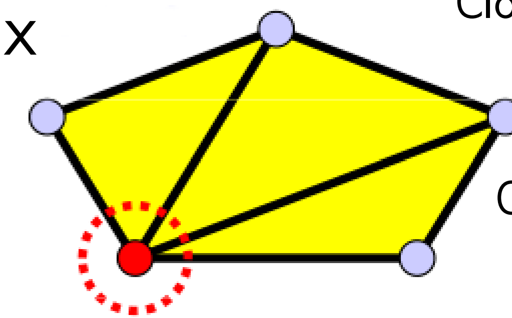
These shapes are individually connected, but not connected to each other

Manifold vs Non-Manifold

- A mesh is manifold if
  1) Every edge is adjacent to 1 or 2 faces
  2) The faces around every vertex form a closed or open fan

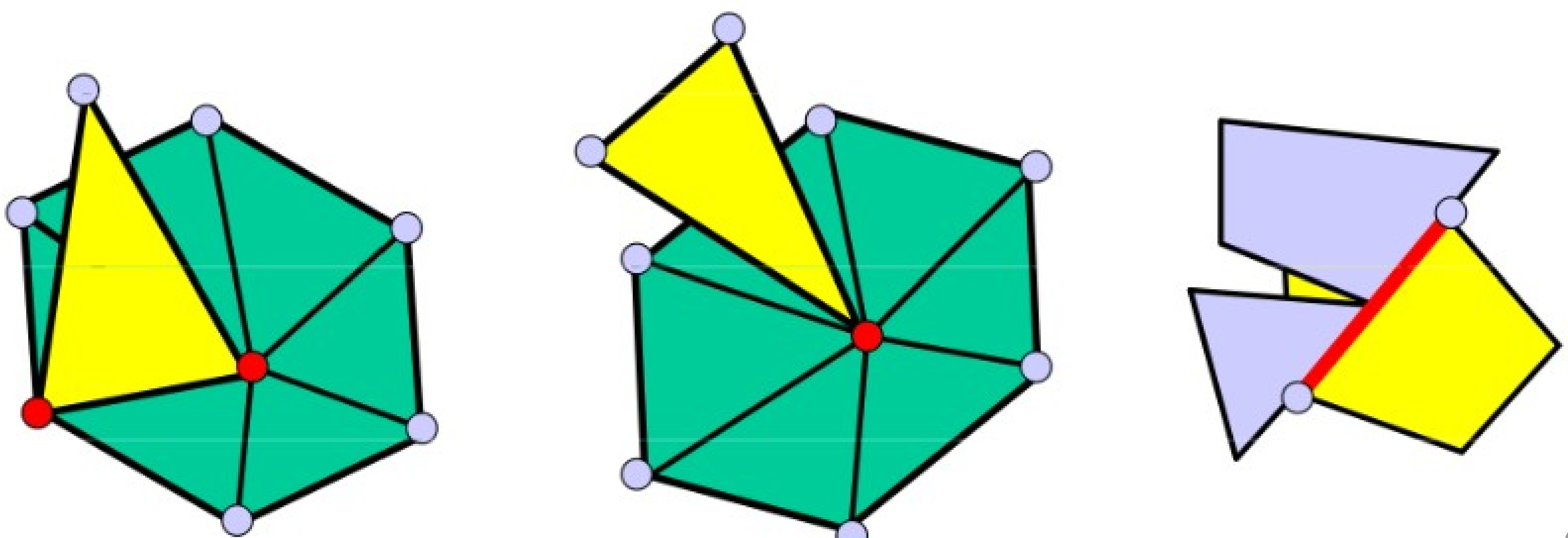


Closed fan



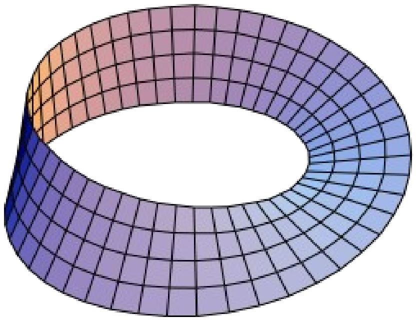
Open fan

Not Manifold

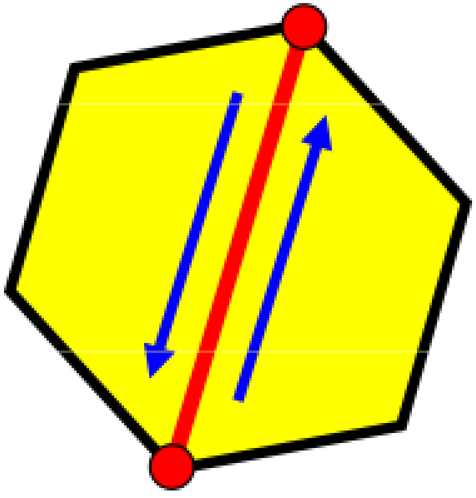


Orientable vs Non-Orientable

- Two adjacent faces are compatible if their vertices wind the same way (both counter-clockwise or both clockwise) around their boundaries
  - In other words, if their boundaries traverse the shared edge in opposite directions
- A mesh is orientable if all pairs of adjacent faces are compatible



Not orientable



Compatible

Storing a mesh in RAM

- What might we need?
  - Fast **iteration** (over vertices, faces, edges...)
  - Fast graph **traversal**
    - Jump from element to adjacent element, e.g. edge to neighboring faces
  - Stored **attributes** (normals, colors, texture coordinates, features...)
  - Efficient use of **space**
  - Other considerations, e.g. caching adjacent elements in nearby memory locations

A simple memory format

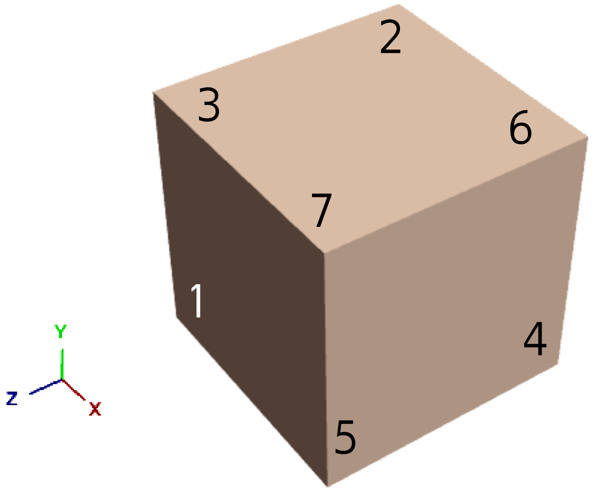
```
vertices = {
   { 0.0, 0.0, 0.0 },
   { 0.0, 0.0, 1.0 },
   { 0.0, 1.0, 0.0 },
   { 0.0, 1.0, 1.0 },
   { 1.0, 0.0, 0.0 },
   { 1.0, 0.0, 1.0 },
   { 1.0, 1.0, 0.0 },
   { 1.0, 1.0, 1.0 },
};
```

In practice, maybe a `vector<Vec3>`

```
quads = {
   { 0, 2, 6, 4 },
   { 0, 1, 3, 2 },
   { 2, 3, 7, 6 },
   { 4, 6, 7, 5 },
   { 0, 4, 5, 1 },
   { 1, 5, 7, 3 },
};
```

References to vertex list

In practice, maybe a `vector< array<long, 4> >`

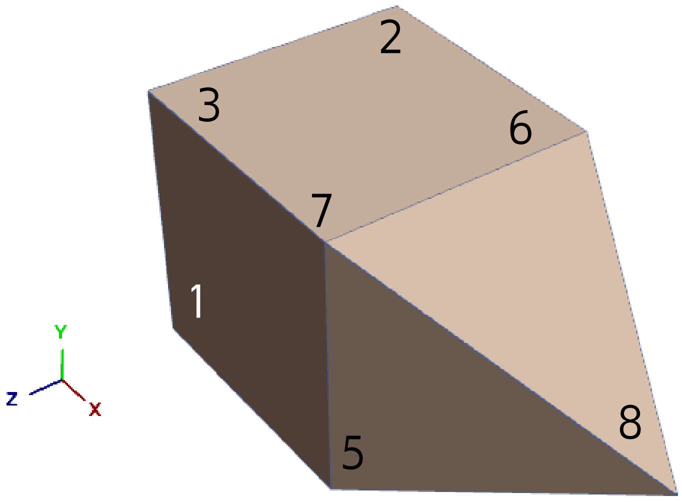


A simple memory format

```
vertices = {
  { 0.0, 0.0, 0.0 },
  { 0.0, 0.0, 1.0 },
  { 0.0, 1.0, 0.0 },
  { 0.0, 1.0, 1.0 },
  { 1.0, 0.0, 0.0 },
  { 1.0, 0.0, 1.0 },
  { 1.0, 1.0, 0.0 },
  { 1.0, 1.0, 1.0 },
  { 2.0, 0.5, 0.5 },
};
```

```
quads = {
  { 0, 2, 6, 4 },
  { 0, 1, 3, 2 },
  { 2, 3, 7, 6 },
  { 4, 6, 7, 5 },
  { 0, 4, 5, 1 },
  { 1, 5, 7, 3 },
};
```

```
triangles = {
  { 7, 8, 6 },
  { 5, 8, 7 },
  { 4, 8, 5 },
  { 6, 8, 4 },
};
```



Pros and Cons

- Fast iteration, good for rendering

```
glBegin(GL_QUADS);
  for (size_t i = 0; i < quads.size(); ++i)
    for (size_t j = 0; j < 4; ++j) {
      Vec3 const & v = vertices[quads[i][j]];
      glVertex3f(v.x, v.y, v.z);
    }
glEnd();
```

  - Directly maps to GPU vertex and index buffer formats
- Compact use of space
  - Higher-degree polys are usually rare and can be stored in separate list (such as a `vector< vector<long> >`)
- Bad for traversal
  - How would you go from a vertex to its neighbors?
  - How would you go from a vertex to its adjoining faces?

Adjacencies

- Let's explicitly store the graph structure
  - Every vertex will store its incident faces and edges
  - Every edge will store its two endpoints, and its adjoining faces
  - Every face will store its vertices and edges

```
class Vertex {
  Vec3 position;
  Vec3 normal;

  list<Face *> faces;
  list<Edge *> edges;
};
```

(Constructors, accessors and other functions omitted)

```
class Edge {
  double length;

  Vertex * endpoints[2];
        // ^^^ unordered

  list<Face *> faces;
};
```

```
class Face {
  Vec3 normal;

  // Invariant:
  // vertices[i] =
  //   edges[i]->endpoint[0 or 1]
  list<Vertex *> vertices;
  list<Edge *> edges;
};
```

```
Mesh = [ list<Vertex>, list<Edge>, list<Face> ]
```

Pros and Cons

- Fast iteration
  - ... over any standard subset of elements (all vertices, or vertices around a face, or edges at a vertex...)
- Great for traversal
  - Can go from any element to its adjoining elements (of any type) in O(1) time
- Ok use of space
  - Typically a constant-factor overhead
- Such adjacency-heavy representations are good for geometric algorithms

Analysis of storage overhead

- For a manifold surface
  - Each edge has (at most) two adjacent faces
    - … so #edge-face incidences $\leq 2E$
  - Number of vertices around a face = number of edges around the face
    - … so #vertex-face incidences $\leq 2E$
  - Each edge has two endpoints
    - … so #edge-vertex incidences $\leq 2E$
  - So total overhead of the adjacency information = $O(E)$
    - $\ldots = O(V + F)$, for small genus

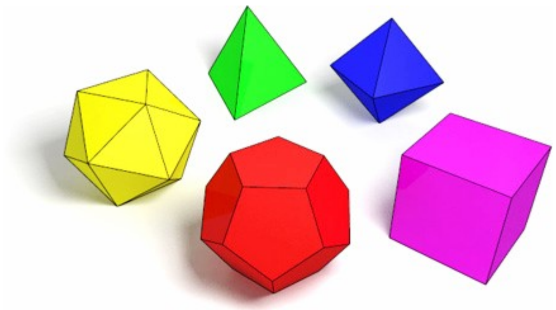
Euler-Poincaré formula

- For a closed polygonal mesh with V vertices, E edges and F faces

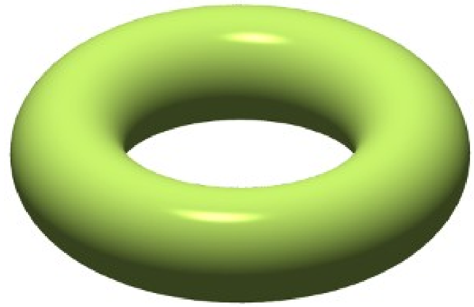
$$V - E + F = \chi$$

- χ is the **Euler characteristic** of the surface
  - For a closed, connected, orientable 2-manifold, $\chi = 2(1 - g)$
  - g is the **genus** of the surface
    - Number of holes/handles
    - More formally, the number of cuttings along simple closed loops on the surface that do not disconnect it

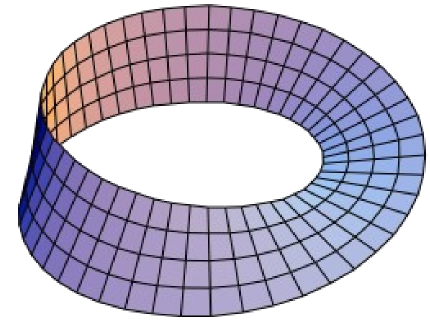
Euler Characteristic



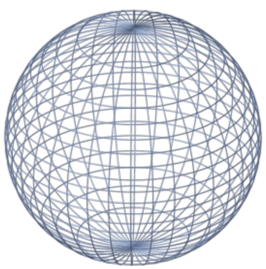
Platonic solids (and other convex polyhedra): $g = 0$, $\chi = 2$



Torus: $g = 1$, $\chi = 0$

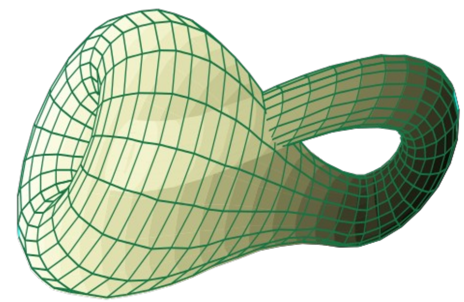


Möbius strip: $\chi = 0$

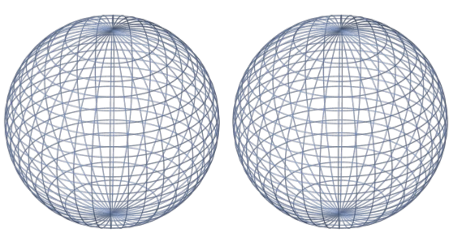


Sphere: $g = 0$, $\chi = 2$

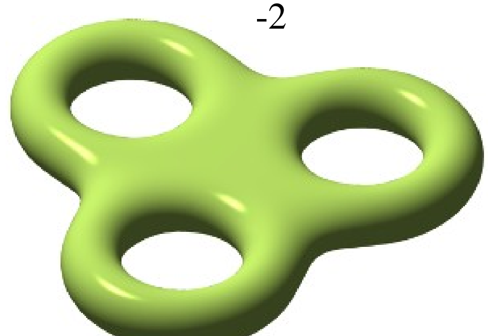
Double torus: $g = 2$, $\chi =$ -2



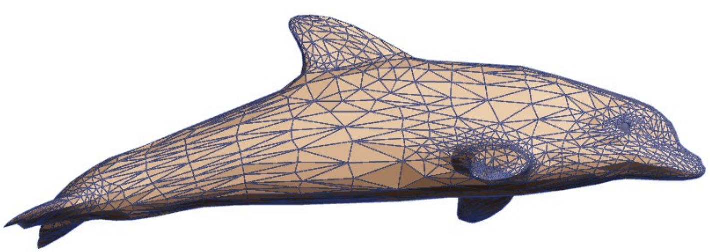
Klein bottle: $\chi = 0$



Two spheres: $\chi = 4$



Triple torus: $g = 3$, $\chi =$ -4



$g = 0$, $\chi = 2$ (if you don't model the alimentary canal)

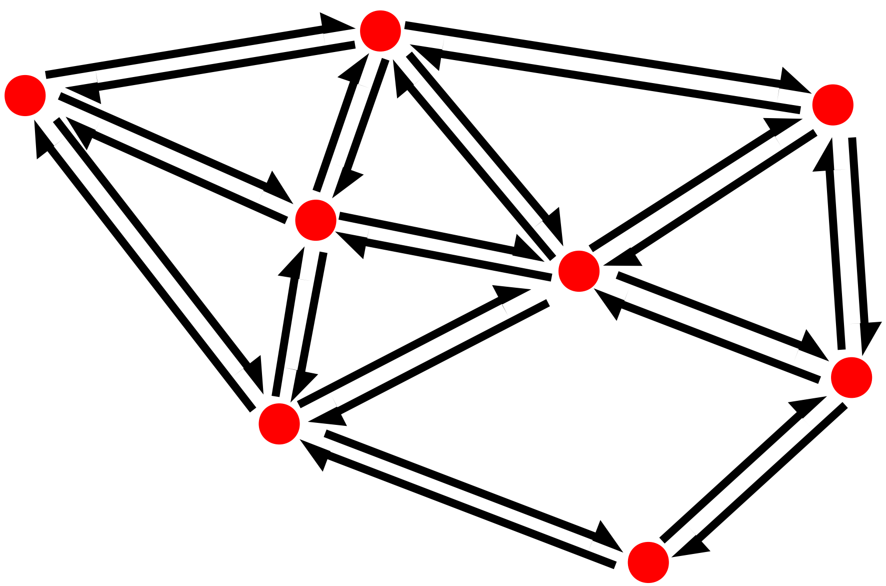
greatlittleminds.com, Wikipedia

Euler-Poincaré formula

- For a closed polygonal mesh with V vertices, E edges and F faces, $V - E + F = \chi$
- For small genus/characteristic, gives $E \approx V + F$
- Consider a closed manifold mesh with only triangles
  - Each edge borders two faces, each face borders 3 edges
    - … so $2E = 3F$
    - … and plugging this into the formula, $V = E/3 + \chi$
  - Hence, the vertex, edge and face counts are all (asymptotically) the same, for fixed characteristic
    - $O(V) = O(E) = O(F)$

Space-efficient adjacencies

- Can we store adjacencies more efficiently?
  - Yes! For manifold, orientable surfaces, we can store the graph with a constant storage overhead *per-element* (no arbitrary-size lists)
  - ... without changing the complexity of traversal



Half-Edge Data Structure

aka Winged Edge Data Structure, aka Doubly-Connected Edge List (DCEL)

- Only for manifold, orientable surfaces
- Instead of an edge, store two opposing half-edges linked to each other
  - Every half-edge links to its twin
- For every vertex, store one half-edge exiting it
  - The half-edge also links back to this source vertex
- For every face, store one half-edge on its boundary that traverses it counter-clockwise
  - The half-edge links back to this adjacent face
  - … and also to the next half-edge along the boundary of the same face

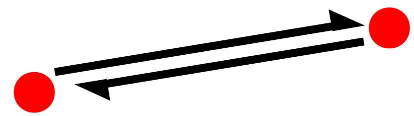
```
class Vertex {
  Vec3 position;

  HalfEdge * half_edge;
};
```

```
class HalfEdge {
  HalfEdge * twin;
  HalfEdge * next;
  Vertex * source;
  Face * face;
};
```

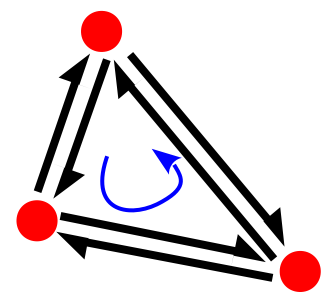
```
class Face {
  HalfEdge * half_edge;
};
```

Traversal Building Blocks

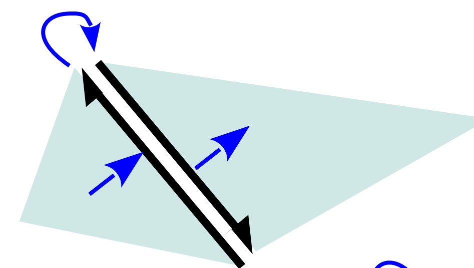
- The tip of a half-edge E
  - `E→twin→source`



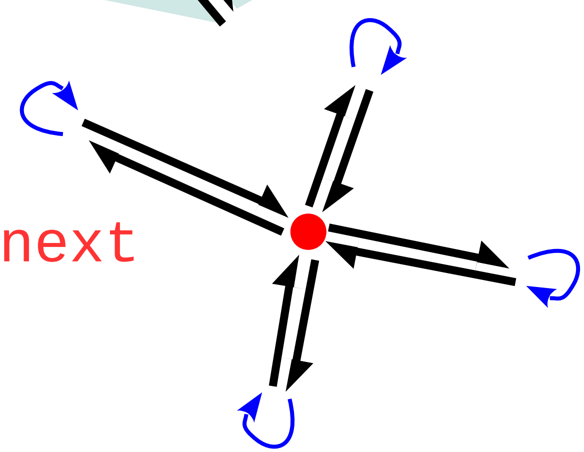
- The boundary of a face
  - Follow the `next` pointer



- From a face F to an adjacent face
  - `F→half_edge→twin→face`



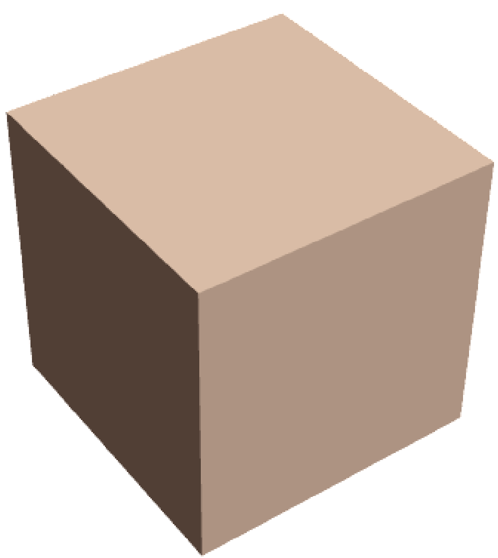
- All edges at a vertex V
  - Start from `V→half_edge`, follow `twin→next`



- …

Storing a mesh on disk

- OBJ: a simple and common file format
  - Plain text, easy to hand-review and edit if needed
  - Also see: OFF, PLY, STL



cube.obj

Vertex positions, one per line

```
v  0.0  0.0  0.0
v  0.0  0.0  1.0
v  0.0  1.0  0.0
v  0.0  1.0  1.0
v  1.0  0.0  0.0
v  1.0  0.0  1.0
v  1.0  1.0  0.0
v  1.0  1.0  1.0

f  1  3  7  5
f  1  2  4  3
f  3  4  8  7
f  5  7  8  6
f  1  5  6  2
f  2  6  8  4
```

List of vertex indices for each face, one face per line (indices are 1-based)

(Many more tags not listed here, see
https://en.wikipedia.org/wiki/Wavefront_.obj_file
http://www.martinreddy.net/gfx/3d/OBJ.spec)