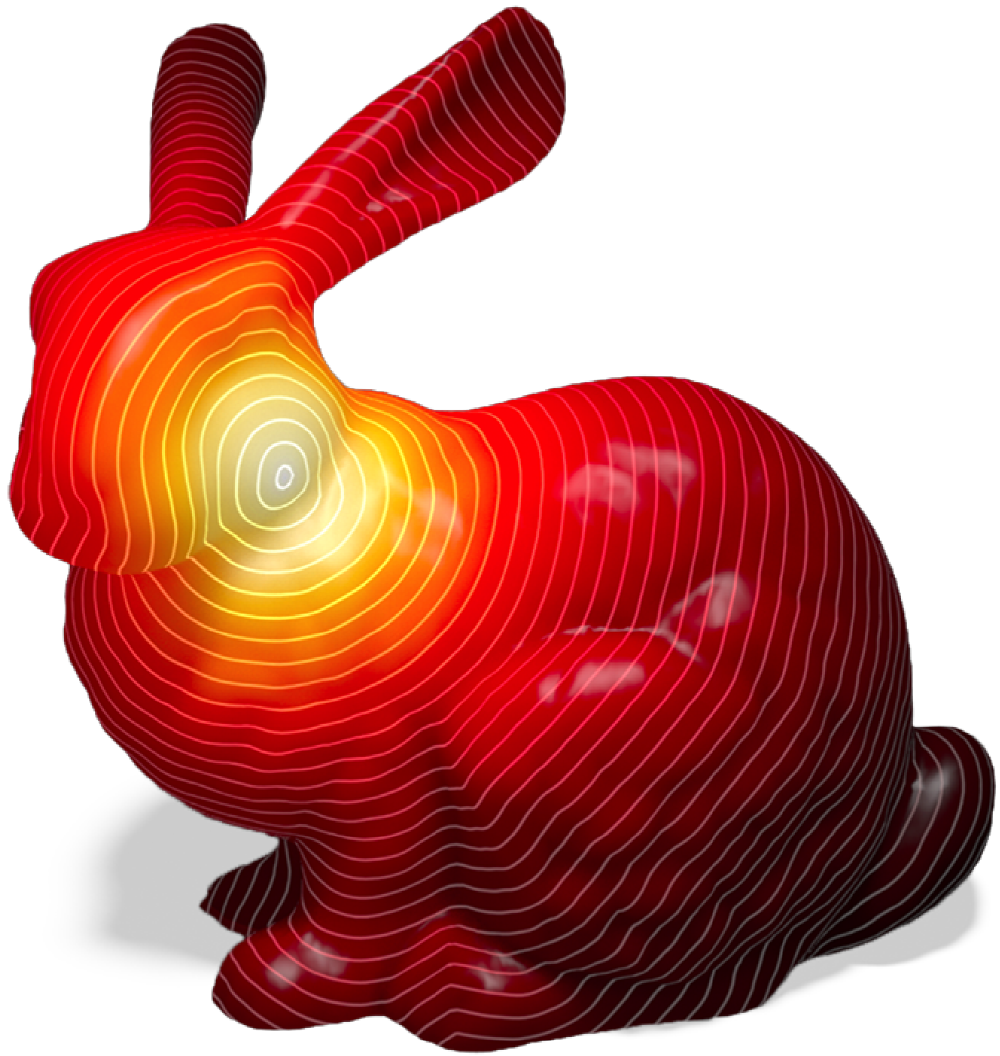
Crane, Weischedel, Wardetzky



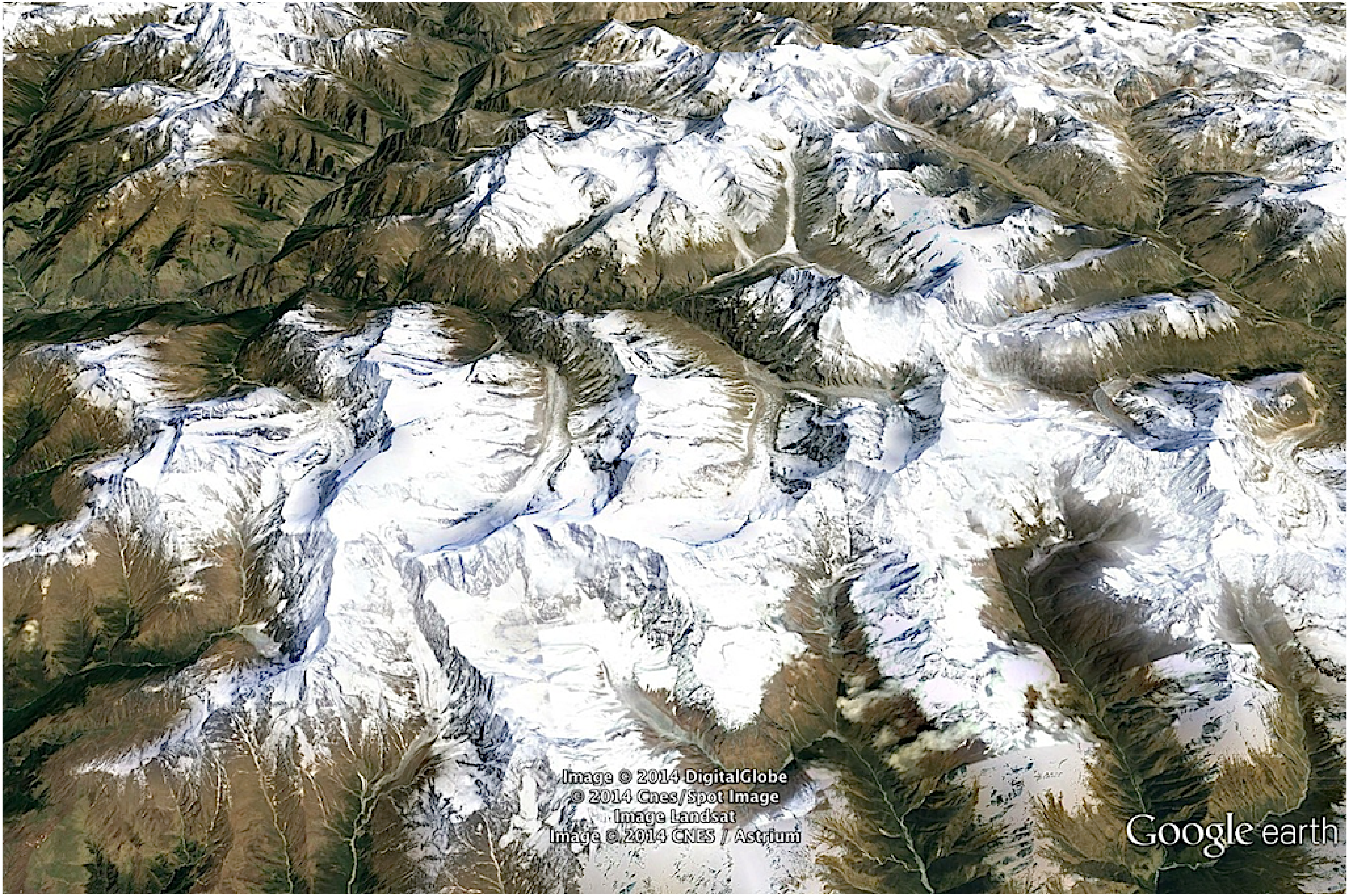
Distances on Surfaces

Siddhartha Chaudhuri http://www.cse.iitb.ac.in/~cs749

Biswaroop Mukherjee

John Porter


Steven House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Nanda Devi
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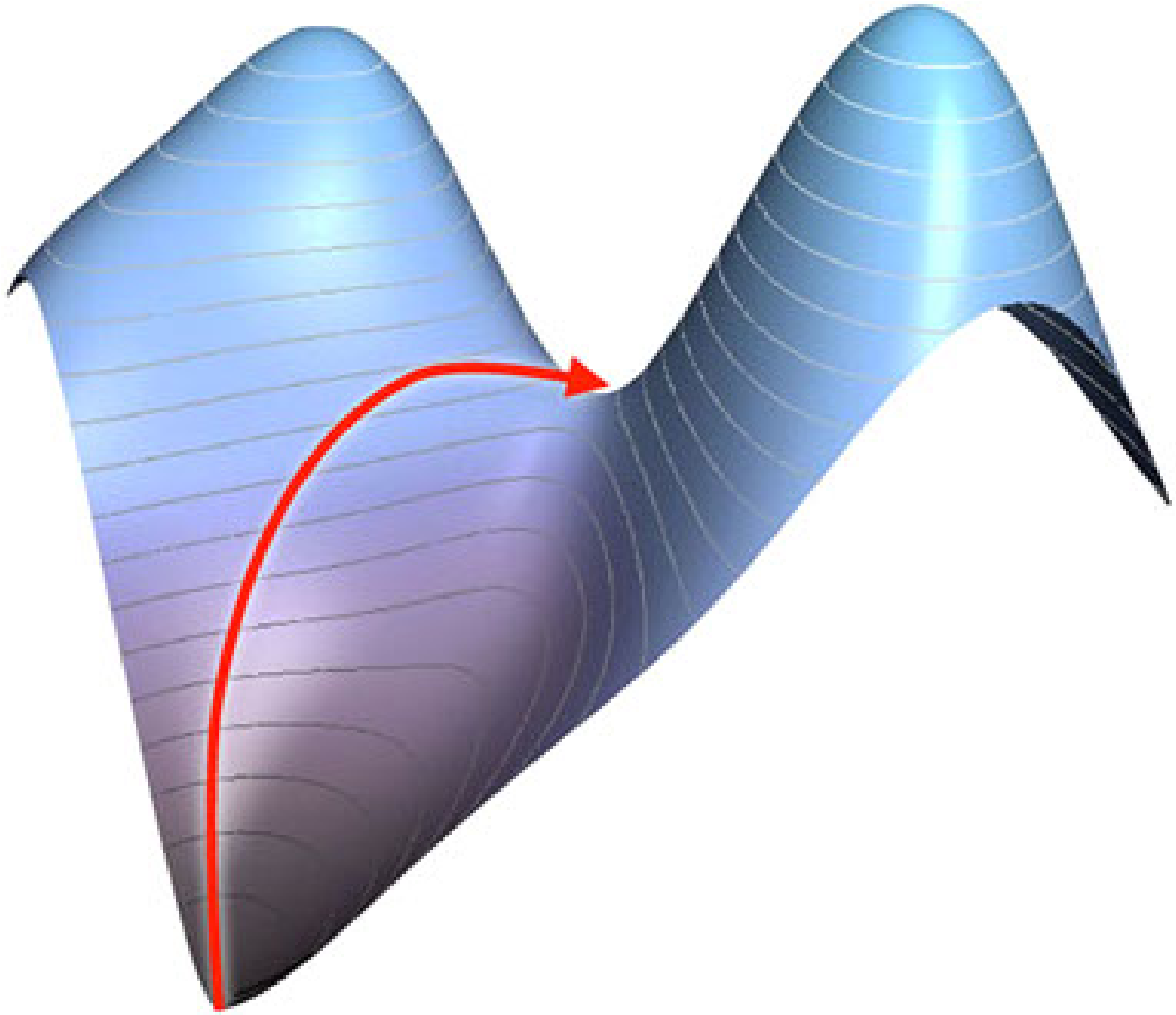
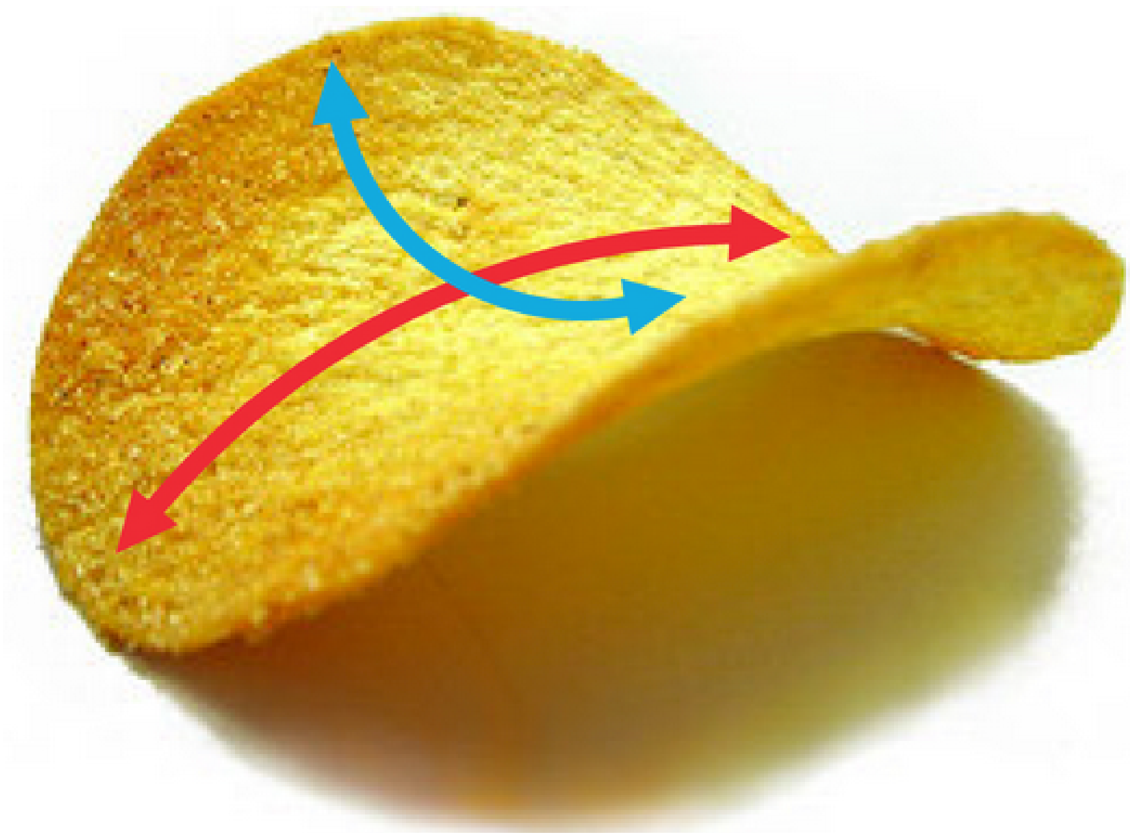
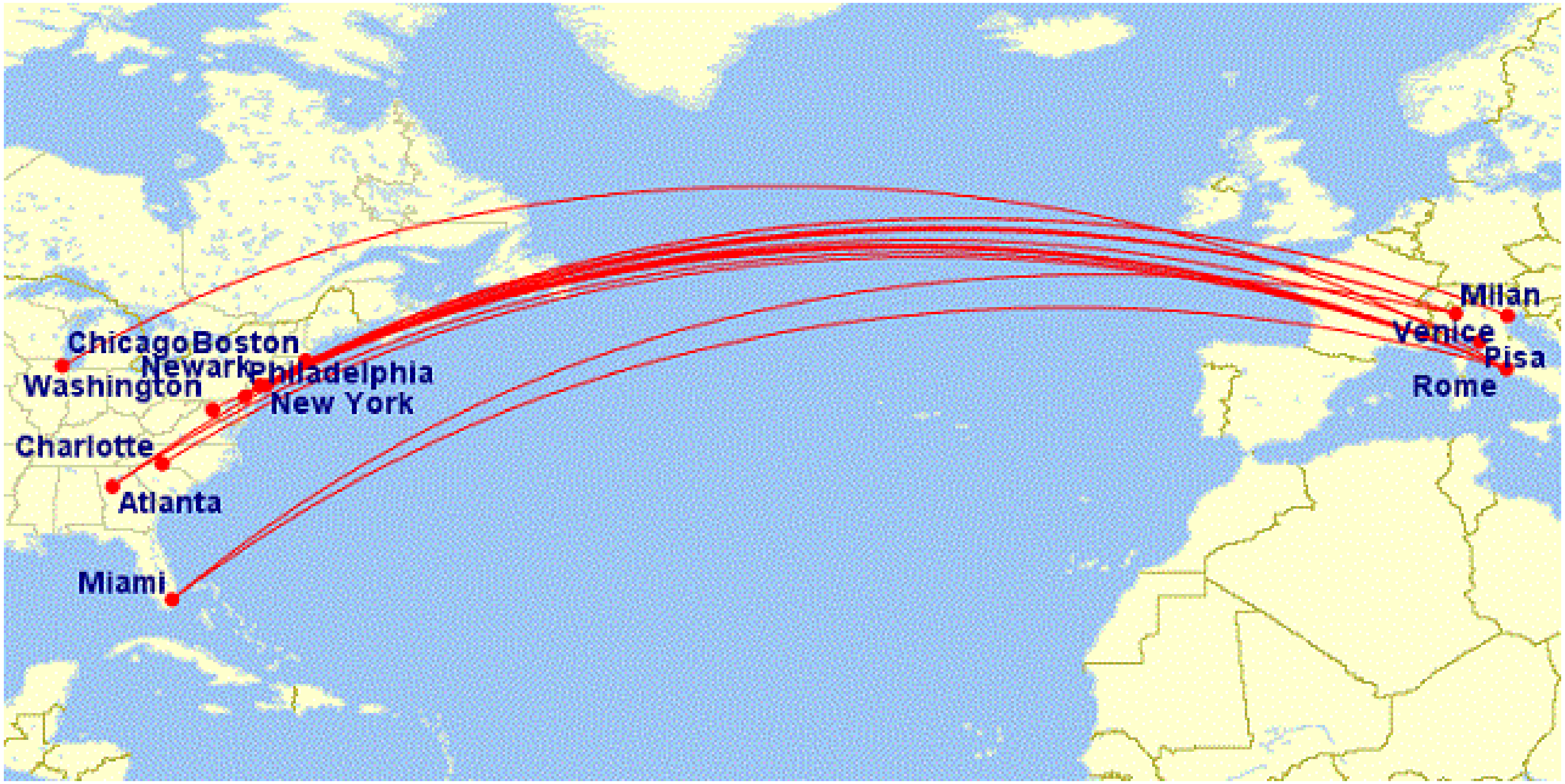

Mousseau and Derreumaux, 2008

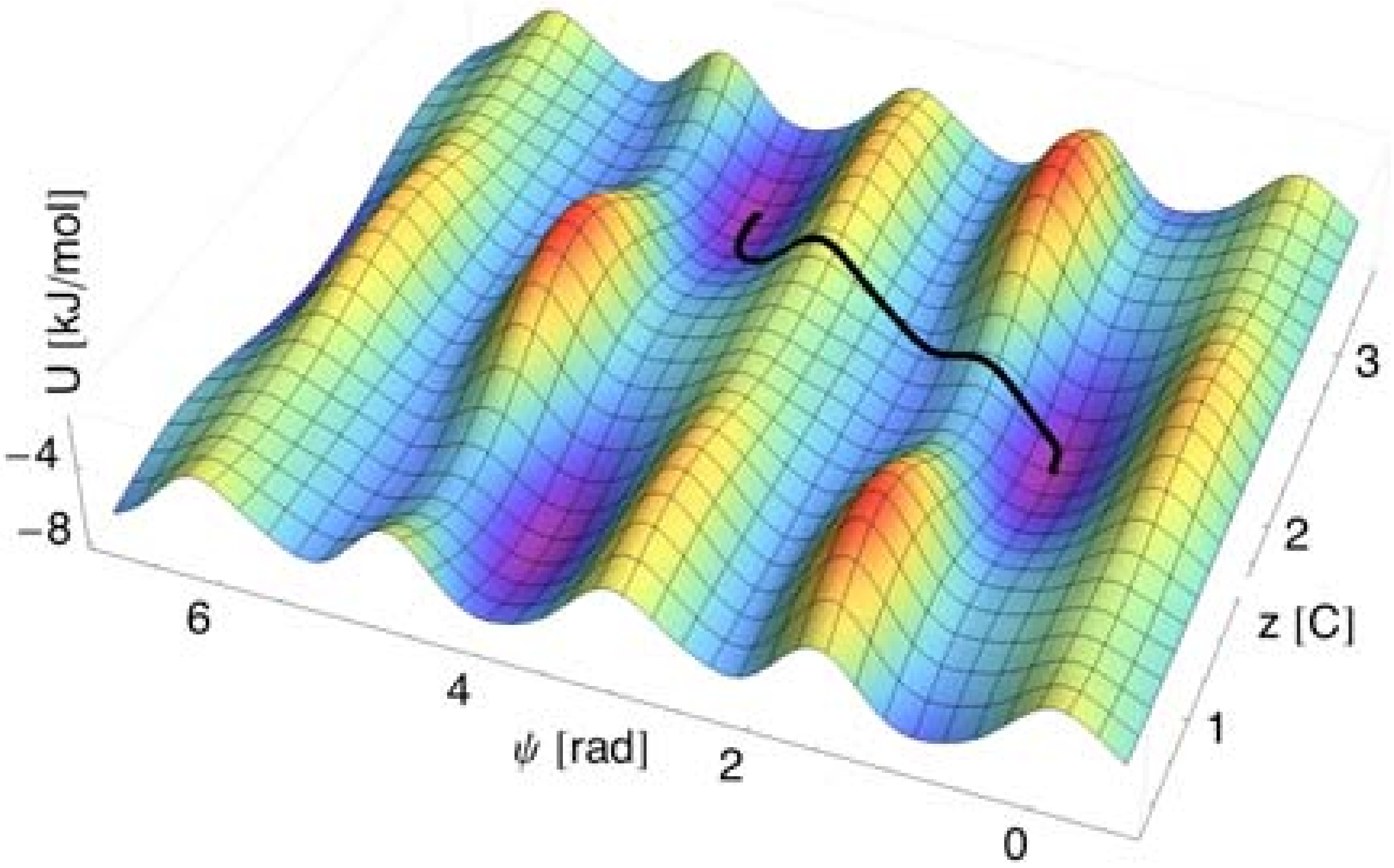


Stephanie K. Fleming

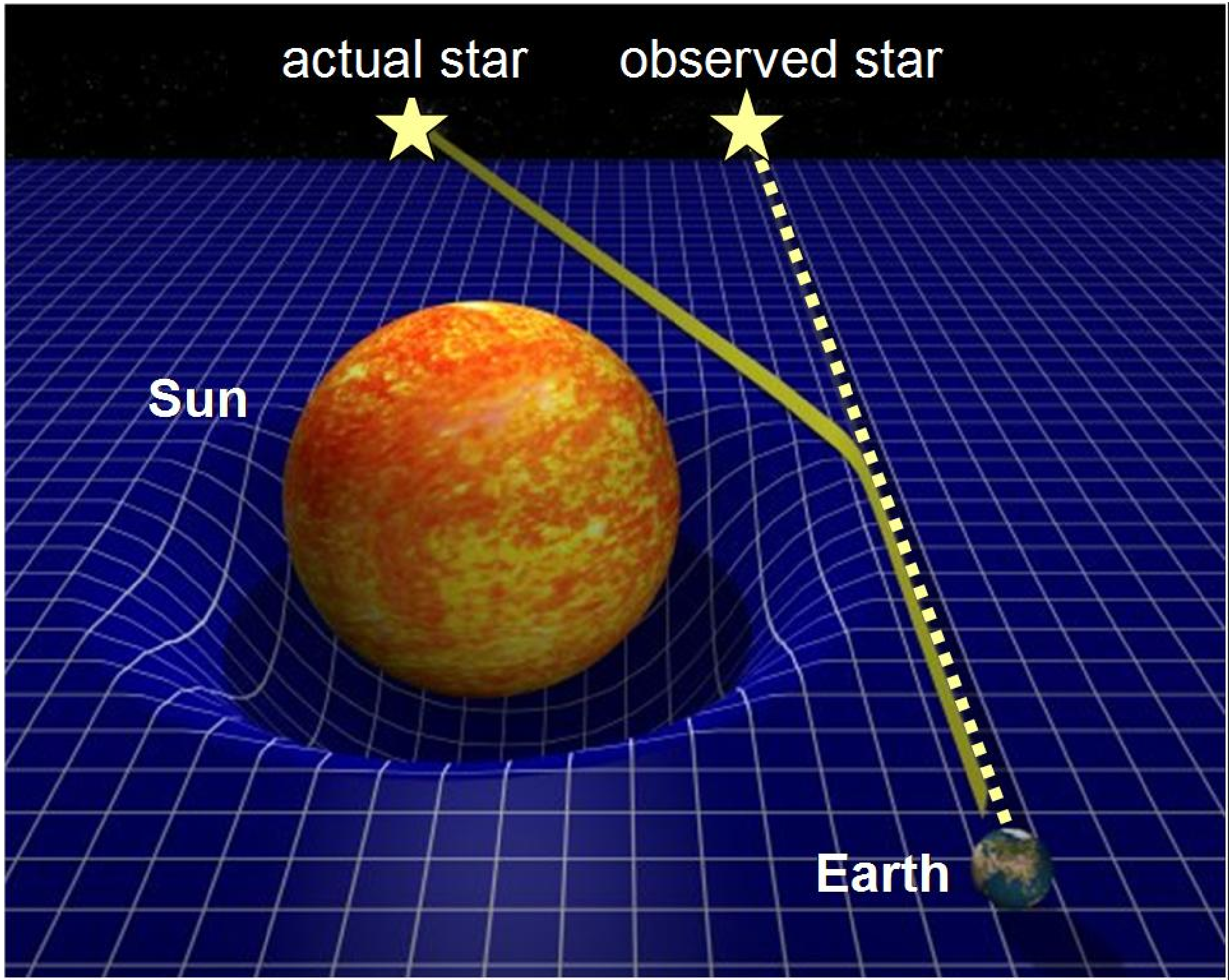

Chicago
Boston
Newark
Philadelphia
Washington
New York
Charlotte
Atlanta
Miami
Milan
Venice
Pisa
Rome

Steven Strogatz, NYTimes

Konrad Polthier, http://page.mi.fu-berlin.de/polthier/video/Geodesics/Scenes.html



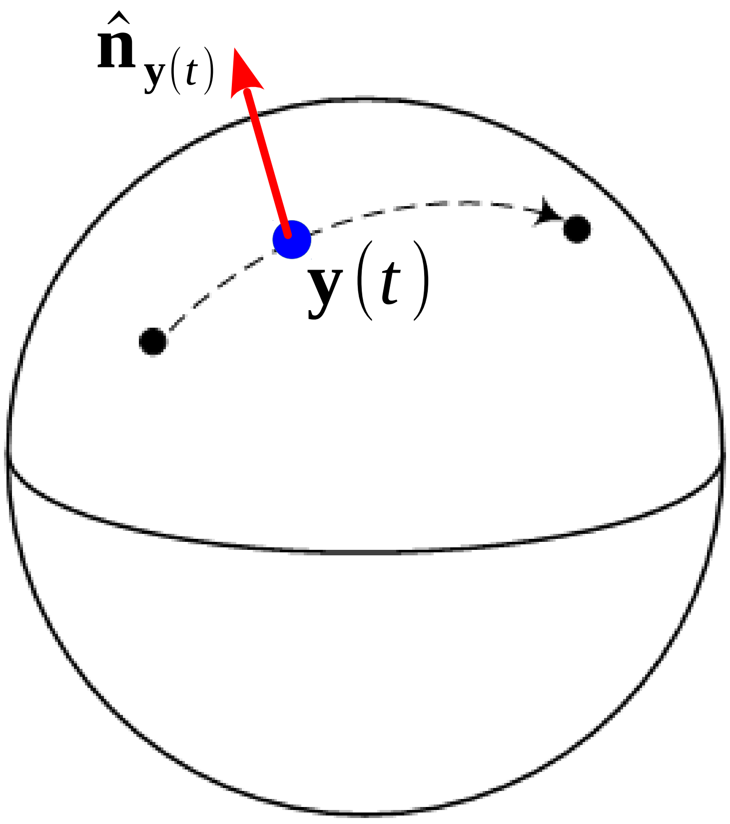
Milner Research Group, PSU



www.physics.uwyo.edu

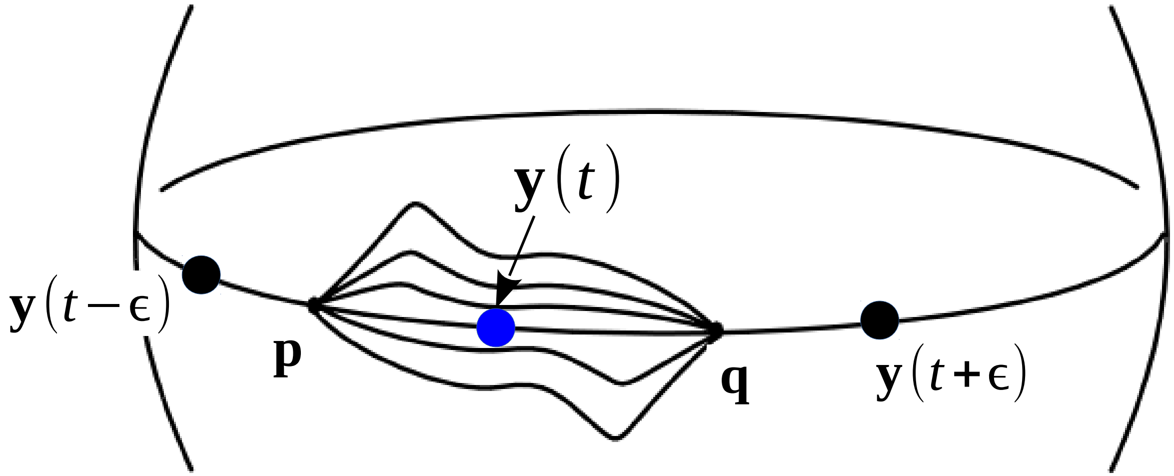
Geodesic Curve

- A **geodesic curve** on a surface (technically, a Riemannian manifold) is a curve $\mathbf{y}(t)$ such that:
  - **Definition 1:** It describes the motion of a particle with acceleration along the surface normal $\ddot{\mathbf{y}}(t) = c\,\hat{\mathbf{n}}_{\mathbf{y}(t)}$
    - Implies that geodesics have constant speed: $\| \dot{\mathbf{y}}(\mathrm{t}) \| = s$



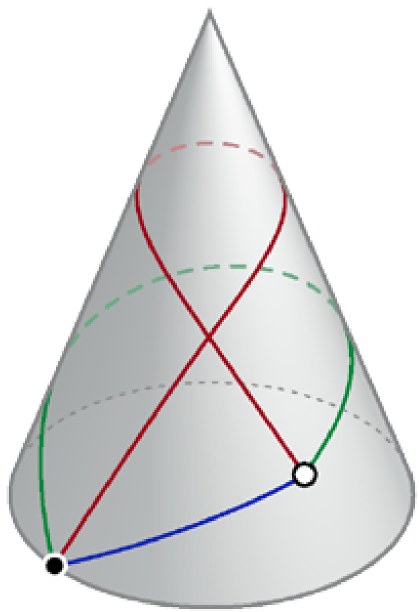
Geodesic Curve

- A **geodesic curve** on a surface (technically, a Riemannian manifold) is a curve $\mathbf{y}(t)$ such that:
  - **Definition 2:** It is locally length-minimizing:
    - Around any point $\mathbf{y}(t)$, there is a neighborhood $B_t = (t - \varepsilon, t - \varepsilon)$ such that the curve is the shortest path between any two points $\mathbf{p}$, $\mathbf{q}$ in $\mathbf{y}(t \in B_t)$



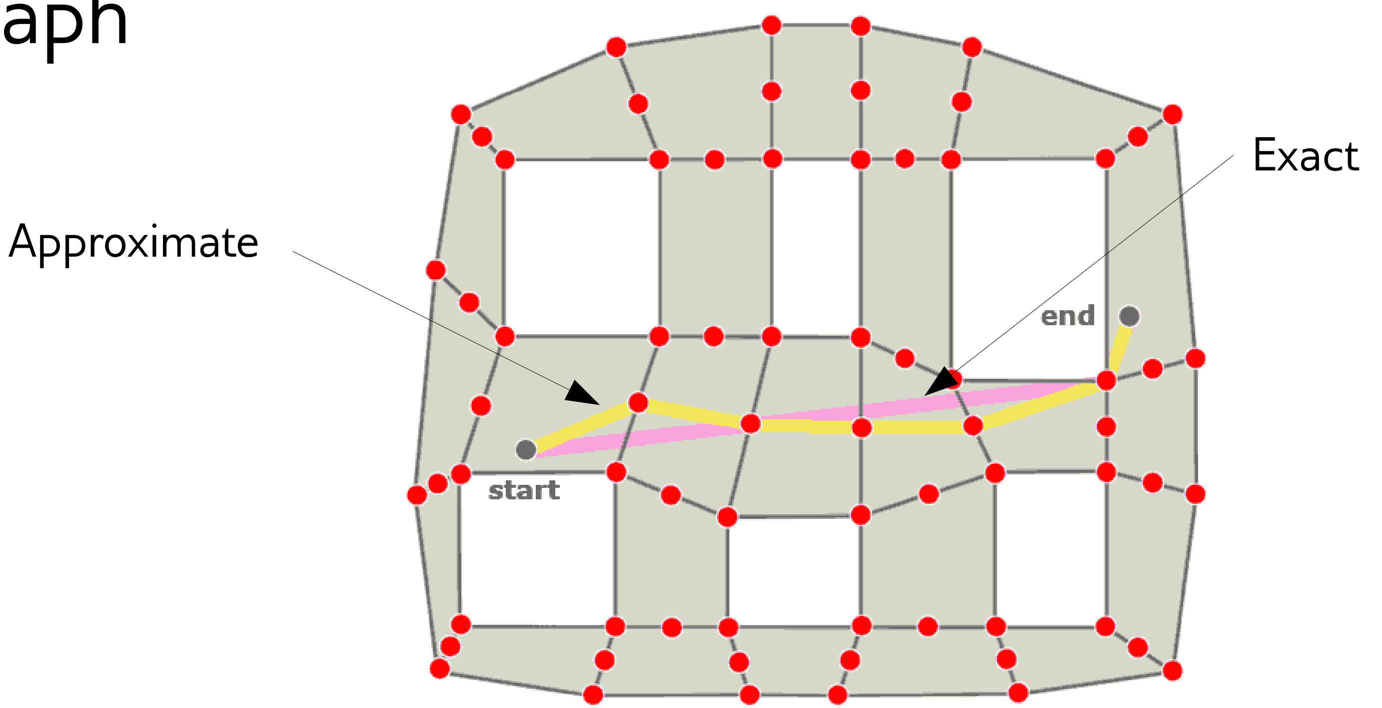
Geodesics ≠ Shortest Paths

- A geodesic is not necessarily the shortest path between two points
- ... but the shortest path is always a geodesic



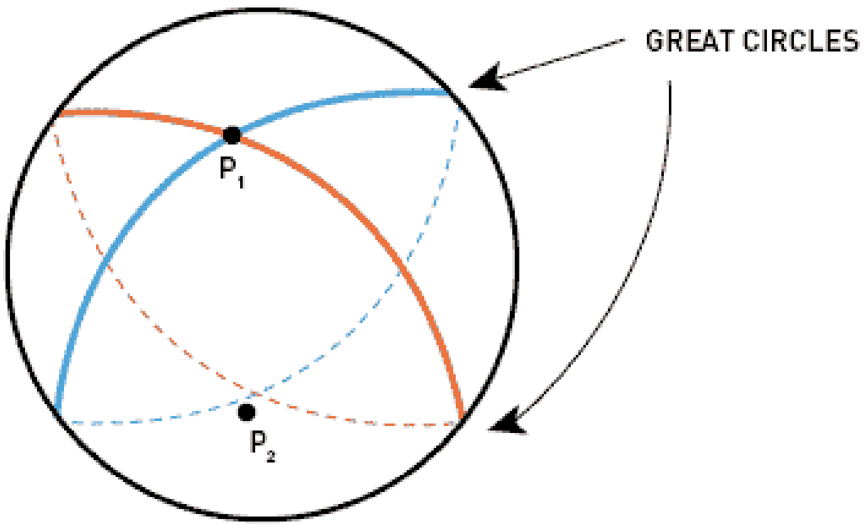
But in common usage…

- … we often use “geodesic” and “shortest path” interchangeably (and hence inaccurately)
- The shortest path between two points on a mesh is approximated by the distance along the edge graph

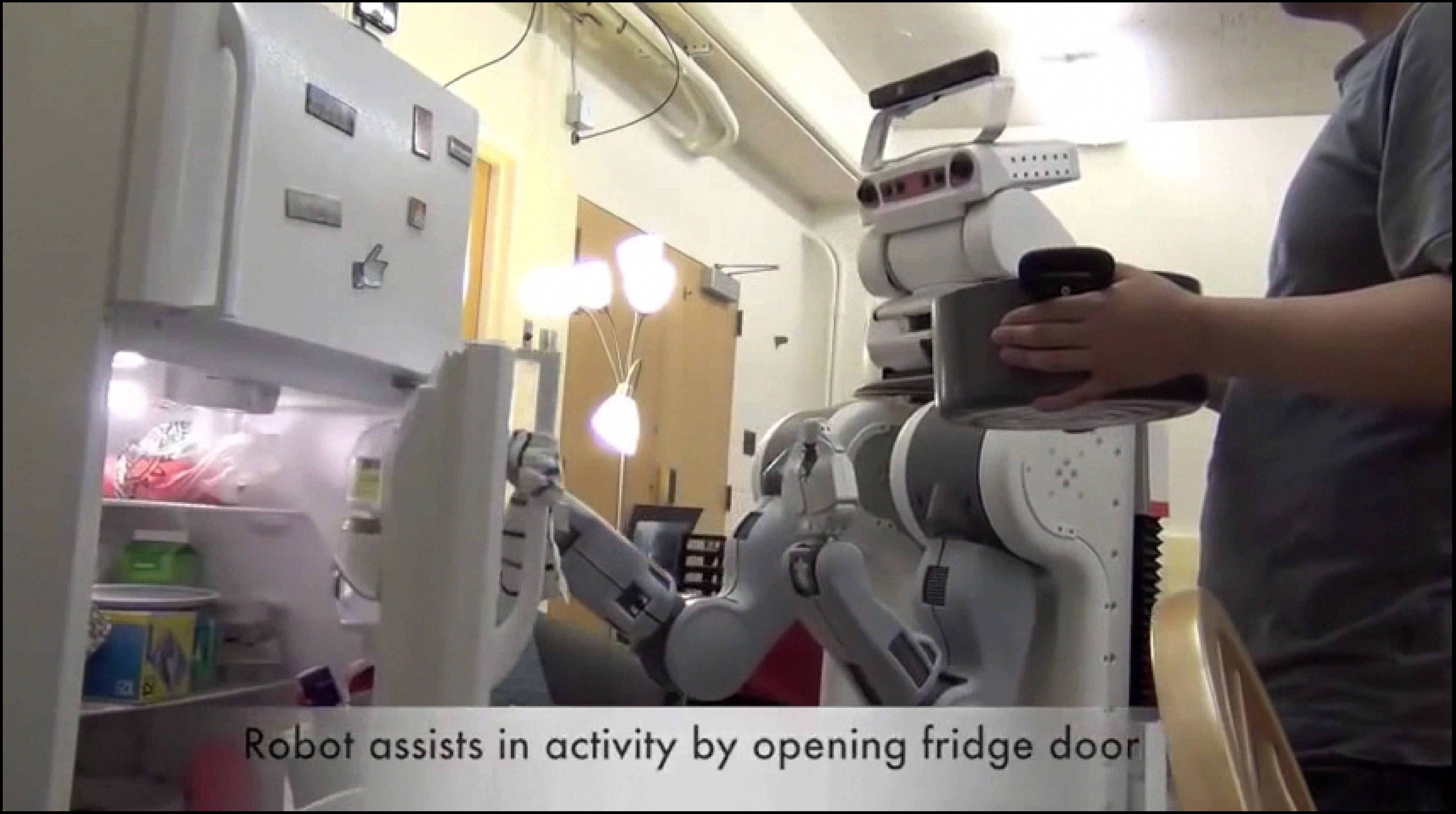


Existence and Uniqueness

- *(Roughly)* On a smooth manifold surface, if we're given a point $\mathbf{p}$ and a vector $\mathbf{v}$ in the tangent plane at $\mathbf{p}$, then there is exactly one geodesic through $\mathbf{p}$, with direction (tangent) $\mathbf{v}$
- There can be multiple geodesics through the same point, for different $\mathbf{v}$

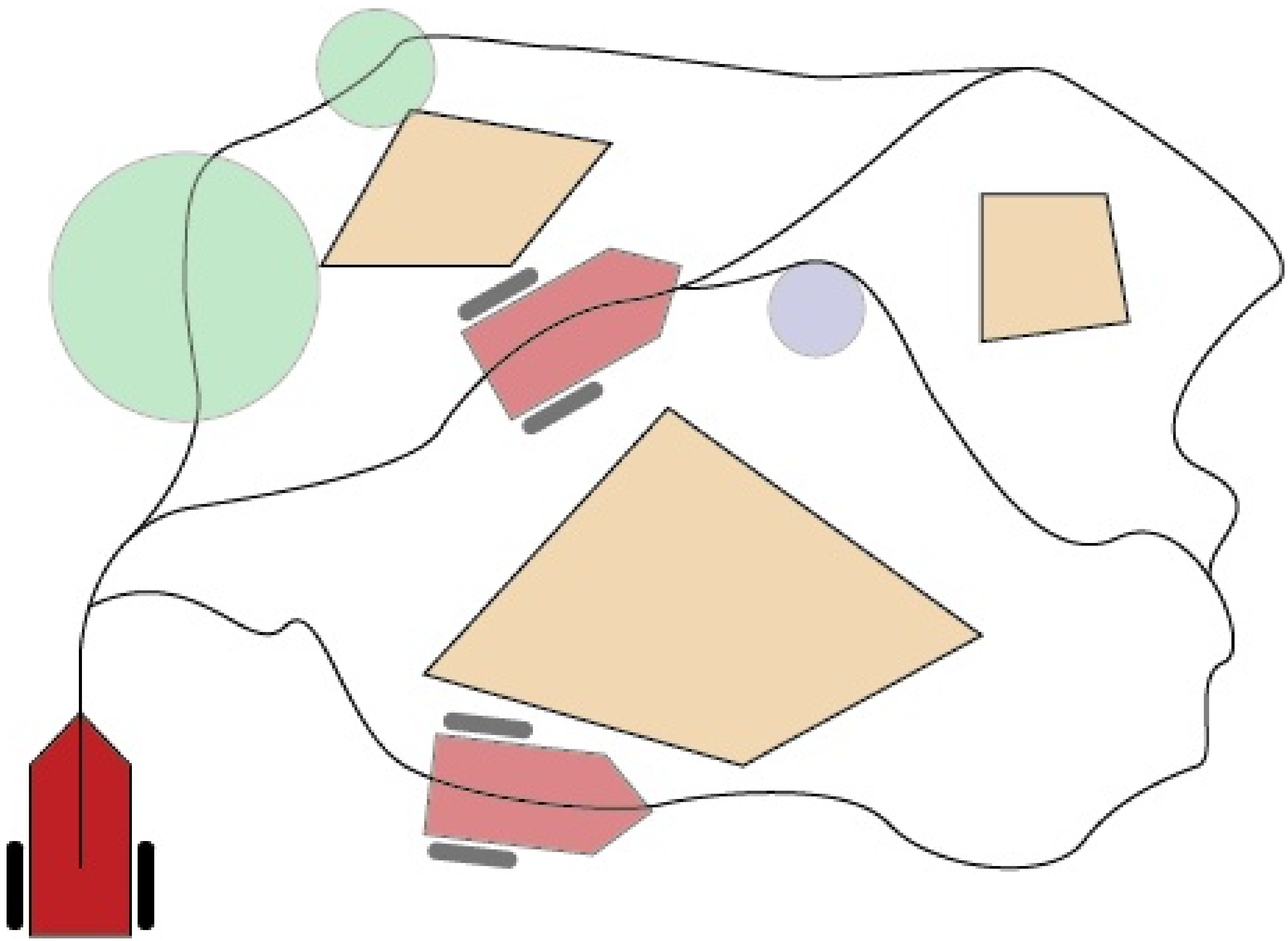


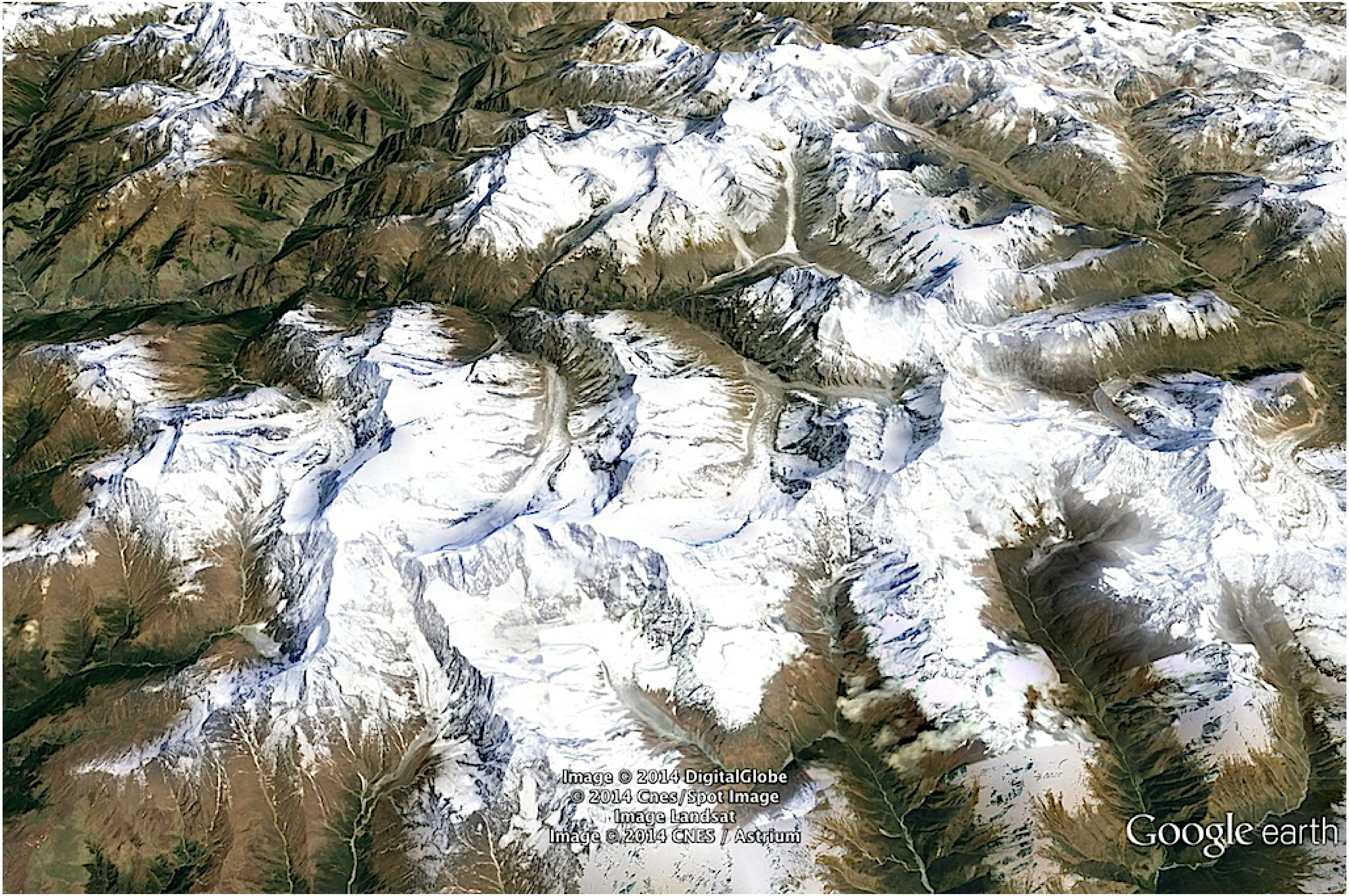
www.learner.org



Koppula and Saxena, Cornell, 2013, http://pr.cs.cornell.edu/anticipation/


Robot assists in activity by opening fridge door

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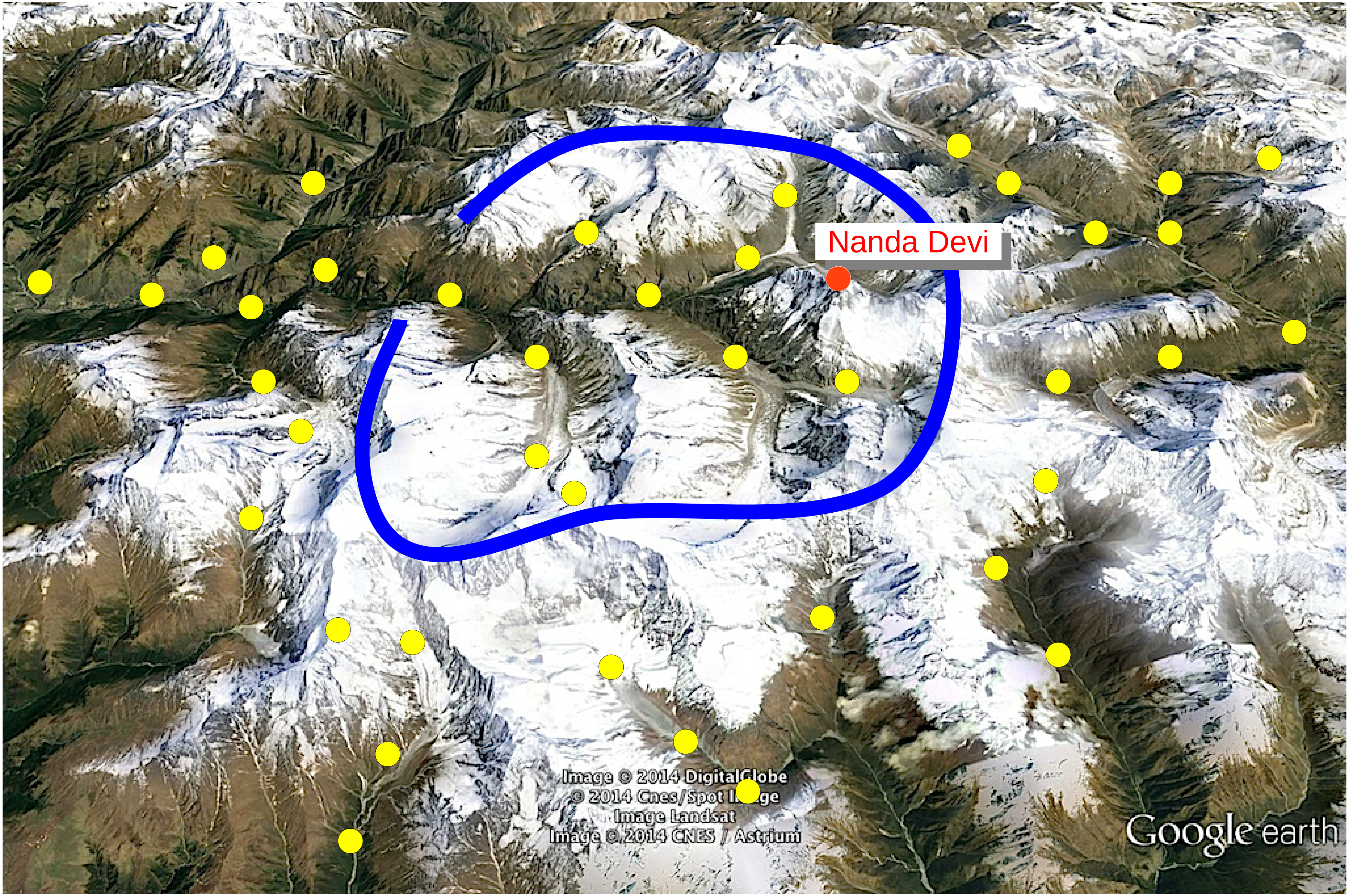
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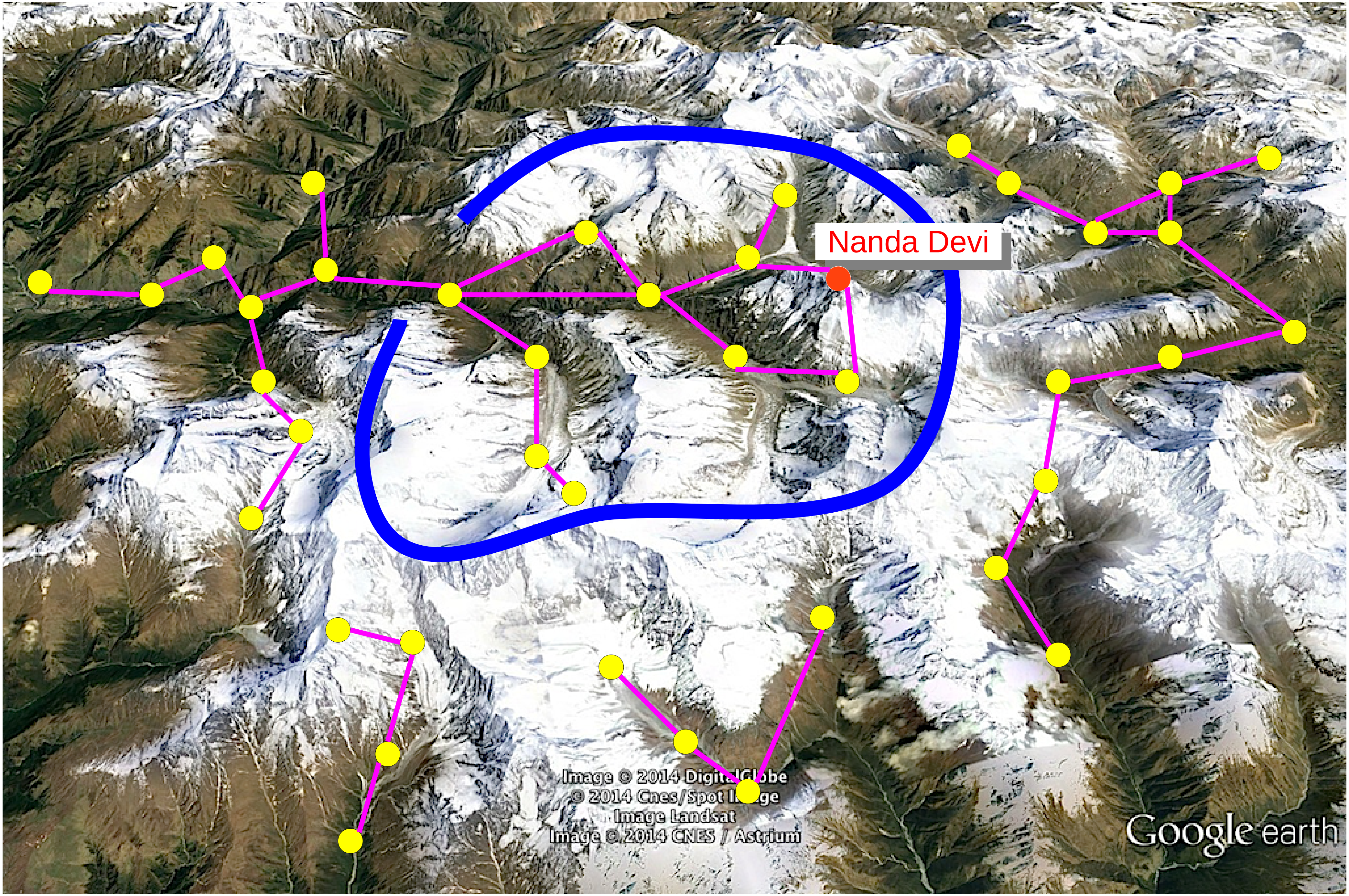
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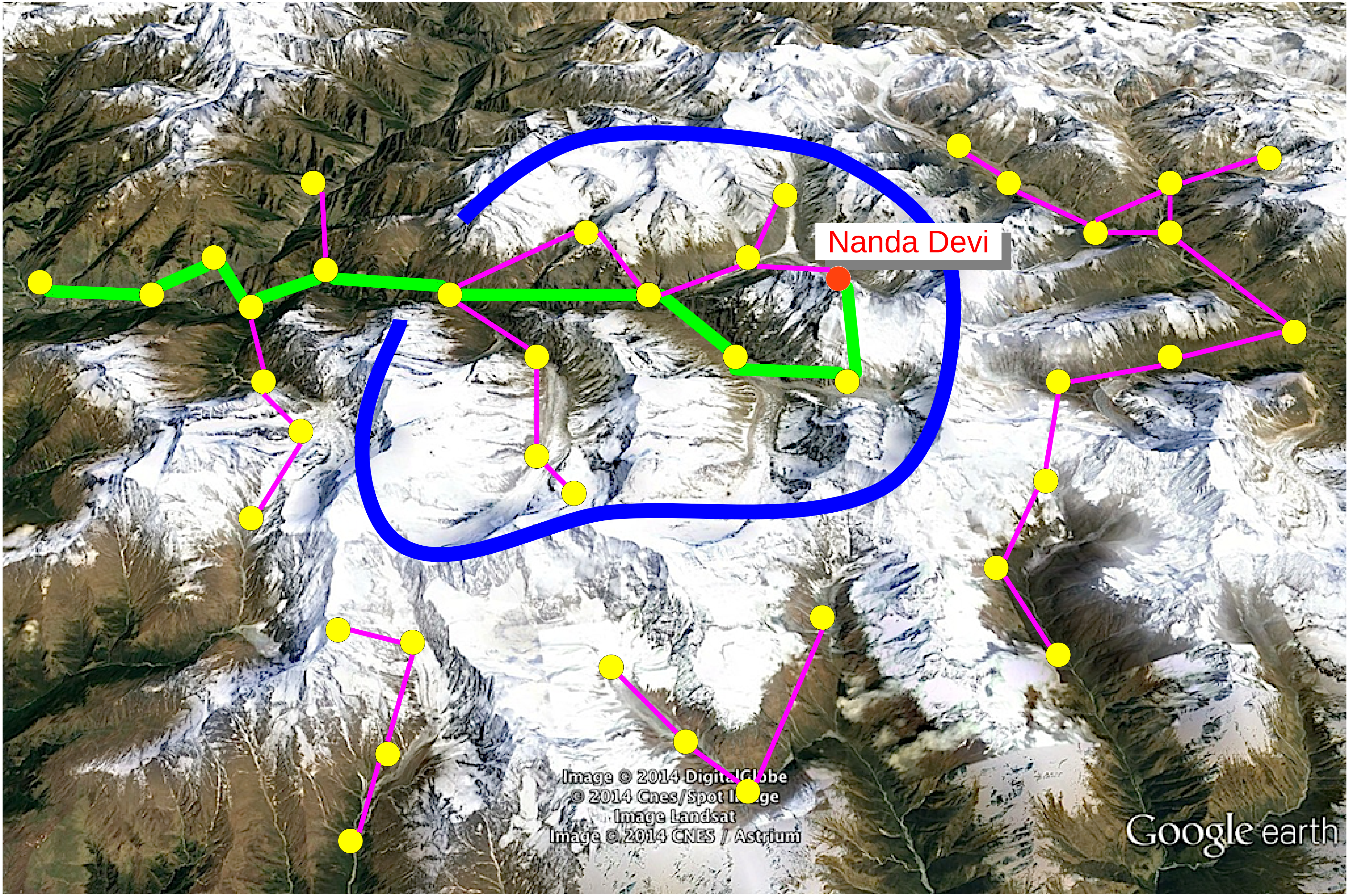
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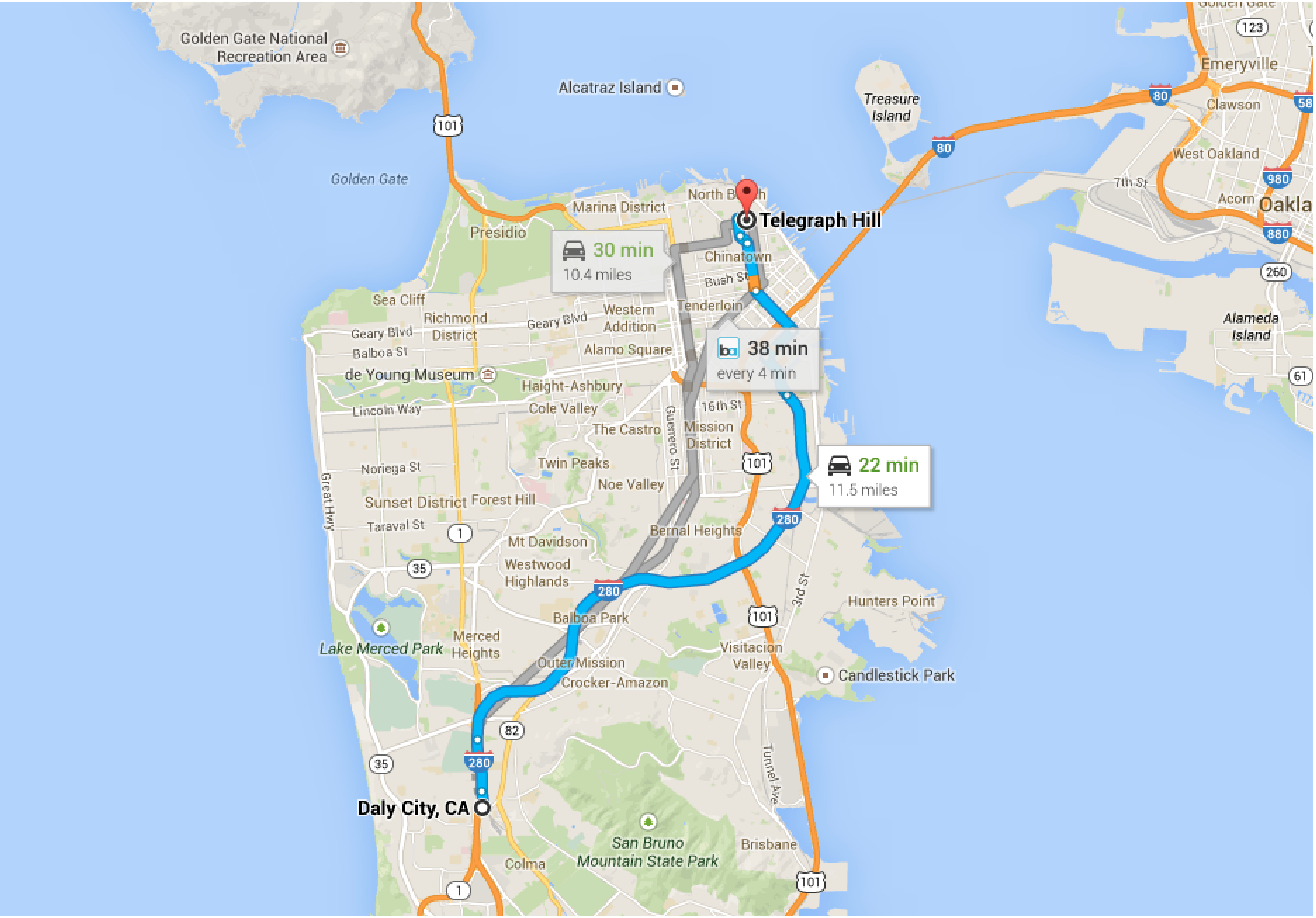
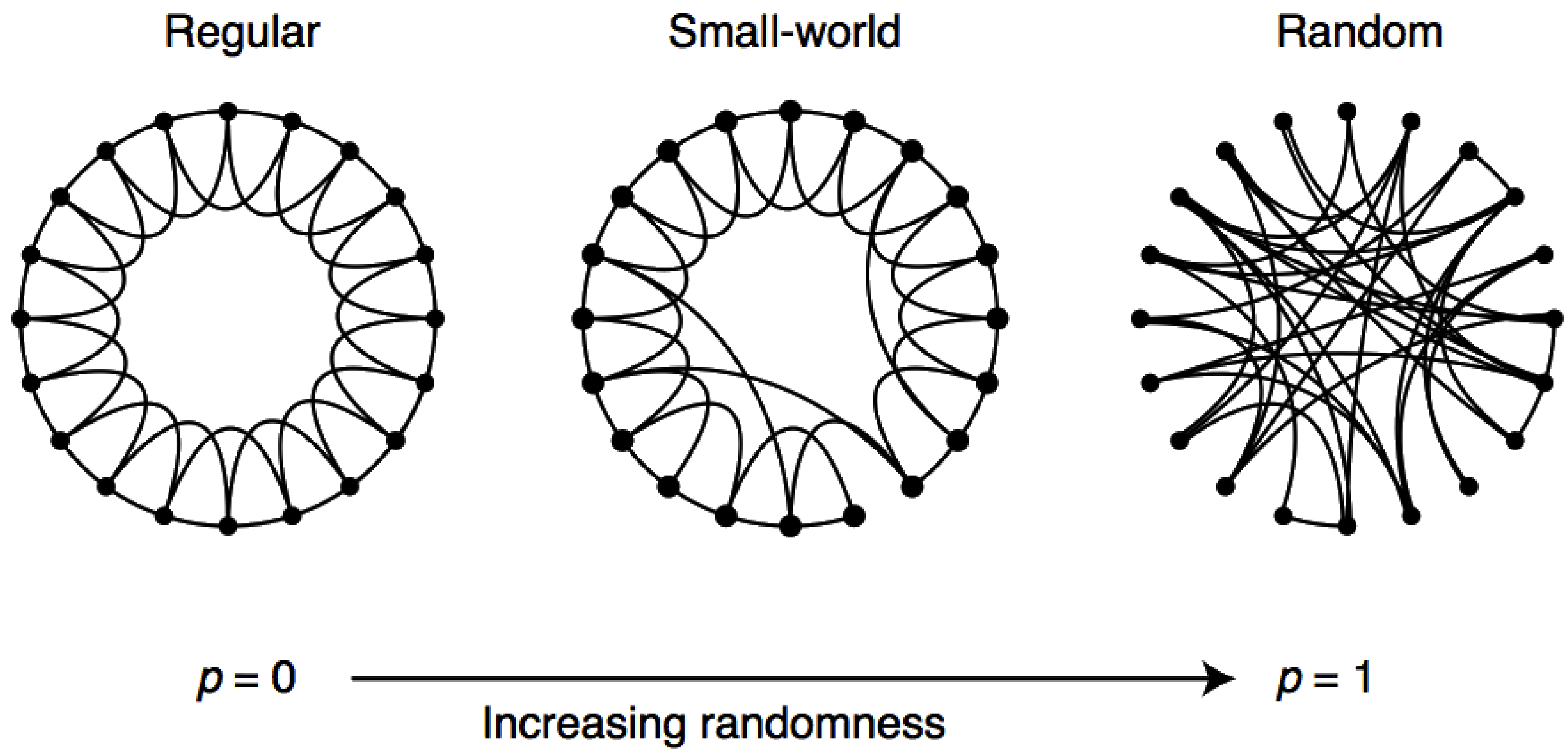


Golden Gate National Recreation Area
Alcatraz Island
Treasure Island
Emeryville
Clawson
West Oakland
7th St
Acorn
Oakla
Golden Gate
Marina District
North Beach
Telegraph Hill
Presidio
30 min
10.4 miles
Chinatown
Bush St
Tenderloin
Sea Cliff
Richmond District
Geary Blvd
Balboa St
Western Addition
Alamo Square
38 min
every 4 min
de Young Museum
Haight-Ashbury
Lincoln Way
Cole Valley
16th St
The Castro
Mission District
Guerrero St
22 min
11.5 miles
Noriega St
Twin Peaks
Noe Valley
Great Hwy
Sunset District
Forest Hill
Taraval St
Mt Davidson
Bernal Heights
Westwood Highlands
Alameda Island
Hunters Point
3rd St
Balboa Park
Lake Merced Park
Merced Heights
Visitacion Valley
Outer Mission
Crocker-Amazon
Candlestick Park
Daly City, CA
Tunnel Ave
San Bruno Mountain State Park
Brisbane
Colma



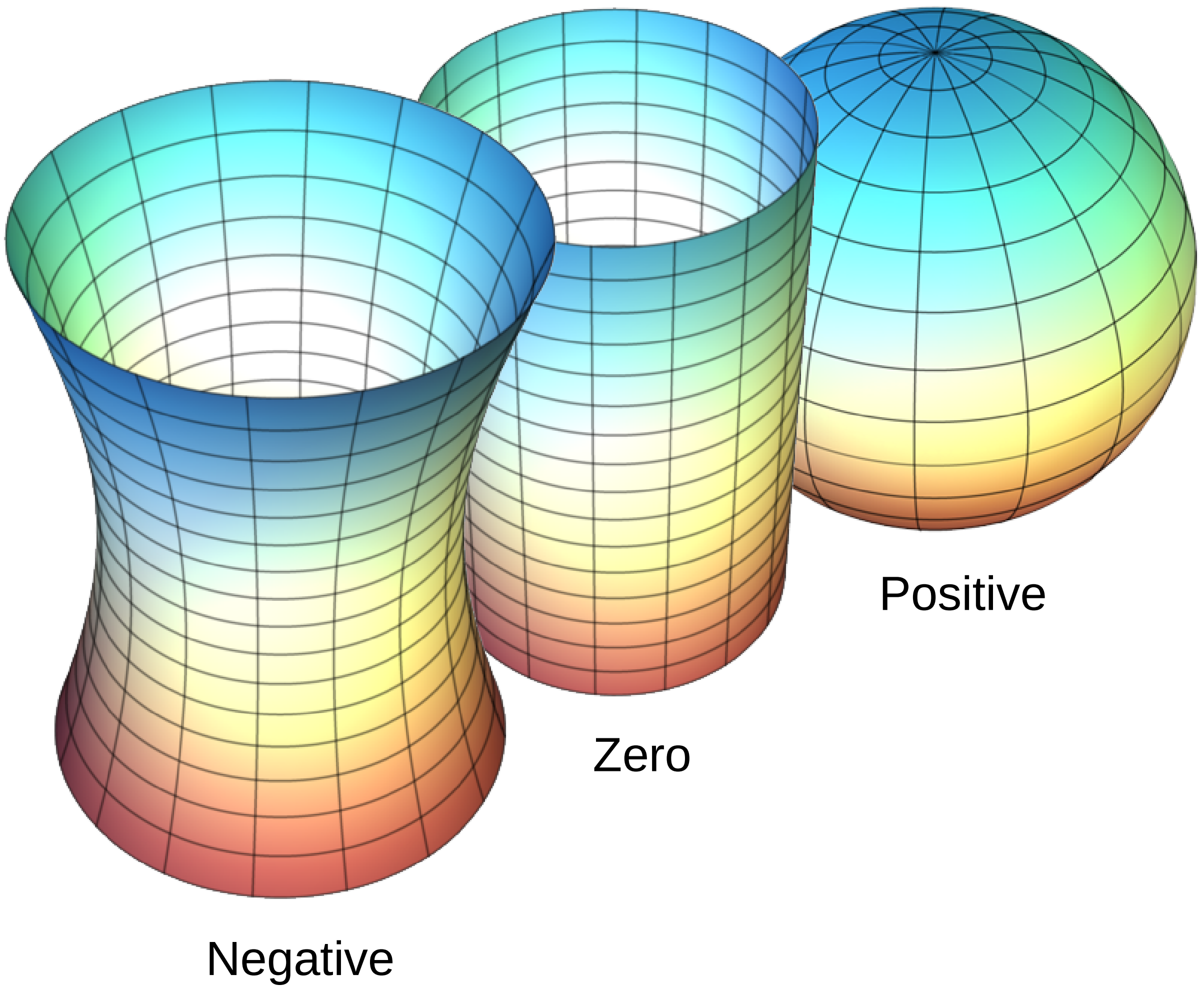
Watts and Strogatz, Cornell, 1998

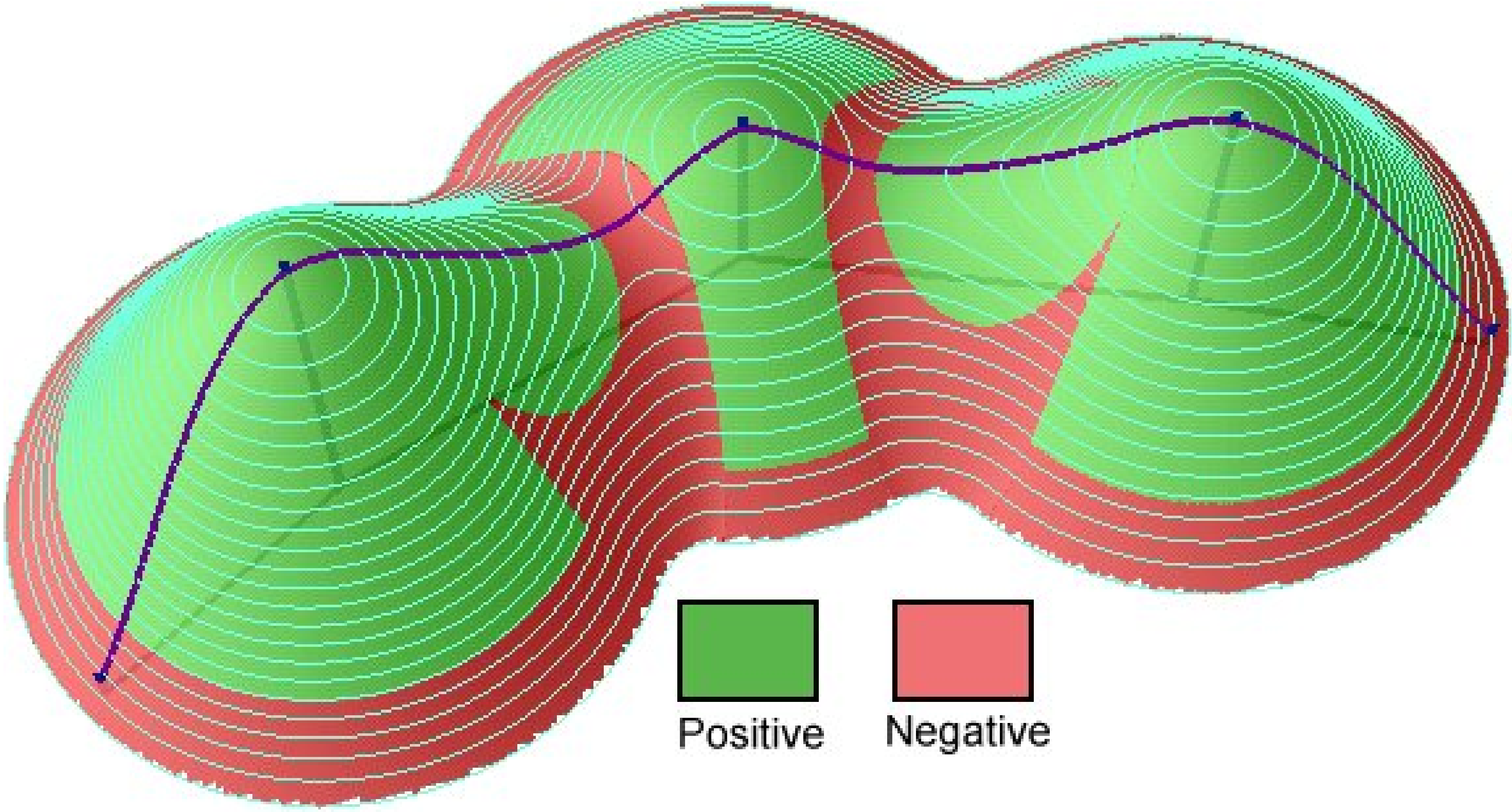
Average path lengths

- 225k Film actors: 3.65
- 5k nodes on US power grid: 18.7
- 282 neurons of C. elegans: 2.65
- 721m Facebook users: 4.74

Watts and Strogatz, 1998; Facebook, 2011

If geometry tells us about distances, what do distances tell us about geometry?


Positive
Zero
Negative


Positive
Negative

Can a 2D ant on a 2D surface tell if it lives in a space of positive, negative or zero curvature?

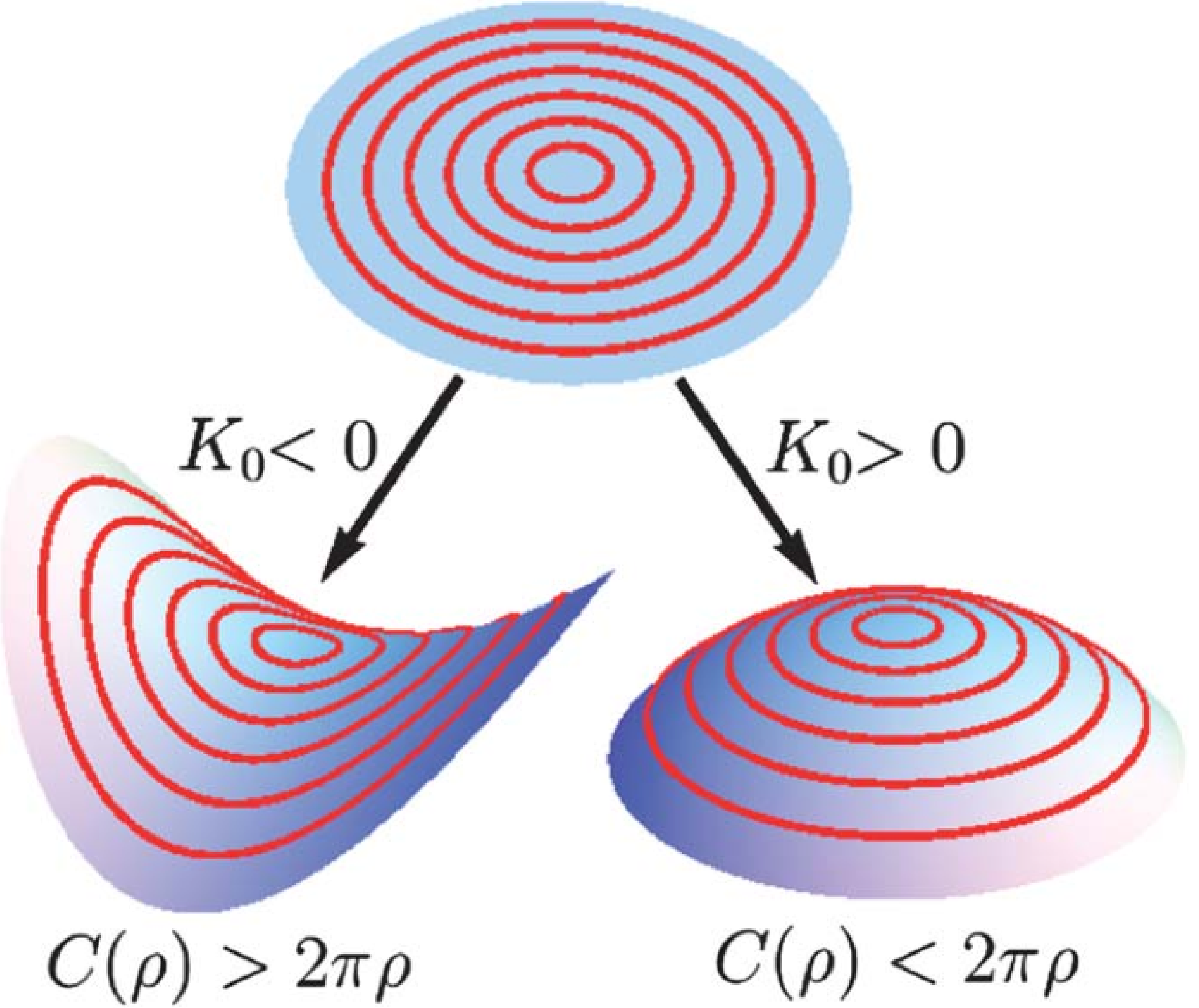
Can a 2D ant on a 2D surface tell if it lives in a space of positive, negative or zero curvature?

Can a person, in 3D?

Can a 2D ant on a 2D surface tell if it lives in a space of positive, negative or zero curvature?

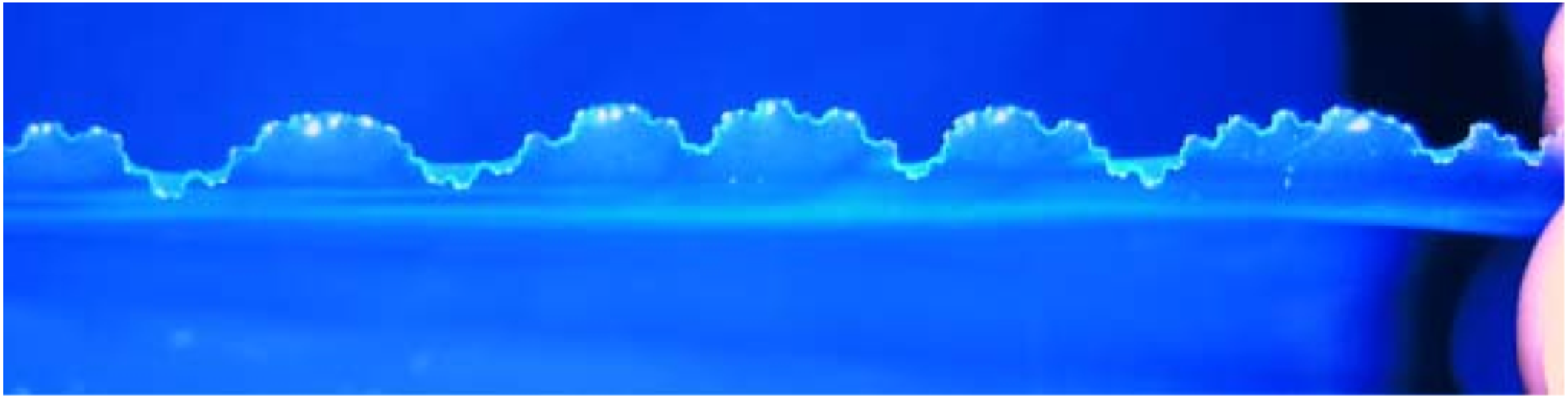
Can a person, in 3D?

Yes, by measuring distances!

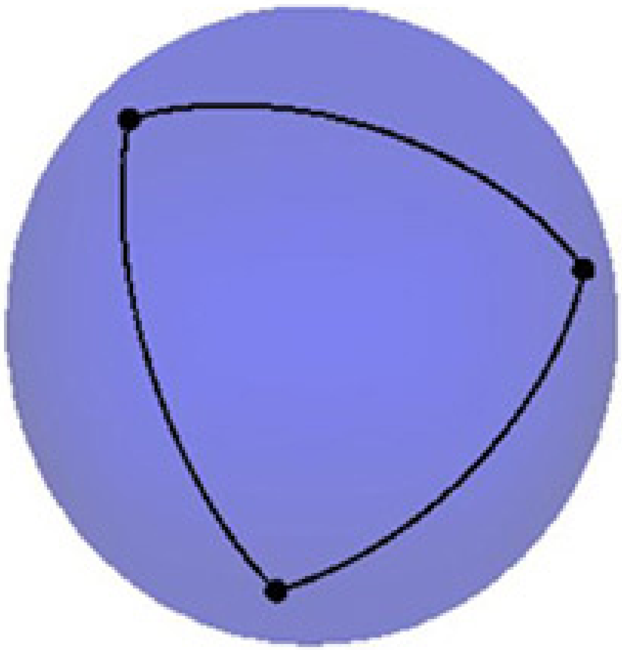


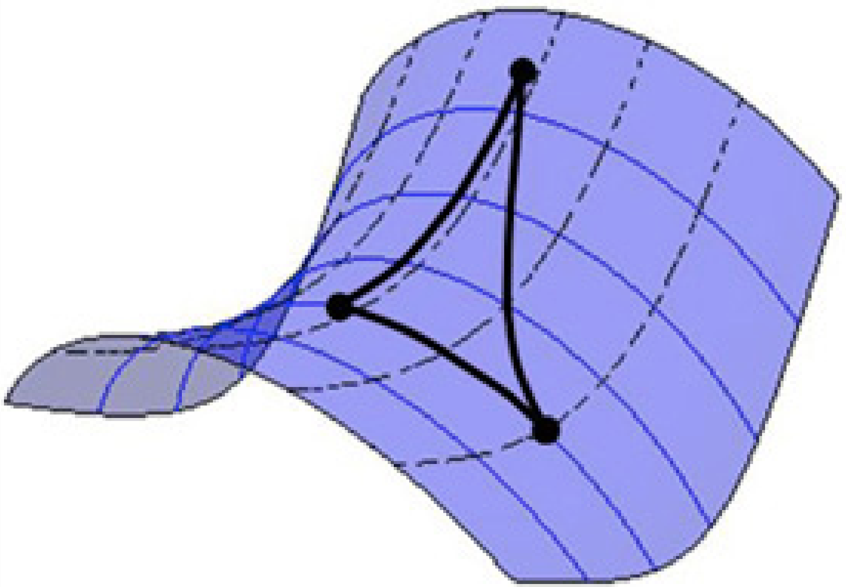
Gemmer and Venkataramani, 2013

Sharon et al. 2004



Sharon et al. 2004

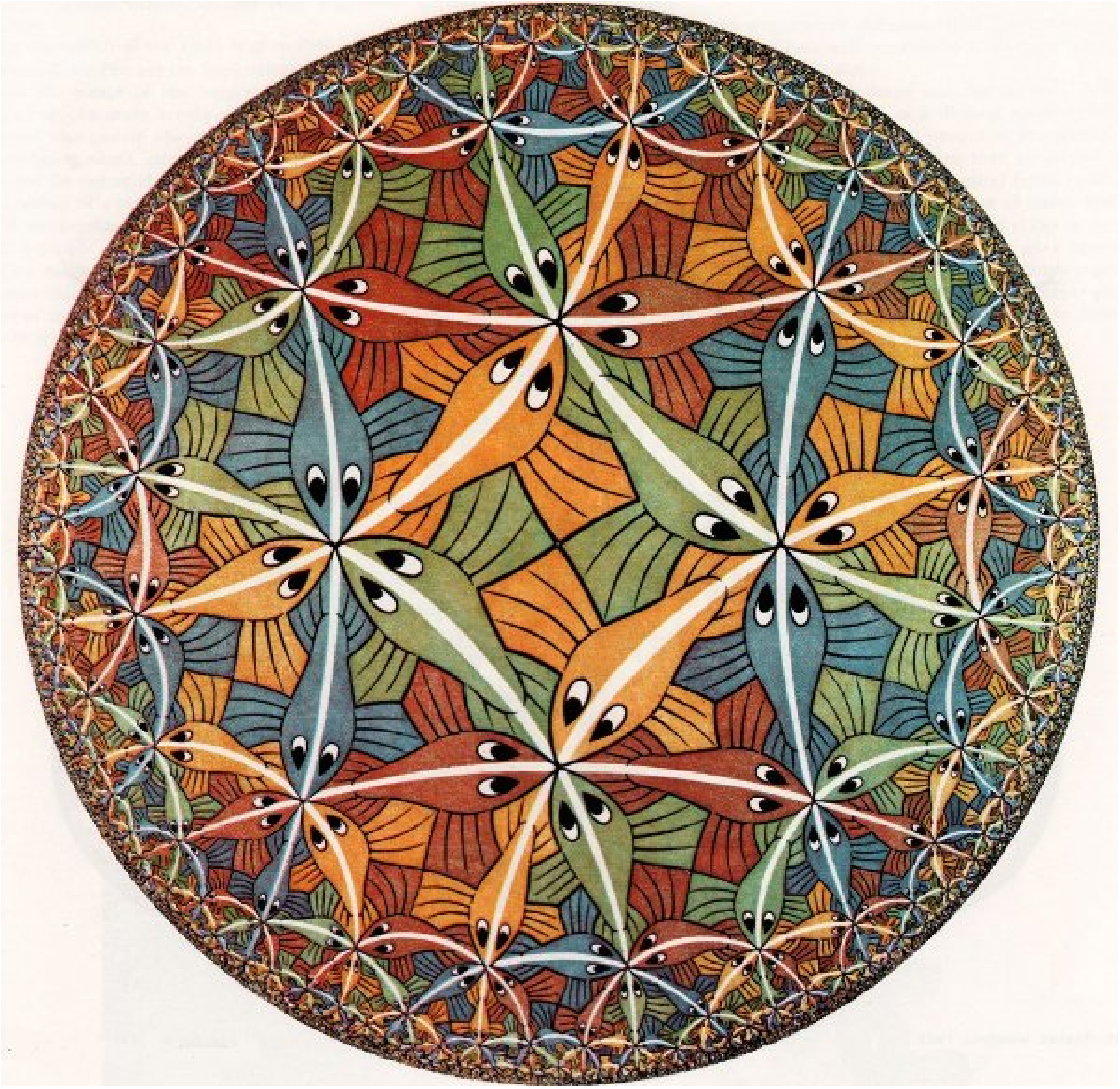




Sum of angles $> 180°$

Sum of angles $< 180°$

Blayne Heckel

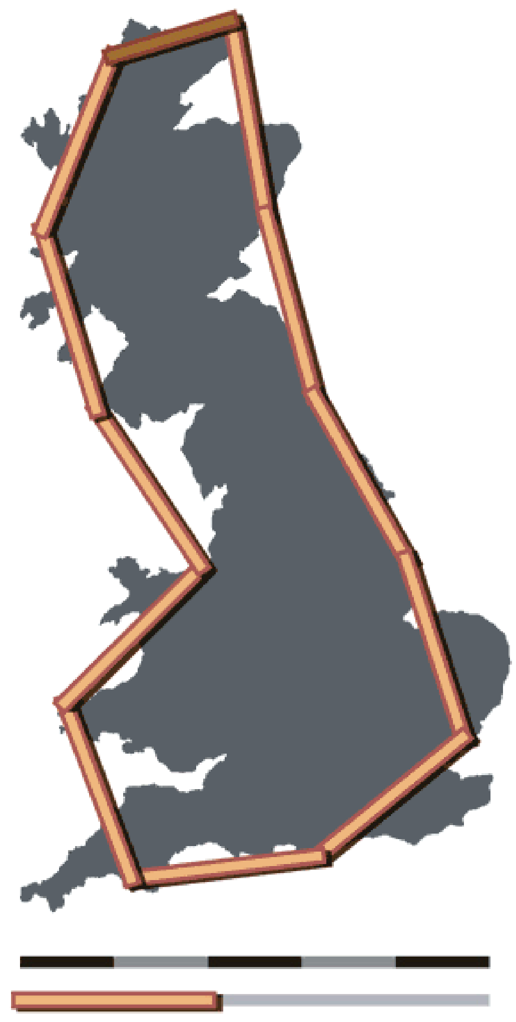


M. C. Escher, *Circle Limit III*

How long is the coastline?

www.british-towns.net

2400km

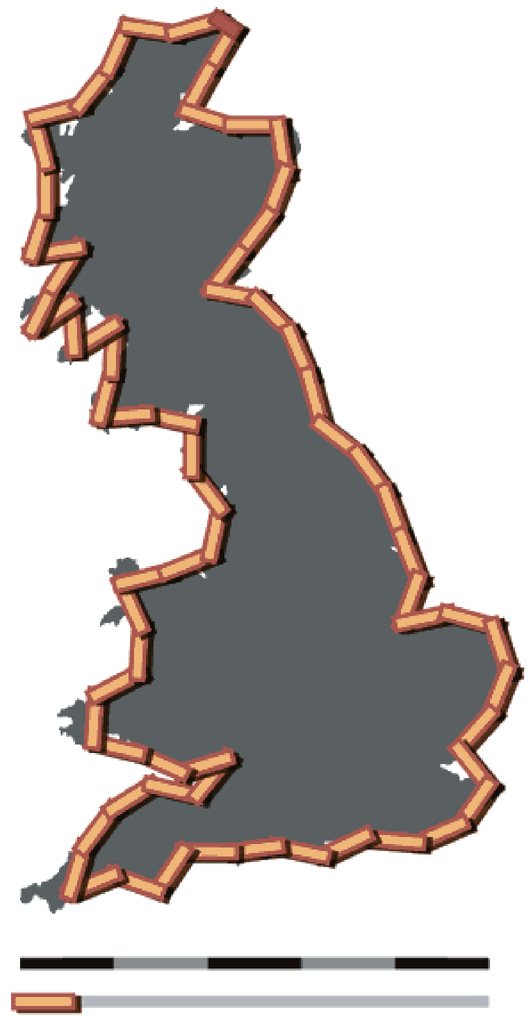


Avsa@wikipedia

2800km

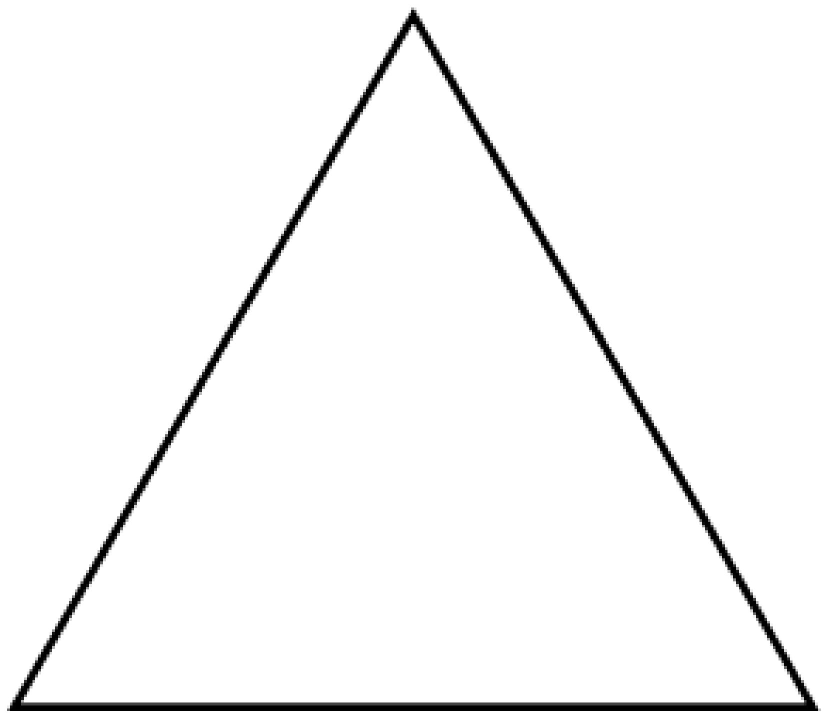
Avsa@wikipedia

3450km



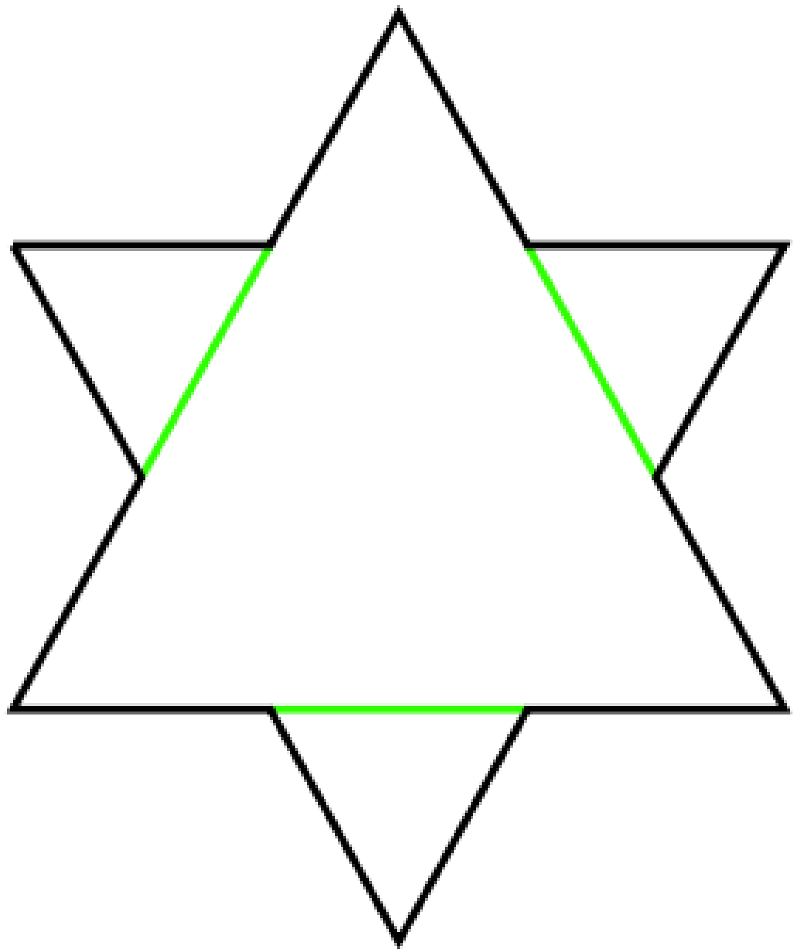
Avsa@wikipedia

The Koch snowflake



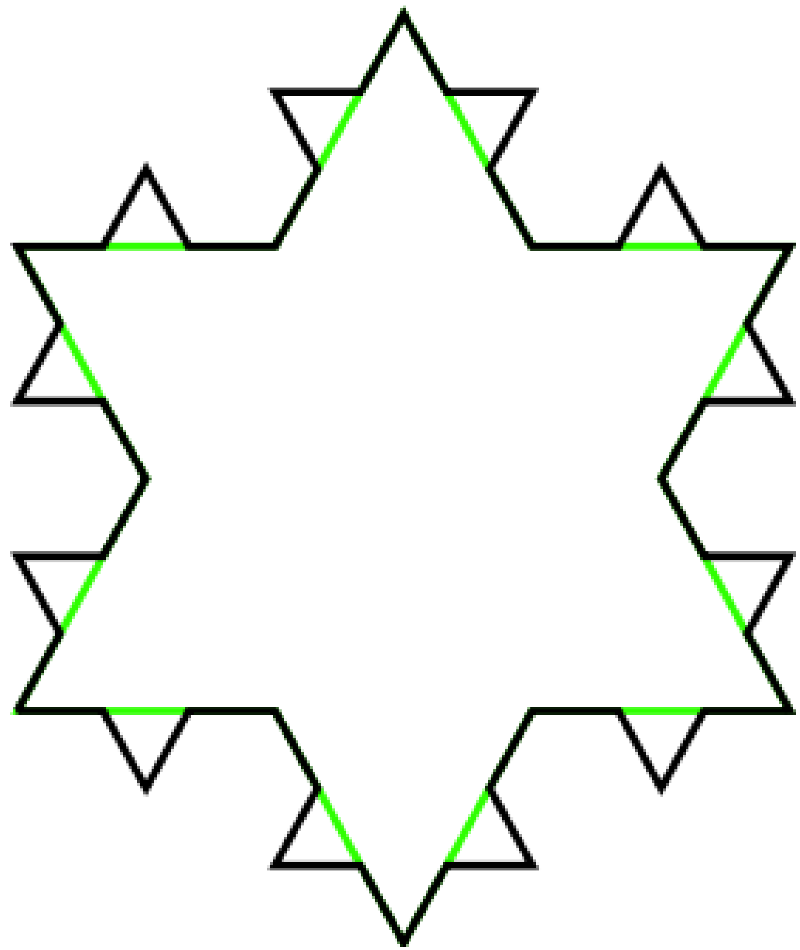
Wikipedia

The Koch snowflake



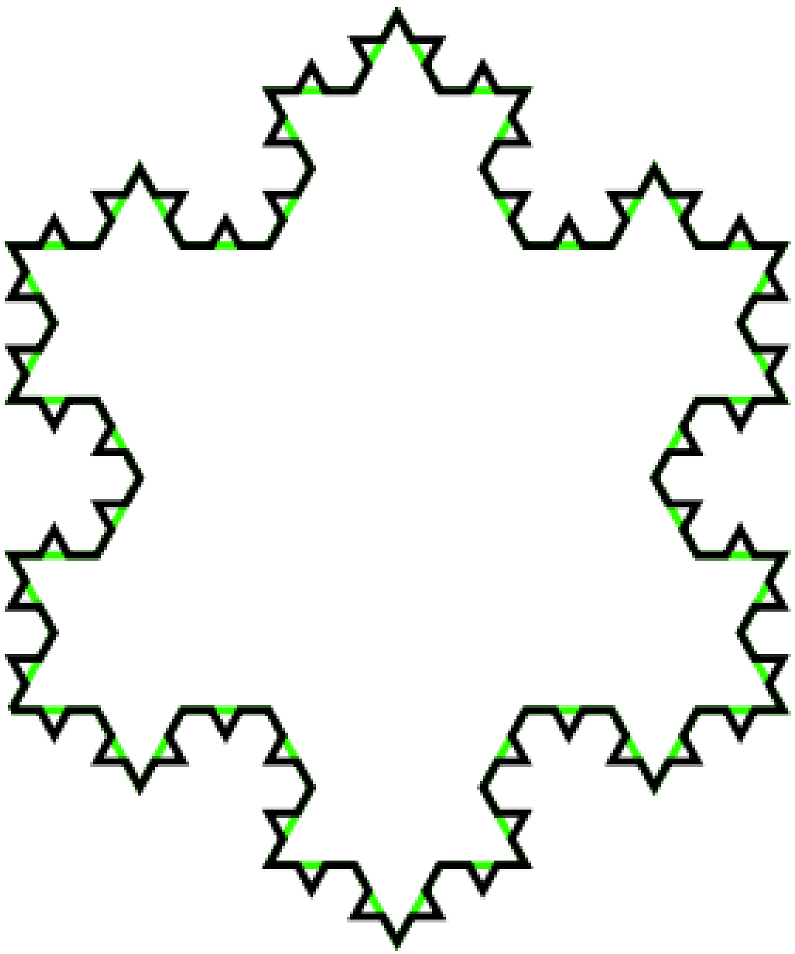
Wikipedia

The Koch snowflake



Wikipedia

The Koch snowflake



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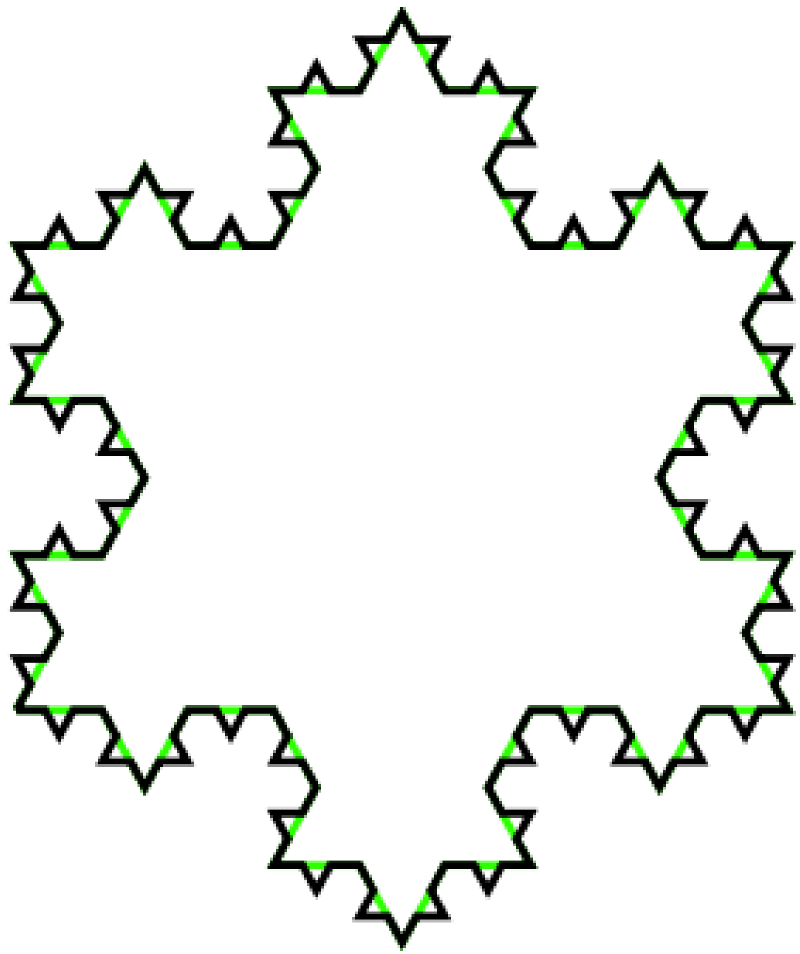
The Koch snowflake



In the limit: bounded area, unbounded perimeter

Wikipedia