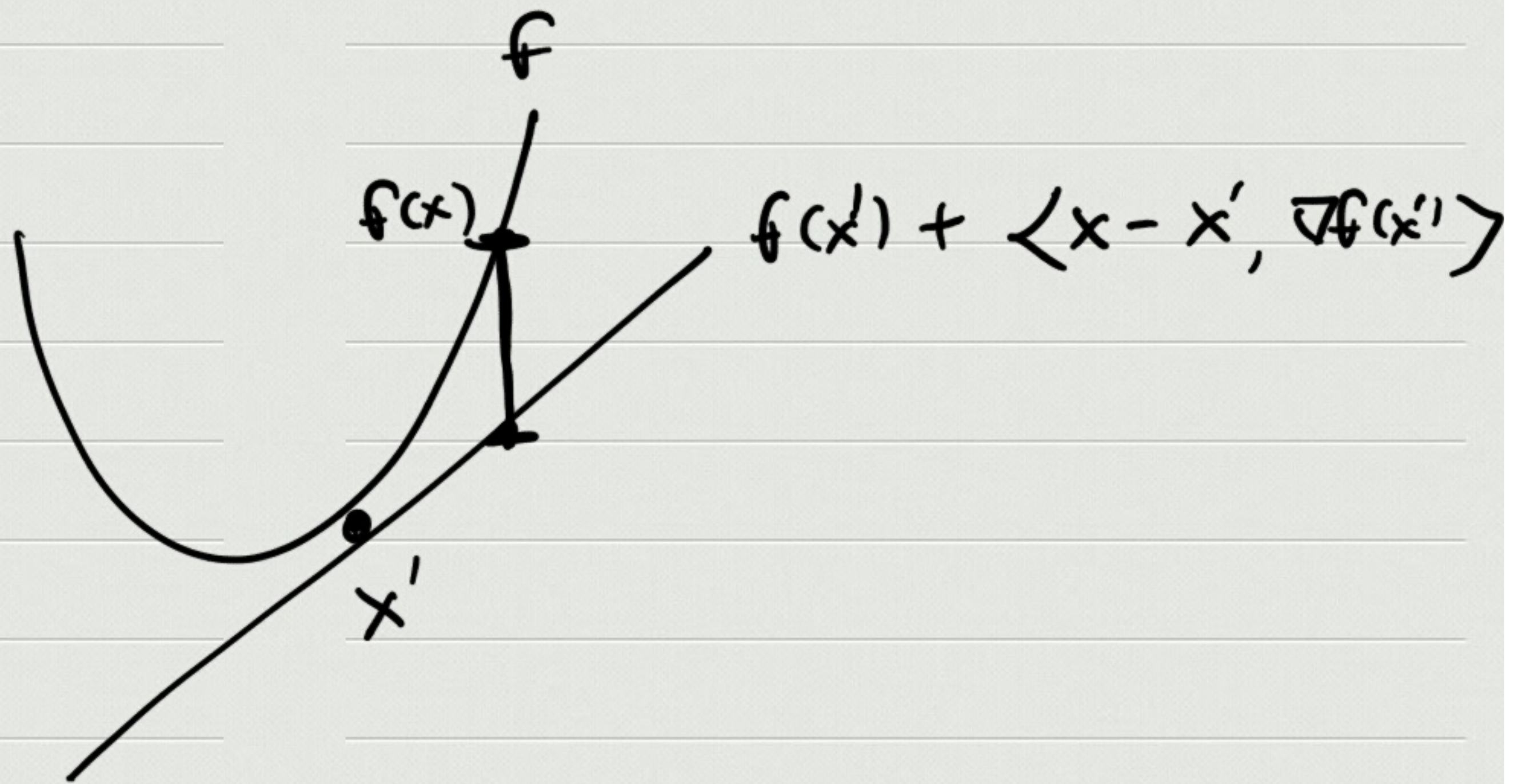


BREGMAN BOX LEMMA

$$\Delta_f(x, y) + \Delta_f(y, z) = \Delta_f(x, z) + \langle \nabla f(z) - \nabla f(y), x - y \rangle$$



$$\Delta_f(x, x') = f(x) - f(x') - \langle x - x', \nabla f(x') \rangle$$

$$J(\omega) = \sum_{t=1}^T f_t(\omega)$$

Ω - DOMAIN

ψ - PROX FUNCTION

$\nabla \psi$

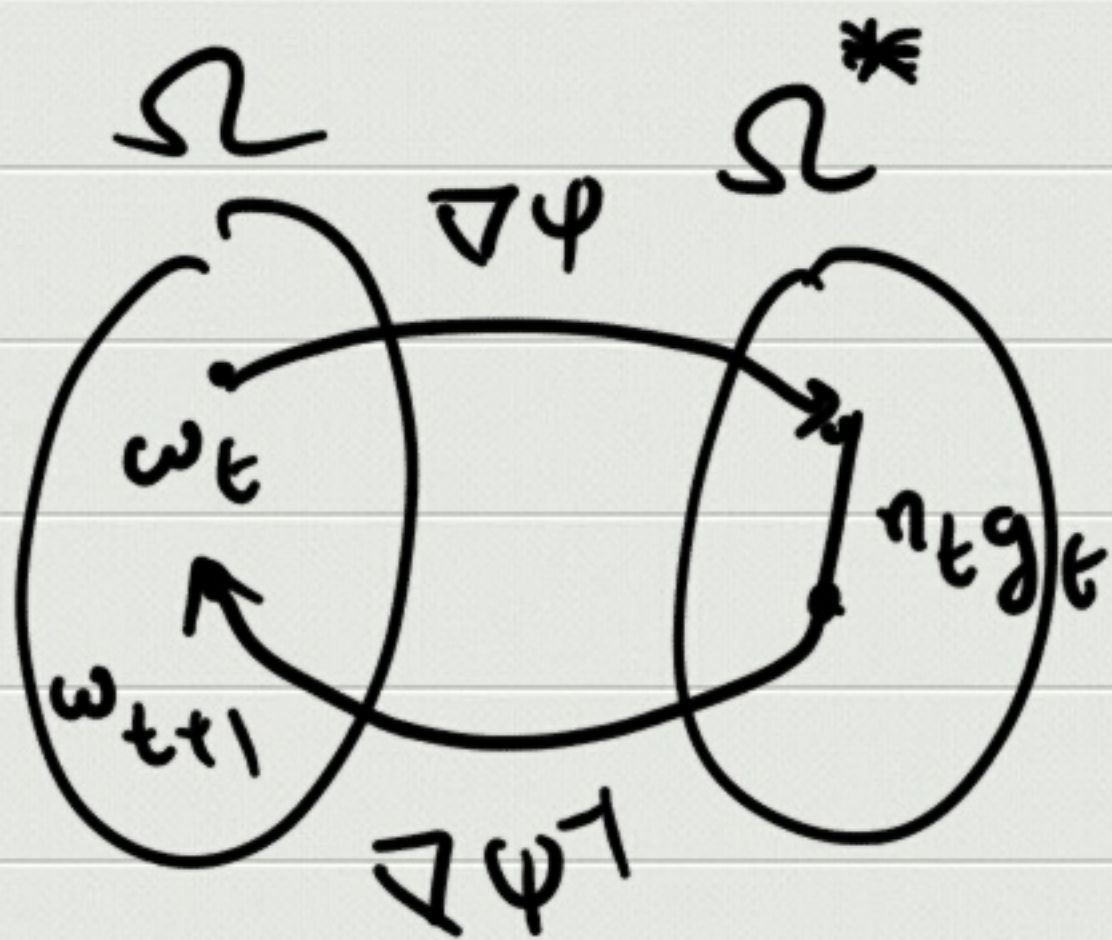
$\nabla \psi^{-1}$

$$P_{\Omega}^{\psi}(\omega) = \omega^* = \underset{\omega' \in \Omega}{\operatorname{arg\,min}} \Delta_{\psi}(\omega', \omega)$$

$$\hat{\omega}_{t+1} = \nabla\psi^{-1}(\nabla\psi(\omega_t) - \eta_t g_t)$$

$$\omega_{t+1} = P_{\Omega}^{\psi}(\hat{\omega}_{t+1})$$

$$g_t = \nabla f_t(\omega_t)$$



$$F = \text{diam}(\Omega) = \max_{\omega, \omega'} \Delta_{\psi}(\omega, \omega')$$

Ω - SIMPLEX

CASE 1: $\psi = \frac{1}{2} \|\phi\|^2$

CASE 2: $\psi =$ RELATIVE ENTROPY

f s.c w.r.t g

$$\Delta_f(x, x') \geq \lambda \Delta_g(x, x')$$

LEMMA:

$$\Delta_\psi(\omega, \omega_{t+1}) \leq (1 - \eta_t \lambda) \Delta_\psi(\omega, \omega_t) - \eta_t (f_t(\omega_t) - f_t(\omega)) + \frac{\eta_t^2}{2\sigma} \|g_t\|^2$$

f_t S.E w.r.t ψ WITH MODULUS $\lambda \geq 0$

ψ S.C WITH MODULUS $\sigma > 0$

REGRET:

$$\sum_{t=1}^T (f_t(\omega_t) - f_t(\omega^*)) \leq \frac{L^2}{2\sigma\lambda} (1 + \log(T)) \quad \text{S.C}$$
$$\leq (F + \frac{L^2}{\sigma}) \sqrt{T}$$

(Note: A bracket under the sum in the first equation is labeled with $t=1$ and an arrow pointing to the sum.)

$$L = \max_t \|g_t\|$$