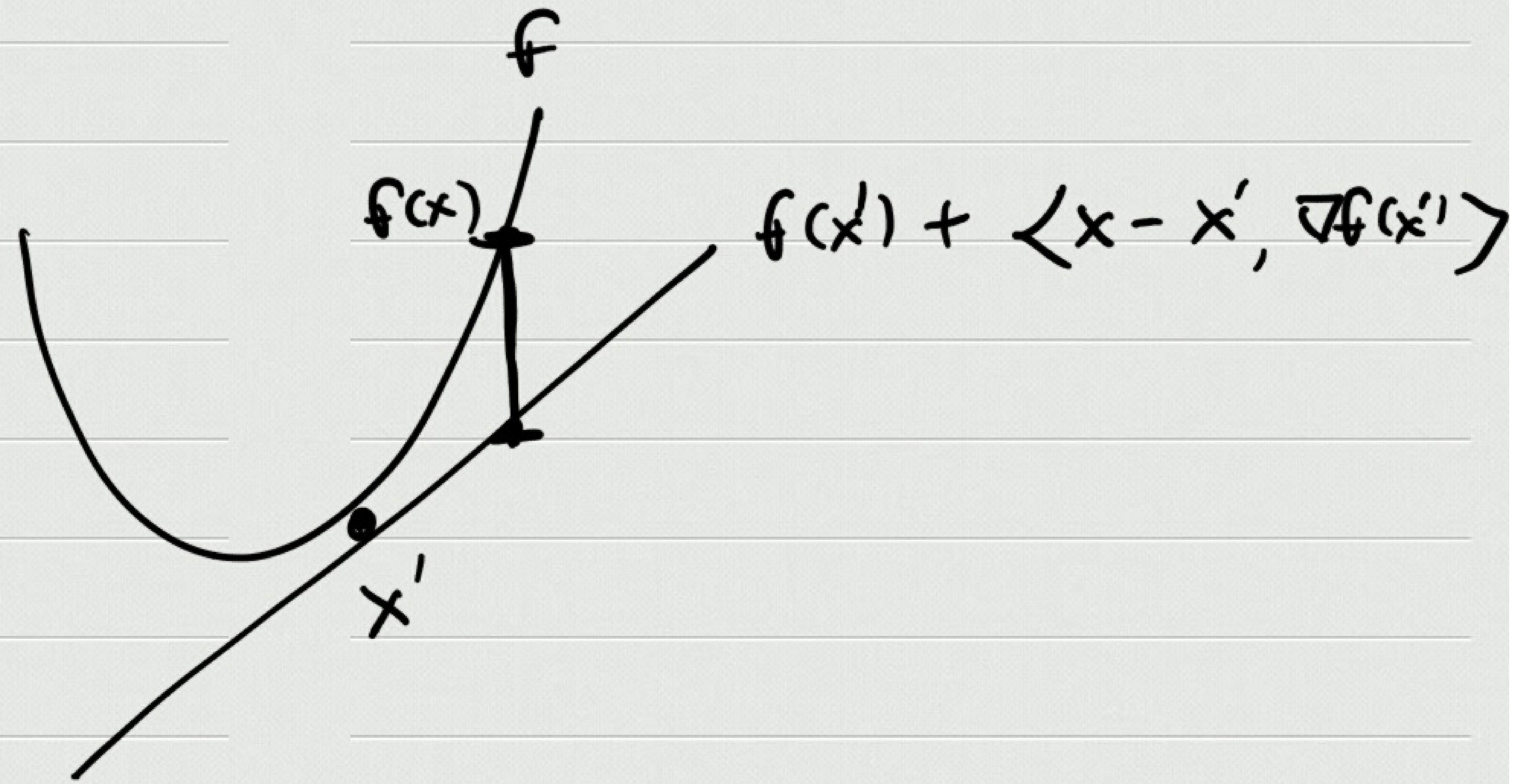
BREGMAN BOX LEMMA

$$\Delta_f(x, y) + \Delta_f(y, z) = \Delta_f(x, z) + \langle \nabla f(z) - \nabla f(y), x - y \rangle$$



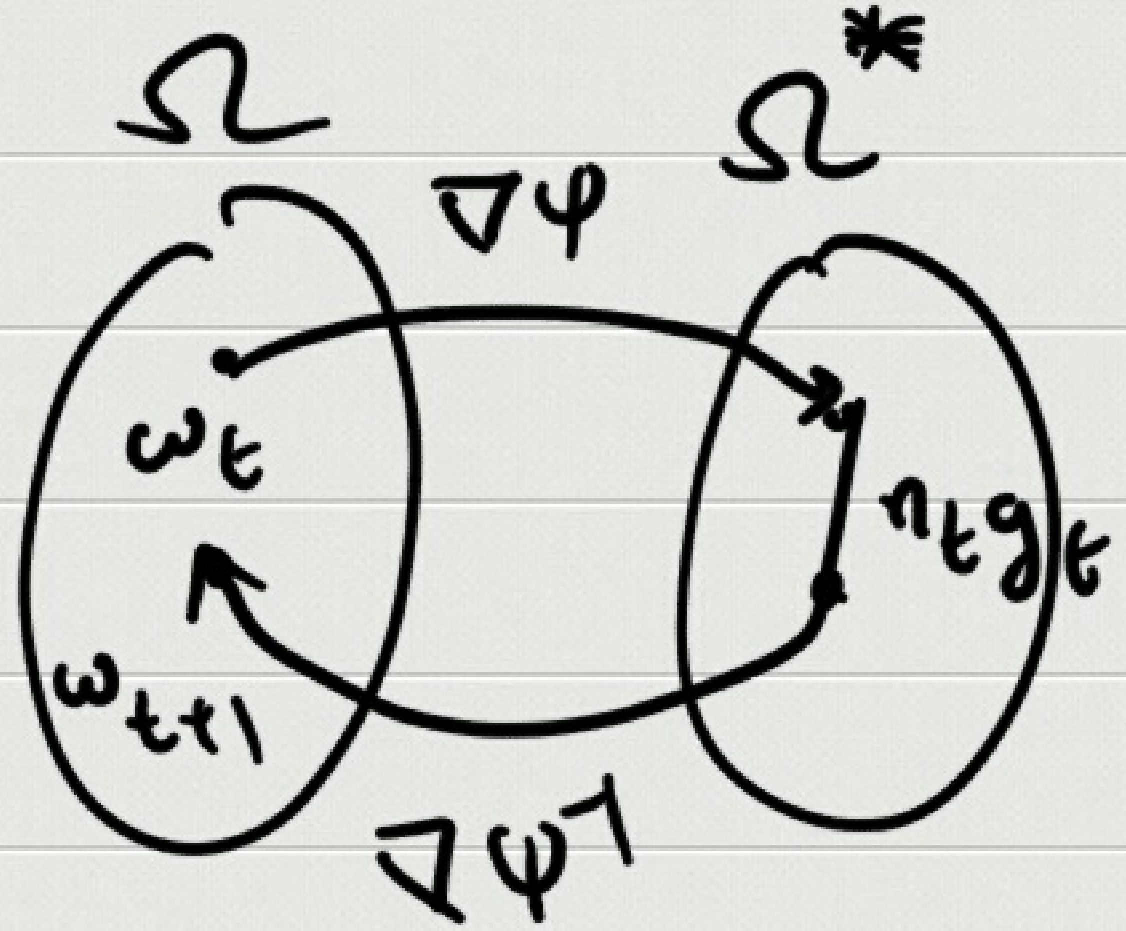
$$\Delta_f(x, x') = f(x) - f(x') - \langle x - x', \nabla f(x') \rangle$$

$$J(\omega) = \sum_{t=1}^{T} f_t(\omega)$$

Ω - DOMAIN

ψ - PROX FUNCTION $\qquad \nabla\psi \qquad \nabla\psi^{-1}$

$$P_{\Omega}^{\psi}(\omega) = \omega^{*} = \arg\min_{\omega' \in \Omega} \Delta_{\psi}(\omega', \omega)$$



$$\hat{\omega}_{t+1} = \nabla\psi^{-1}\left(\nabla\psi(\omega_t) - \eta_t\, g_t\right)$$

$$\omega_{t+1} = P_{\Omega}^{\psi}(\hat{\omega}_{t+1})$$

$$g_t = \nabla f_t(\omega_t)$$

$$F = \operatorname{diam}(\Omega) = \max_{\omega, \omega'} \Delta_\psi(\omega, \omega')$$

Ω - SIMPLEX

CASE 1: ψ - $\frac{1}{2}\|\cdot\|^2$

CASE 2: ψ - RELATIVE ENTROPY

f s.c w.r.t g

$$\Delta_f(x, x') \geq \lambda \Delta_g(x, x')$$

LEMMA:

$$\Delta_\psi(\omega, \omega_{t+1}) \leq (1-\eta_t\lambda)\Delta_\psi(\omega, \omega_t) - \eta_t\left(f_t(\omega_t) - f_t(\omega)\right) + \frac{\eta_t^2}{2\sigma}\|g_t\|^2$$

f_t S.E w.r.t ψ WITH MODULUS $\lambda \geq 0$

ψ S.C WITH MODULUS $\sigma > 0$

REGRET:

$$\underbrace{\sum_{t=1}^{T}\left(f_t(w_t) - f_t(w^*)\right)}$$

$$\leq \frac{L^2}{2\sigma\lambda}\left(1 + \log(T)\right) \quad \text{S.C}$$

$$\leq \left(F + \frac{L^2}{\sigma}\right)\sqrt{T}$$

$$L = \max_t \|g_t\|$$