Improved Expected Running Time for MDP Planning

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Overview

1. MDP Planning
2. Solution strategies
   - Linear programming
   - Value iteration
   - Policy iteration
   - **Our contribution**: Planning by Guessing and Policy Improvement (PGPI)
3. PGPI algorithm
   - A total order on the set of policies
   - Guessing game
   - Algorithm
4. Discussion

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MDP Planning

Markov Decision Problem: general abstraction of sequential decision making.

An MDP comprises a tuple (S, A, R, T), where

- S is a set of states (with $|S| = n$),
- A is a set of actions (with $|A| = k$),
- $R(s, a)$ is a bounded real number, $\forall s \in S, \forall a \in A$, and
- $T(s, a)$ is a probability distribution over S, $\forall s \in S, \forall a \in A$.

A policy $\pi : S \to A$ specifies an action from each state. The value of a policy π from state s is:

$$V^\pi(s) = \mathbb{E}\left[\sum_{t=0}^{\infty} \gamma^t r_t \mid s_0 = s, a_t = \pi(s_t), t = 0, 1, 2, \ldots\right], \text{where}$$

$\gamma \in [0, 1)$ is a discount factor.

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Planning problem: Given S, A, R, T, and γ, find a policy $\pi^\star$ from the set of all policies Π such that

$$V^{\pi^\star}(s) \geq V^{\pi}(s), \forall s \in S, \forall \pi \in \Pi.$$

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Linear Programming

- The optimal value function $V^{\pi^\star} \stackrel{\text{def}}{=} V^\star$ is unique solution of Bellman's Optimality Equations: $\forall s \in S$:

$$V^\star(s) = \max_{a \in A} \left(R(s,a) + \gamma \sum_{s' \in S} T(s,a,s') V^\star(s') \right).$$

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- $V^\star$ can be obtained by solving an equivalent linear program:

$$\begin{aligned} &\text{maximise} && \sum_{s \in S} V(s) \\ &\text{subject to} && V(s) \geq \left(R(s,a) + \gamma \sum_{s'} T(s,a,s') V(s') \right), \forall s \in S, \forall a \in A. \end{aligned}$$

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- n variables, nk constraints (or *dual* with nk variables, n constraints).

Solution time: poly(n, k, B),

where B is the number of bits used to represent the MDP.

Value Iteration

- Classical dynamic programming approach.

$V_0 \leftarrow$ Arbitrary, element-wise bounded, n-length vector.
$t \leftarrow 0$.
Repeat:
  For $s \in S$**:**
    $V_{t+1}(s) \leftarrow \max_{a \in A} \left(R(s,a) + \gamma \sum_{s' \in S} T(s,a,s') V_t(s') \right)$.
    $t \leftarrow t + 1$.
Until $V_t = V_{t-1} = V^\star$ (up to machine precision).

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- Convergence to $V^\star$ guaranteed using a max-norm contraction argument.

Number of iterations: poly$(n, k, B, \frac{1}{1-\gamma})$.

Bellman's Equations and Policy Evaluation

Recall that

$$V^{\pi}(s) = \mathbb{E}[r_1 + \gamma r_2 + \gamma^2 r_3 + \ldots | s_0 = s, a_i = \pi(s_i)].$$

Bellman's Equations ($\forall s \in S$):

$$\boxed{V^{\pi}(s) = \sum_{s' \in S} T(s, \pi(s), s') \left[R(s, \pi(s), s') + \gamma V^{\pi}(s')\right].}$$

V^{π} is called the value function of π.

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Define ($\forall s \in S, \forall a \in A$):

$$Q^{\pi}(s, a) = \textstyle\sum_{s' \in S} T(s, a, s') \left[R(s, a, s') + \gamma V^{\pi}(s')\right].$$

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$V^{\pi}(s) = Q^{\pi}(s, \pi(s))$.

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Thus, given S, A, T, R, γ, and a fixed policy π, we can solve Bellman's equations efficiently to obtain, $\forall s \in S, \forall a \in A$, $V^\pi(s)$ and $Q^\pi(s, a)$.

Policy Iteration

- For a given policy π:

$$I(\pi) \stackrel{\text{def}}{=} \left\{ s \in S : Q^{\pi}(s, \pi(s)) < \max_{a \in A} Q^{\pi}(s, a) \right\}.$$

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- Assume $I^{\pi} \neq \emptyset$. Let $C(\pi)$ be an arbitrarily "chosen" non-empty subset of $I(\pi)$.
- Define a policy π' as follows.

$$\pi'(s) \stackrel{\text{def}}{=} \begin{cases} \arg\max_{a \in A} Q^{\pi}(s, a) & \text{if } s \in C(\pi), \\ \pi(s) & \text{otherwise.} \end{cases}$$

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  (1) $\forall s \in S : V^{\pi'}(s) \geq V^{\pi}(s)$, and
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(2) $\exists s \in S : V^{\pi'}(s) > V^{\pi}(s)$. [Policy improvement]

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$\pi_0 \leftarrow$ Arbitrary policy.
$t \leftarrow 0$.
Repeat:
  Evaluate π^t; derive $I(\pi^t)$.
  If $I(\pi^t) \neq \emptyset$, select $C(\pi^t) \subset I(\pi^t)$ and improve π^t to π^{t+1}.
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- Howard's Policy Iteration: $C(\pi) = I(\pi)$.

Number of iterations: $O\left(\frac{k^n}{n}\right)$.

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Expected number of iterations:
$O\left(2^{0.78n}\right)$ for k = 2; $O\left(\left(\left(1 + \frac{2}{log(k)}\right)\frac{k}{2}\right)^n\right)$ for general k.

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- References: Howard(1960), Mansour and Singh (1999), Hollanders et al. (2014).

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- References: Howard(1960), Mansour and Singh (1999), Hollanders et al. (2014).
- Note that bounds do not depend on B and γ!

Our Contribution

Algorithm	**Computational complexity**
Linear Programming	$\text{poly}(n, k, B)$
Value Iteration	$\text{poly}\left(n, k, B, \frac{1}{1-\gamma}\right)$
Policy Iteration	$\text{poly}(n) \cdot O\left(\left(\left(1 + \frac{2}{log(k)}\right)\frac{k}{2}\right)^n\right)$ (expected)

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Key ingredient: a **total order** on the set of policies.

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Even tighter bounds by combining PGPI with Randomised Policy Iteration!

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- Total order $\succ$:

> For $\pi_1, \pi_2 \in \Pi$, define $\pi_1 \succ \pi_2$ iff
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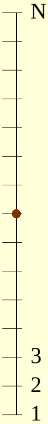
- Observe that if policy improvement to π yields π', then $\pi' \succ \pi$.

$$\forall s \in S : V^{\pi'}(s) \geq V^{\pi}(s) \text{ and } \exists s \in S : V^{\pi'}(s) > V^{\pi}(s)$$
$$\implies V(\pi') > V(\pi)$$
$$\implies \pi_1 \succ \pi_2.$$

Guessing Game



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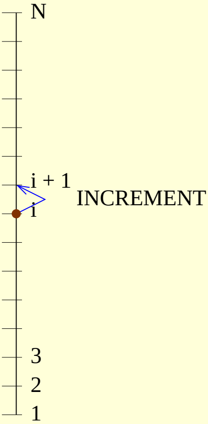

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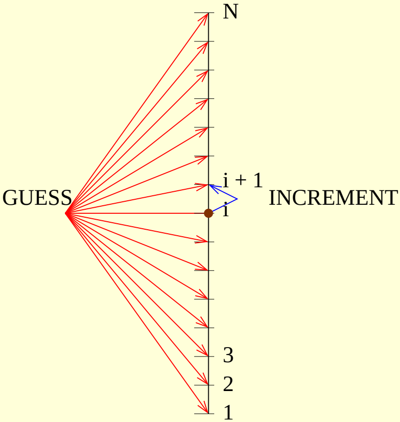

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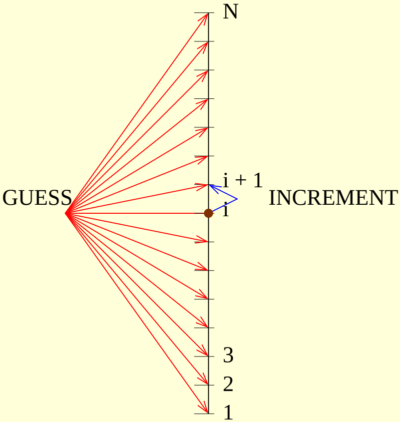

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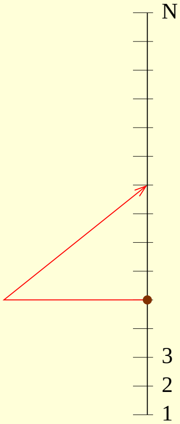

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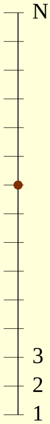

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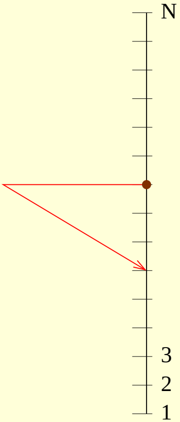

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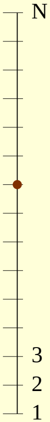

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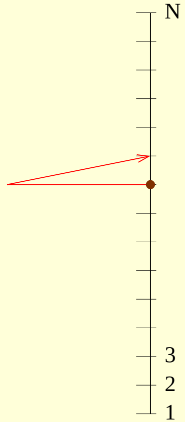

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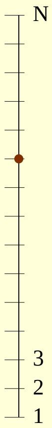

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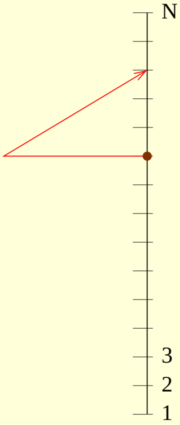

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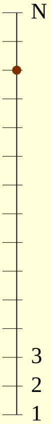

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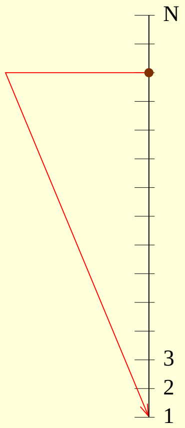

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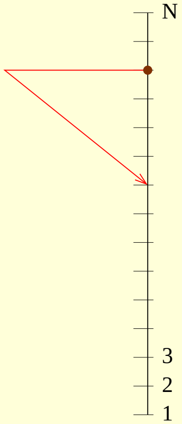

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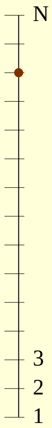

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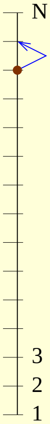

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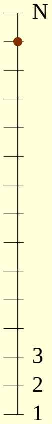

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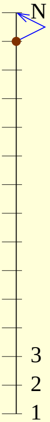

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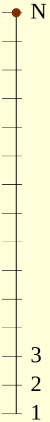

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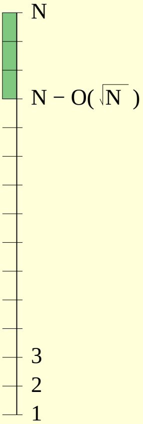

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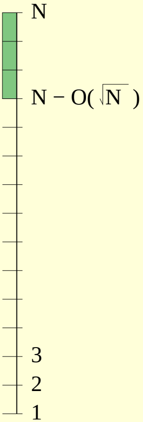
Guessing Game



- How many operations needed to reach N?
- Pick the best of $\sqrt{N}$ guesses, and then increment up to N.
- Expected number of operations: $O(\sqrt{N})$.

PGPI Algorithm

$\pi \leftarrow$ Arbitrary policy.
Repeat $k^{\alpha n}$ times:
  Draw π' from Π uniformly at random.
  If $\pi' \succ \pi$, $\pi \leftarrow \pi'$.
While π is not optimal:
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$k = 2, \alpha = 0.46$, randomised policy improvement;
Expected number of iterations: $O\left(2^{0.46n}\right)$.

Overview

1. MDP Planning
2. Solution strategies
   - Linear programming
   - Value iteration
   - Policy iteration
   - **Our contribution**: Planning by Guessing and Policy Improvement (PGPI)
3. PGPI algorithm
   - A total order on the set of policies
   - Guessing game
   - Algorithm
4. Discussion

Discussion

- PGPI: improved complexity bound solely in terms of n and k.

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Thank you!