# CS626: Speech, NLP and the Web 

## POS Tagging

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## Agenda

- Rule Based POS Tagging
- Statistical ML based POS Tagging (Hidden Markov Model, Support Vector Machine)
- Neural (Deep Learning) based POS Tagging




## Multilayer neural net



Output layer (m o/p neurons)

Hidden layers

Input layer
(n i/p neurons)

- NLP pipeline $\leftarrow \rightarrow$ NN layers
- Discover bigger structures bottom up, starting from character?
- Words. POS. Parse. Sentence.


## Subwords (for "jaauMgaa", जाऊंगा)

- Characters: "j+aa+u+M+g+aa"
- Morphemes: "jaa"+"uMgaa"
- Syllables: "jaa"+"uM"+"gaa"
- Orthographic syllables: "jaau"+"Mgaa"
- BPE (depends on corpora, statistically frequent patterns): both "jaa" and "uMgaa" are likely


## NLP Layer

What a gripping movie was Three_Idiots!
What/WP a/DT gripping/JJ movie/NN was/VBD Three_Idiots/NNP !/!

```
Parse
(ROOT
    (FRAG
        (SBAR
            (WHNP
            (WP What))
            (S
                (NP
                            (DT a)
            (JJ gripping)
            (NN movie)
                )
                (VP
                    (VBD was)
                    (NP
                    (NNP Three_idiots)))))
                (.!)
```

Universal dependencies dobj(Three_Idiots-6, What-1) det(movie-4, a-2)<br>amod(movie-4, gripping-3) nsubj(Dangal-6, movie-4) cop(Dangal-6, was-5) root(ROOT-0, Three_idiots-6)

## Part of Speech Tagging

- Attach to each word a tag from Tag-Set
- Standard Tag-set : Penn Treebank (for English).


## POS ambiguity instances

best ADJ ADV NP V better ADJ ADV V DET
close ADV ADJ V N (running close to the competitor, close escape, close the door, towards the close of the play)

```
cut V N VN VD
even ADV DET ADJ V
grant NP N V -
hit V VD VN N
lay ADJ V NP VD
left VD ADJ N VN
like CNJ V ADJ P -
near P ADV ADJ DET
open ADJ V N ADV
past N ADJ DET P
present ADJ ADV V N
read V VN VD NP
right ADJ N DET ADV
second NUM ADV DET N
set VN V VD N -
that CNJ V WH DET
```


## POS Ambiguity

- E.g.

Adjective

1. What a gripping movie was Abhiman!
2. He is gripping it firm.


## Linguistic fundamentals

- A word can have two roles
- Grammatical role (Dictionary POS tag)
- Functional role (Contextual POS tag)
- E.g. Golf stick
- POS tag of "Golf"
- Grammatical: Noun
- Functional: Adjective (+ al)


## The "al" rule!

- If a word has different functional POS tag than its grammatical pos then add "al" to the functional POS tag
- E.g. Golf stick

Adjective + al

Adjectival

| Noun +al | $=$ Nominal |
| :--- | :--- |
| Verb + al | $=$ Verbal |
| Adjective + al | $=$ Adjectival |
| Adverb + al | $=$ Adverbial |

## Dictionary meaning of "Golf"

## noun

- a game in which clubs with wooden or metal heads are used to hit a small, white ball into a number of holes, usually 9 or 18, in succession,
- situated at various distances over a course having natural or artificial obstacles, the object being to get the ball into each hole in as few strokes as possible.
- a word used in communications to represent the letter $G$.


## Golf stick

verb
(used without object) to play golf.
We golfed the whole day in the weekend

## The "al" rule cntd.

- Examples:
- Nominal
- Many don't understand the problem of hungry.
- Adverbial
- Come quick.
- Verbal?
adjective, hun• gri-er, hun-gri•est.
having a desire, craving, or need for food; feeling hunger. indicating, characteristic of, or characterized by hunger:

He approached the table with a hungry look.
strongly or eagerly desirous.
lacking needful or desirable elements; not fertile; poor:
hungry land.
marked by a scarcity of food:
The depression years were hungry times.

## Learning POS Tags

- Question
- Is one instance of example enough for ML?
- E.g. common example of "people"

People $\rightarrow$ Noun
,

- But it can be verb as well People $\rightarrow$ Verb (to populate)
- Answer
- We need at least as many instances as number of different labels \#POS tags-1 to make decision.


## Disambiguation of POS tag

- If no ambiguity, learn a table of words and its corresponding tags.
- If ambiguity, then look for the contextual information i.e. look-back or look-ahead.


## Data for "present"

He gifted me the/a/this/that present_NN.

They present_VB innovative ideas.

He was present_JJ in the class.

## Rules for disambiguating "present"

- For Present_NN (look-back)
- If present is preceded by determiner (the/a) or demonstrative (this/that), then POS tag will be noun.
- Does this rule guarantee $100 \%$ precision and $100 \%$ recall?
- False positive:
- The present_ADJ case is not convincing.

Adjective preceded by "the"

- False negative:
- Present foretells the future.

Noun but not preceded by "the"

## Rules for disambiguating "present"

- For Present_NN (look-back and look ahead)
- If present is preceded by determiner (the/a) or demonstrative (this/that) or followed by a verb, then POS tag will be noun.
- E.g.
- Present_NN will tell the future.
- Present_NN fortells the future.
- Does this rule guarantee $100 \%$ precision and $100 \%$ recall?


## Need for ML in POS tagging

- Rules are challenged by new data
- Need a robust system.
- Machine learning based POS tagging:
- HMM (Accuracy increased by 10-20\% against rule based systems)
- Jelinek's work inspired from ASR


## Noisy Channel Model



$$
\left(w_{n}, w_{n-1}, \ldots, w_{1}\right)
$$

$\left(t_{m}, t_{m-1}, \ldots, t_{1}\right)$

## Sequence $W$ is transformed into sequence $T$

$$
\begin{aligned}
& \mathrm{T}^{*}=\underset{\mathrm{T}}{\operatorname{argmax}}(\mathrm{P}(\mathrm{~T} \mid \mathrm{W})) \\
& \mathrm{W}^{*}=\underset{\mathrm{W}}{\operatorname{argmax}}(\mathrm{P}(\mathrm{~W} \mid \mathrm{T}))
\end{aligned}
$$

## Mathematics of POS tagging

## Argmax computation (1/2)

Best tag sequence
$=\mathrm{T}^{*}$
$=\operatorname{argmax} \mathrm{P}(\mathrm{T} \mid \mathrm{W})$
$=\operatorname{argmax} \mathrm{P}(\mathrm{T}) \mathrm{P}(\mathrm{W} \mid \mathrm{T})$
(by Baye's Theorem)
$P(T)=P\left(t_{0}=^{\wedge} t_{1} t_{2} \ldots t_{n+1}=.\right)$
$=P\left(t_{0}\right) P\left(t_{1} \mid t_{0}\right) P\left(t_{2} \mid t_{1} t_{0}\right) P\left(t_{3} \mid t_{2} t_{1} t_{0}\right) \ldots$ $P\left(t_{n} \mid t_{n-1} t_{n-2} \ldots t_{0}\right) P\left(t_{n+1} \mid t_{n} t_{n-1} \ldots t_{0}\right)$
$=P\left(t_{0}\right) P\left(t_{1} \mid t_{0}\right) P\left(t_{2} \mid t_{1}\right) \ldots P\left(t_{n} \mid t_{n-1}\right) P\left(t_{n+1} \mid t_{n}\right)$
$\mathrm{N}+1$
$\overline{\bar{i}=0} \prod_{0} P\left(\mathbf{t}_{\mathbf{i}} \mid \mathbf{t}_{\mathrm{i}-1}\right)$
Bigram Assumption

## Argmax computation (2/2)

$P(W \mid T)=P\left(w_{0} \mid t_{0}-t_{n+1}\right) P\left(w_{1} \mid w_{0} t_{0}-t_{n+1}\right) P\left(w_{2} \mid w_{1} w_{0} t_{0}-t_{n+1}\right) \ldots$

$$
P\left(w_{n} \mid w_{0}-w_{n-1} t_{0}-t_{n+1}\right) P\left(w_{n+1} \mid w_{0}-w_{n} t_{0}-t_{n+1}\right)
$$

Assumption: A word is determined completely by its tag. This is inspired by speech recognition

$$
\begin{aligned}
& =P\left(w_{0} \mid t_{0}\right) P\left(w_{1} \mid t_{1}\right) \ldots P\left(w_{n+1} \mid t_{n+1}\right) \\
& =\prod_{i=0}^{n+1} P\left(w_{i} \mid t_{i}\right) \\
& =\prod_{i=1}^{n+1} P\left(w_{i} \mid t_{i}\right) \quad \text { (Lexical Probability Assumption) }
\end{aligned}
$$

## Generative Model



This model is called Generative model.
Here words are observed from tags as states.
This is similar to HMM.

## Typical POS tag steps

- Implementation of Viterbi - Unigram,

Bigram.

- Five Fold Evaluation.
- Per POS Accuracy.
- Confusion Matrix.


## Screen shot of typical Confusion Matrix



# Computation of POS tags 

## DECODING




Probability of a path (e.g. Top most path) $=P(\mathrm{~T}){ }^{*} P(\mathrm{~W} \mid \mathrm{T})$ $P\left({ }^{\wedge}\right) \cdot P(N N / \wedge) . P(N N S / N N) . P(V B D / N N S) . P(N N / V B D)$. $P(D T / N N) . P(N N / D T) . P(. \mid N N) . P($.
$P(\wedge / \wedge) . P(b r o w n / N N) . P(f o x e s / N N S) . P(j u m p e d / V B D)$. P(over/NN) . P(the/DT) . P(fence/NN) . P(...)

## Questions?

-Where do tags come from?

- Tag set
- How to get probability values i.e. P(.)?
- Annotated corpora

After modeling of the problem, emphasis should be on the corpus

## Computing $\mathrm{P}($.$) values$

Let us suppose annotated corpus has the following sentence

| I | have | a | brown | bag |
| :---: | :---: | :---: | :---: | :---: |
| PRN | VB | DT | JJ | NN |

$$
P(N N \mid J J)=\frac{\text { Number_of_times_}_{-} J J_{-} \text {followed_by_} N N}{N u m b e r_{-} \text {of_times_JJ_appeared }}
$$

$$
P(B r o w n \mid J J)=\frac{\text { Number_of_times_Brown_tagged_as_JJ }_{\text {Number_of_times_JJ_appeared }}}{\text { Nut }}
$$

## Why Ratios?

- This way of computing parameter probabilities: is this correct?
- What does "correct" mean?
- Is this principled?
- We are using Maximum Likelihood Estimate (MLE)
- Assumption: underlying distribution is multinomial


## Explanation with coin tossing

- A coin is tossed 100 times, Head appears 40 times
- $P(H)=0.4$
- Why?
- Because of maximum likelihood


## $N$ tosses, K Heads, parameter $P(H)=p$

- Construct Maximum Likelihood Expression
- Take log likelihood and take derivative
- Equate to 0 and Get $p$

$$
\begin{aligned}
& L=p^{K}(1-p)^{N-K} \\
& \Rightarrow L L=\log (L)=K \log p+(N-K) \log (1-p) \\
& \Rightarrow \frac{d(L L)}{d p}=\frac{K}{p}-\frac{N-K}{1-p} \\
& \Rightarrow \frac{d(L L)}{d p}=0 \text { gives } p=\frac{K}{N}
\end{aligned}
$$

## Exercise

- Following the process for finding the probability of Head from N tosses of coin yielding K Heads, prove that the transition probabilities can be found from MLE
- Most important: get the likelihood expression
- Use chapter 2 of the book
- Pushpak Bhattacharyya: Machine translation, CRC Press, Taylor \& Francis Group, Boca Raton, USA, 2015, ISBN: 978-1-4398-9718-8


## Next question?

- How to decode efficiently?
- E.g.
- T: Tags
- W: Words
- Two special symbol: ‘^’ and '.'

Find out number of paths in the tree given word sequence.

Exponential w.r.t. number of wor
Number of path $=$ Number of leaves in the tree.

$$
O\left(T^{n}\right)
$$

## We do not need exponential work!

- Suppose our tags are - DT, NN, VB, JJ, RB and OT

| $\wedge$ | The | black | dog | barks | . |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\wedge$ | DT | DT | DT | DT |  |
|  | NN | NN | NN | NN |  |
|  | VB | VB | VB | VB |  |
|  | JJ | JJ | JJ | JJ |  |
|  | RB | RB | RB | RB |  |
|  | OT | OT | OT | OT |  |

DT- determiner<br>NN- Noun<br>VB- Verb<br>JJ- Adjective<br>RB- Adverb<br>OT- others<br>Possible tags

So, $6^{4}$ possible path

dog: $6^{3}$
barks: $6^{4}$
Total $6^{4}$ paths

- Now consider the paths that end in NN after seeina input "The black"

|  | The | black |  |
| :---: | :---: | :---: | :---: |
| $\wedge$ | DT | NN |  |
| $\wedge$ | NN | NN | $\boldsymbol{P}(\mathrm{T}) \cdot \boldsymbol{P}(\mathrm{W} \mid \mathrm{T})=P(\mathrm{NN} \mid \wedge) . P(\mathrm{NN} \mid \mathrm{NN}) . P(\mathrm{The} \mid \mathrm{NN})$. $P($ Black $\mid \mathrm{NN})$ |
| $\wedge$ | VB | NN | $\boldsymbol{P}(\mathrm{T}) \cdot \boldsymbol{P}(\mathrm{W} \mid \mathrm{T})=P\left(\left.\mathrm{VB}\right\|^{\wedge}\right) . P(\mathrm{NN} \mid \mathrm{VB}) . P(\mathrm{The} \mid \mathrm{VB})$. $P($ Black $\mid$ NN $)$ |
| $\wedge$ | JJ | NN | $\boldsymbol{P}(\mathrm{T}) \cdot \boldsymbol{P}(\mathrm{W} \mid \mathrm{T})=P\left(\mathrm{JJ} \mid{ }^{\wedge}\right) \cdot P(\mathrm{NN} \mid \mathrm{JJ}) \cdot P(\mathrm{The} \mid \mathrm{JJ})$ $P($ Black $\mid \mathrm{NN})$ |
| $\wedge$ | RB | NN | $\boldsymbol{P}(\mathrm{T}) \cdot \boldsymbol{P}(\mathrm{W} \mid \mathrm{T})=P\left(\left.\mathrm{RB}\right\|^{\wedge}\right) \cdot P(\mathrm{NN} \mid \mathrm{RB}) \cdot P(\mathrm{The} \mid \mathrm{RB})$ $P($ Black $\mid \mathrm{NN})$ |
| $\wedge$ | OT | NN | $\boldsymbol{P}(\mathrm{T}) \cdot \boldsymbol{P}(\mathrm{W} \mid \mathrm{T})=P\left(\left.\mathrm{OT}\right\|^{\wedge}\right) \cdot P(\mathrm{NN} \mid \mathrm{OT}) \cdot P($ The $\mid \mathrm{OT})$ $P$ (Black\|NN) |

Complexity $=W_{n} * T T_{\text {For each tag, only path } \text { with highest probability }}^{\text {valtained, others are simply discarded. }}$

## Machine Translation v/s POS

 tagging!- Similarity
- POS
- Every word in a sentence has one corresponding tag.
- MT
- Every word in a sentence has one (or more) corresponding translated word.
- Difference
- Order: Order of translated word may change.
- Fertility: One word corresponds to many. Many to one also possible.


## Complexity

- POS and HMM
- Linear time complexity
- MT and Bean search
- Exponential time complexity
- Permutation of words produces exponential searc space
- However, for related languages, MT is like POS tagging


## Properties of related languages

## 1. Order preserving

## 2. Fertility ~ 1

## 3. Morphology preserving

| Hindi | Jaaunga |
| :--- | :--- |
| Bengali | Jaabo |
| English | Will go |

Hindi \& Bengali $\uparrow$ Hindi \& English $\downarrow$

## Properties of related languages

4. Syncretism: Suffix features should be similarly loaded

| Hindi | Main jaaunga | Hum jaayenge |
| :--- | :--- | :--- |
| Bengali | Ami jaabo | Aamra jaabo |
|  |  |  |

5. Idiomaticity: Literal translation should be high

| Hindi | Aap Kaise Ho? |
| :--- | :--- |
| Bengali | Aapni Kemon Achen? |
| English | How do you do? |

Hindi \& Bengali $\uparrow$
Hindi \& English $\downarrow$


Cemeqoilifuglipak

## A Motivating Example

Colored Ball choosing


## Example (contd.)

Given :

|  | $U_{1}$ | $U_{2}$ | $U_{3}$ |
| :--- | :--- | :--- | :--- |
| $U_{1}$ | 0.1 | 0.4 | 0.5 |
| $U_{2}$ | 0.6 | 0.2 | 0.2 |
| $U_{3}$ | 0.3 | 0.4 | 0.3 |

Transition probability table

and |  | $R$ | $G$ | $B$ |
| :--- | :--- | :--- | :--- |
| $U_{1}$ | 0.3 | 0.5 | 0.2 |
| $U_{2}$ | 0.1 | 0.4 | 0.5 |
| $U_{3}$ | 0.6 | 0.1 | 0.3 |

Emission probability table

Observation : RRGGBRGR
State Sequence : ??

Not so Easily Computable.

## Diagrammatic representation (1/2)



## Diagrammatic representation (2/2)



## Classic problems with respect to HMM

1.Given the observation sequence, find the possible state sequences- Viterbi
2. Given the observation sequence, find its probability- forward/backward algorithm
3.Given the observation sequence find the HMM prameters.- Baum-Welch algorithm

## Illustration of Viterbi

- The "start" and "end" are important in a sequence.
- Subtrees get eliminated due to the Markov Assumption.

POS Tagset

- N (noun), V (verb), O(other) [simplified]
- ^ (start), . (end) [start $\&$ end states]

Illustration of Viterbi
Lexicon
people: N, V
laugh: N, V

Corpora for Training
$\qquad$ ${ }^{\wedge} W_{11-} t_{11} W_{12-} t_{12} W_{13-} t_{13}$ $\qquad$ $\mathrm{W}_{1 \mathrm{k} \_1-} \mathrm{t}_{1 \mathrm{k} \_1}$.
${ }^{\wedge} W_{21-} t_{21} W_{22-} t_{22} W_{23-} t_{23}$ $\qquad$ $W_{2 k \_2-} t_{2 k \_2}$.
${ }^{\wedge} W_{n 1-} t_{n 1} W_{n 2-} t_{n 2} W_{n 3-} t_{n 3}$ $\qquad$ . $W_{n k \_n-} t_{n k \_n}$.

## Inference



Partial sequence graph

|  | $\wedge$ | $N$ | $V$ | $O$ | . |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\wedge$ | 0 | 0.6 | 0.2 | 0.2 | 0 |
| $N$ | 0 | 0.1 | 0.4 | 0.3 | 0.2 |
| $V$ | 0 | 0.3 | 0.1 | 0.3 | 0.3 |
| 0 | 0 | 0.3 | 0.2 | 0.3 | 0.2 |
| . | 1 | 0 | 0 | 0 | 0 |

This transition table will change from language to language due to language divergences.

## Lexical Probability Table

|  | $\epsilon$ | people | laugh | $\ldots$ | $\ldots$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\wedge$ | 1 | 0 | 0 | $\ldots$ | 0 |
| $N$ | 0 | $1 \times 10^{-3}$ | $1 \times 10^{-5}$ | $\ldots$ | $\ldots$ |
| V | 0 | $1 \times 10^{-6}$ | $1 \times 10^{-3}$ | $\ldots$ | $\ldots$ |
| 0 | 0 | 0 | 0 | $\ldots$ | $\ldots$ |
| . | 1 | 0 | 0 | 0 | 0 |

Size of this table = \# pos tags in tagset $X$ vocabulary size
vocabulary size = \# unique words in corpus

## Inference

New Sentence:
people laugh .

$\mathrm{p}\left({ }^{\wedge} \mathrm{N} N .\left.\right|^{\wedge}\right.$ people laugh .)
$=(0.6 \times 0.1) \times\left(0.1 \times 1 \times 10^{-3}\right) \times\left(0.2 \times 1 \times 10^{-5}\right)$

## Computational Complexity

- If we have to get the probability of each sequence and then find maximum among them, we would run into exponential number of computations.
- If $|\mathrm{s}|=$ \#states (tags $+^{\wedge}+$. ) and $|\mathrm{o}|=$ length of sentence ( words $+^{\wedge}+$.) Then, \#sequences $=s^{|0|-2}$
- But, a large number of partial computations can be reused using Dynamic Programming.


## Dynamic Programming



## Computational Complexity

- Retain only those N / V / O nodes which ends in the highest sequence probability.
- Now, complexity reduces from $|s|^{|0|}$ to $|s| .|0|$
- Here, we followed the Markov assumption of order 1.


## Points to ponder wrt HMM and Viterbi

## Viterbi Algorithm

- Start with the start state.
- Keep advancing sequences that are "maximum" amongst all those ending in the same state


## Viterbi Algorithm

Tree for the sentence: "^ People laugh ."


Claim: We do not need to draw all the subtrees in the algorithm

## Viterbi phenomenon (Markov process)



LAUGH

Next step all the probabilities will be multiplied by identical probability (lexical and transition). So children of N2 will have probability less than the children of N 1 .

## What does $\mathrm{P}(\mathrm{A} \mid \mathrm{B})$ mean?

- $P(A \mid B)=P(B \mid A)$ If $P(A)=P(B)$
- $\mathrm{P}(\mathrm{A} \mid \mathrm{B})$ means??
- Causality?? B causes A??
- Sequentiality?? A follows B?


## Back to the Urn Example

- Here:

$$
\left.\begin{array}{l|l}
-S=\{U 1, U 2, U 3\} \\
-V=\{R, G, B\} \\
\text { - For observation: } \\
& -\mathrm{O}=\left\{\mathrm{o}_{1} \ldots \mathrm{o}_{n}\right\}
\end{array} \right\rvert\,
$$

- And State sequence

$$
-Q=\left\{q_{1} \cdots q_{n}\right\}
$$

- $\pi$ is $_{\pi_{i}=P\left(q_{1}=U_{i}\right)}$

|  | $U_{1}$ | $U_{2}$ | $U_{3}$ |
| :--- | :--- | :--- | :--- |
| $U_{1}$ | 0.1 | 0.4 | 0.5 |
| $U_{2}$ | 0.6 | 0.2 | 0.2 |
| $U_{3}$ | 0.3 | 0.4 | 0.3 |
|  | $R$ | $G$ | $B$ |
| $U_{1}$ | 0.3 | 0.5 | 0.2 |
| $U_{2}$ | 0.1 | 0.4 | 0.5 |
| $U_{3}$ | 0.6 | 0.1 | 0.3 |

## Observations and states

| $\mathrm{O}_{1}$ | $\mathrm{O}_{2}$ | $\mathrm{O}_{3}$ | $\mathrm{O}_{4}$ | $\mathrm{O}_{5}$ | $\mathrm{O}_{6}$ | $\mathrm{O}_{7}$ | $\mathrm{O}_{8}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| OBS: | R | R | G | G | B | R | G |
| R |  |  |  |  |  |  |  |
| State: $\mathrm{S}_{1} \mathrm{~S}_{2}$ | $\mathrm{~S}_{3}$ | $\mathrm{~S}_{4}$ | $\mathrm{~S}_{5}$ | $\mathrm{~S}_{6}$ | $\mathrm{~S}_{7}$ | $\mathrm{~S}_{8}$ |  |

$\mathrm{S}_{\mathrm{i}}=\mathrm{U}_{1} / \mathrm{U}_{2} / \mathrm{U}_{3}$; A particular state
S : State sequence
O: Observation sequence
S* = "best" possible state (urn) sequence
Goal: Maximize $\mathrm{P}\left(\mathrm{S}^{*} \mid \mathrm{O}\right)$ by choosing "best" S

## Goal

- Maximize $\mathrm{P}(\mathrm{S} \mid \mathrm{O})$ where S is the State Sequence and O is the Observation Sequence

$$
S^{*}=\arg \max _{S}(P(S \mid O))
$$

## False Start

|  | $\mathrm{O}_{1}$ | $\mathrm{O}_{2}$ | $\mathrm{O}_{3}$ | $\mathrm{O}_{4}$ | $\mathrm{O}_{5}$ | $\mathrm{O}_{6}$ | $\mathrm{O}_{7}$ | $\mathrm{O}_{8}$ |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| OBS: | R | R | G | G | B | R | G | R |
| State: | $\mathrm{S}_{1}$ | $\mathrm{~S}_{2}$ | $\mathrm{~S}_{3}$ | $\mathrm{~S}_{4}$ | $\mathrm{~S}_{5}$ | $\mathrm{~S}_{6}$ | $\mathrm{~S}_{7}$ | $\mathrm{~S}_{8}$ |
| $P(S \mid O)$ | $=P\left(S_{1-8} \mid O_{1-8}\right)$ |  |  |  |  |  |  |  |
| $P(S \mid O)=$ | $P\left(S_{1} \mid O\right) . P\left(S_{2} \mid S_{1}, O\right) . P\left(S_{3} \mid S_{1-2}, O\right) \ldots P\left(S_{8} \mid S_{1-7}, O\right)$ |  |  |  |  |  |  |  |

By Markov Assumption (a state depends only on the previous state)

$$
P(S \mid O)=P\left(S_{1} \mid O\right) \cdot P\left(S_{2} \mid S_{1}, O\right) \cdot P\left(S_{3} \mid S_{2}, O\right) \ldots P\left(S_{8} \mid S_{7}, O\right)
$$

## Baye's Theorem

$$
P(A \mid B)=P(A) \cdot P(B \mid A) / P(B)
$$

$\mathrm{P}(\mathrm{A})$-: Prior
$P(B \mid A)$-: Likelihood

$$
{\arg \max _{S} P(S \mid O)=\arg \max _{S} P(S) \cdot P(O \mid S), ~}_{\text {S }}
$$

## State Transitions Probability

$$
\begin{aligned}
& P(S)=P\left(S_{1-8}\right) \\
& P(S)=P\left(S_{1}\right) \cdot P\left(S_{2} \mid S_{1}\right) \cdot P\left(S_{3} \mid S_{1-2}\right) \cdot P\left(S_{4} \mid S_{1-3}\right) \ldots P\left(S_{8} \mid S_{1-7}\right)
\end{aligned}
$$

## By Markov Assumption (k=1)

$$
P(S)=P\left(S_{1}\right) \cdot P\left(S_{2} \mid S_{1}\right) \cdot P\left(S_{3} \mid S_{2}\right) \cdot P\left(S_{4} \mid S_{3}\right) \ldots P\left(S_{8} \mid S_{7}\right)
$$

## Observation Sequence probability

$P(O \mid S)=P\left(O_{1} \mid S_{1-8}\right) \cdot P\left(O_{2} \mid O_{1}, S_{1-8}\right) \cdot P\left(O_{3} \mid O_{1-2}, S_{1-8}\right) \ldots P\left(O_{8} \mid O_{1-7, S} S_{1-8}\right)$
Assumption that ball drawn depends only on the Urn chosen
$P(O \mid S)=P\left(O_{1} \mid S_{1}\right) \cdot P\left(O_{2} \mid S_{2}\right) \cdot P\left(O_{3} \mid S_{3}\right) \ldots P\left(O_{8} \mid S_{8}\right)$
$P(S \mid O)=P(S) \cdot P(O \mid S)$
$P(S \mid O)=P\left(S_{1}\right) \cdot P\left(S_{2} \mid S_{1}\right) \cdot P\left(S_{3} \mid S_{2}\right) \cdot P\left(S_{4} \mid S_{3}\right) \ldots P\left(S_{8} \mid S_{7}\right)$.
$P\left(O_{1} \mid S_{1}\right) . P\left(O_{2} \mid S_{2}\right) . P\left(O_{3} \mid S_{3}\right) \ldots P\left(O_{8} \mid S_{8}\right)$

## Grouping terms

| $\mathrm{O}_{0} \quad \mathrm{O}_{1}$ | $\mathrm{O}_{1} \quad \mathrm{O}_{2}$ | $\mathrm{O}_{3}$ | $\mathrm{O}_{4}$ | $\mathrm{O}_{5}$ | $\mathrm{O}_{6}$ | $\mathrm{O}_{7}$ | $\mathrm{O}_{8}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Obs: $\varepsilon$ R | R R | G | G | B | R | G | R |
| State: $\mathrm{S}_{0} \mathrm{~S}_{1}$ | $\mathrm{S}_{1} \quad \mathrm{~S}_{2}$ | $\mathrm{S}_{3}$ | $\mathrm{S}_{4}$ | $\mathrm{S}_{5}$ | $\mathrm{S}_{6}$ | $\mathrm{S}_{7}$ | $\mathrm{S}_{8}$ |
| $\mathrm{P}(\mathrm{S}) . \mathrm{P}(\mathrm{O} \mid \mathrm{S})$ |  |  |  | We introduce the states |  |  |  |
| $=\left[P\left(O_{0} \mid S_{0}\right) \cdot \mathrm{P}\left(\mathrm{S}_{1} \mid \mathrm{S}_{0}\right)\right]$. |  |  |  | $\mathrm{S}_{0}$ and $\mathrm{S}_{9}$ as initial |  |  |  |
| [ $\mathrm{P}\left(\mathrm{O}_{1} \mid \mathrm{S}_{1}\right) . \mathrm{P}$ ) $\left.\mathrm{P}\left(\mathrm{S}_{2} \mid \mathrm{S}_{1}\right)\right]$. |  |  |  | and final states |  |  |  |
| $\left.{ }^{2}\left(\mathrm{O}_{2} \mid \mathrm{S}_{2}\right) . \quad \mathrm{P}\left(\mathrm{S}_{3} \mid \mathrm{S}_{2}\right)\right]$. |  |  |  | respectively. |  |  |  |
| $\left[P\left(O_{3} \mid S_{3}\right) \cdot \mathrm{P}\left(\mathrm{S}_{4} \mid \mathrm{S}_{3}\right)\right]$. |  |  |  | After $\mathrm{S}_{8}$ the next state is |  |  |  |
| $\left[P\left(O_{4} \mid S_{4}\right) \cdot \mathrm{P}\left(\mathrm{S}_{5} \mid \mathrm{S}_{4}\right)\right]$ ]. |  |  |  | $\mathrm{S}_{9}$ with probability 1 , |  |  |  |
| $\left.{ }^{[P( }\left(\mathrm{O}_{6} \mid \mathrm{S}_{6}\right) \cdot \mathrm{P}\left(\mathrm{S}_{7} \mid \mathrm{S}_{6}\right)\right]$. |  |  |  | i.e., $\mathrm{P}\left(\mathrm{S}_{9} \mid \mathrm{S}_{8}\right)=1$ |  |  |  |
| $\left[\mathrm{P}\left(\mathrm{O}_{7} \mid \mathrm{S}_{7}\right) \cdot \mathrm{P}\left(\mathrm{S}_{8} \mid \mathrm{S}_{7}\right)\right]$. |  |  |  | $\mathrm{O}_{0}$ is $\varepsilon$-transition |  |  |  |
|  | ${ }^{[P}\left(\mathrm{O}_{8} \mid \mathrm{S}_{8}\right)$. P | $\left.\left.\mathrm{S}_{9} \mid \mathrm{S}_{8}\right)\right]$ |  |  |  |  |  |

## Introducing useful notation

| $\mathrm{O}_{0}$ | $\mathrm{O}_{1}$ | $\mathrm{O}_{2}$ | $\mathrm{O}_{3}$ | $\mathrm{O}_{4}$ | $\mathrm{O}_{5}$ | $\mathrm{O}_{6}$ | $\mathrm{O}_{7}$ | $\mathrm{O}_{8}$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| : $\mathrm{\varepsilon}$ | R | R | G | G | B | R | G | R |  |
| $\mathrm{S}_{0}$ | $\mathrm{~S}_{1}$ | $\mathrm{~S}_{2}$ | $\mathrm{~S}_{3}$ | $\mathrm{~S}_{4}$ | $\mathrm{~S}_{5}$ | $\mathrm{~S}_{6}$ | $\mathrm{~S}_{7}$ | $\mathrm{~S}_{8}$ | $\mathrm{~S}_{9}$ |



$$
P\left(O_{k} / S_{k}\right) \cdot P\left(S_{k+1} / S_{k}\right)=P\left(S_{k} \xrightarrow{O_{k}} S_{k+1}\right)
$$

## Probabilistic FSM



The question here is:
"what is the most likely state sequence given the output sequence seen"


## Developing the tree



## Tree structure contd...



The problem being addressed by this tree is $S^{*}=\arg \max P\left(S \mid a_{1}-a_{2}-a_{1}-a_{2, \mu}\right)$ $\mathrm{a} 1-\mathrm{a} 2-\mathrm{a} 1-\mathrm{a} 2$ is the output sequence and $\mu$ the model or the machine

Path found:
(working backward)


Problem statement: Find the best possible sequence

$$
S^{*}=\arg \max P(S \mid O, \mu)
$$

where, $S \rightarrow$ State Seq, $O \rightarrow$ Output Seq, $\mu \rightarrow$ Model or Machine


T is defined as $P\left(S_{i} \xrightarrow{a_{k}} S_{j}\right) \quad \forall_{i, j, k}$

## Tabular representation of the tree

|  | $€$ | $\mathrm{a}_{1}$ | $\mathrm{a}_{2}$ | $\mathrm{a}_{1}$ | $\mathrm{a}_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{S}_{1}$ | 1.0 | $\begin{gathered} (1.0 * 0.1,0.0 * 0.2) \\ =(0.1,0.0) \end{gathered}$ | $\begin{gathered} \hline(0.02, \\ 0.09) \end{gathered}$ | (0.009, 0.012) | $\begin{gathered} (0.0024, \\ \mathbf{0 . 0 0 8 1 )} \end{gathered}$ |
| $S_{2}$ | 0.0 | $\begin{aligned} & (1.0 * 0.3,0.0 * 0.3) \\ & \quad=(0.3,0.0) \end{aligned}$ | (0.04,0.06 <br> ) | (0.027,0.018) | $\begin{gathered} (0.0048,0.005 \\ 4) \end{gathered}$ |

Note: Every cell records the winning probability ending in that state
Final winner
The bold faced values in each cell shows the sequence probability ending in that state. Going backward from final winner sequence which ends in state $S_{2}$ (indicated By the $2^{\text {nd }}$ tuple), we recover the sequence.

## Algorithm

(following James Alan, Natural Language Understanding (2 ${ }^{\text {nd }}$ edition), Benjamin Cummins (pub.), 1995

## Given:

1. The HMM, which means:
a. Start State: $\mathrm{S}_{1}$
b. Alphabet: $A=\left\{a_{1}, a_{2}, \ldots a_{p}\right\}$
c. Set of States: $S=\left\{S_{1}, S_{2}, \ldots S_{n}\right\}$
d. Transition probability which is equal to

$$
P\left(S_{i} \xrightarrow{a_{k}} S_{j}\right) \quad \forall_{i, j, k}
$$

2. The output string $a_{1} a_{2} \ldots a_{T}$

$$
P\left(S_{j}, a_{k} \mid S_{i}\right)
$$

To find:
The most likely sequence of states $\mathrm{C}_{1} \mathrm{C}_{2} \ldots \mathrm{C}_{\mathrm{T}}$ which produces the given output sequence, i.e., $\mathrm{C}_{1} \mathrm{C}_{2} \ldots \mathrm{C}_{\mathrm{T}}=\quad \underset{C}{\arg \max \left[P\left(C \mid a_{1}, a_{2}, \ldots a r, \mu\right]\right.}$

```
\(\operatorname{SEQSCORE}(1,1)=1.0\)
BACKPTR \((1,1)=0\)
For(i=2 to N) do
    SEQSCORE (i,1)=0.0
[expressing the fact that first state is \(\mathrm{S}_{1}\) ]
```


## 2. Iteration

```
For( \(\mathrm{t}=2\) to T ) do
    For( \(\mathrm{i}=1\) to N ) do
        \(\operatorname{SEQSCORE}(\mathrm{i}, \mathrm{t})=\operatorname{Max}_{(\mathrm{j}=1, \mathrm{~N})}\)
```

            \(\left[\operatorname{SEQSCORE}(j,(t-1)) * P\left(S j \xrightarrow{a_{k}} S i\right)\right]\)
    \(\operatorname{BACKPTR}(I, \mathrm{t})=\) index \(j\) that gives the MAX above
    
## 3. Seq. Identification

$\mathrm{C}(\mathrm{T})=\mathrm{i}$ that maximizes $\operatorname{SEQSCORE}(\mathrm{i}, \mathrm{T})$
For ifrom (T-1) to 1 do

$$
\mathrm{C}(\mathrm{i})=\mathrm{BACKPTR}[\mathrm{C}(\mathrm{i}+1),(\mathrm{i}+1)]
$$

## Optimizations possible:

1. BACKPTR can be $1^{*} \top$
2. SEQSCORE can be T*2

Homework:- Compare this with A*, Beam Search [Homework]
Reason for this comparison:
Both of them work for finding and recovering sequence

## Reading List

- https://www.nltk.org/book/ch05.html
- TnT (http://www.aclweb.org/anthology-new/A/A00/A001031.pdf)
- Hindi POS Tagger built by IIT Bombay (http://www.cse.iitb.ac.in/pb/papers/ACL-2006-Hindi-POS-Tagging.pdf)


## Assignment Discussion

## Build a POS Tagger (due date: $5^{\text {th }}$ September, 2020)

- Using
- HMM
-SVM
- Bi-LSTM
- Training corpora (Brown Copus)
- http://www.nltk.org/nltk data/

Project Discussion

## Different Areas of NLP

## Questions from Mohith

1. You mentioned about "ellipsis" during the lecture. In NLP, does "ellipsis" refer to the computational problem that arises due to
skipping a few words in a sentence, or the whole act of skipping words is itself termed as ellipsis?

- Ans: whole act; "where do you live?"- "Delhi" and "your friend? ${ }_{\text {ellipsis }}$ "- "Mumbai"

2. You mentioned about shallow/deep parsing. Was this in the context of dependency/constituency parsing? That is, can I say that dependency parser does shallow parsing and constituency parser does deep parsing, or some other similar relations exist between them?

-     - both constituency and dependency are "deep" parsing tasks; pos tagging, chunking [(the_DT blue_JJ sky_NN) chunk was (vast_JJ and_CONJ deep_JJ) chunk]


## Questions from Mohith

3. In the Learning POS Tags slide, it is mentioned, "We need at least as many instances as number of different labels \#POS tags-1 to make decision". That means, corresponding to every tag(except one) we are giving one example to the learning algorithm. If the algorithm encounters a previously unseen example, give it the last tag. I just wanted to know if my understanding is correct here.
-Ans: Correct; give the remaining tag if none of the tags from the training data is applicable

## Mohith

Besides my doubts, I have also solved the first homework question that you had provided in the class. Could you please let me know if my answers are correct?

- Question 1: Example of Verbal, Answer: "Could you please google this topic?". Here "google" is a noun, but in this context it is being used as a verb. Therefore "google" would be tagged as a "Verbal"
-Ans: No, dictionary definition of "google" includes verb also
I haven't got an example for the second question of finding false positive/negative for rules of "present"
-Ans: The rule is "If present is preceded by determiner (the/a) or demonstrative (this/that) or followed by a verb, then POS tag will be noun." Still fails for "The present situation is comfortable"

