## CS626: Speech, NLP and the Web

Penn TAG Set, HMM and Viterbi Decoding, Other Graphical Models for NLP, SVM Pushpak Bhattacharyya
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## Task vs. Technique Matrix

| Task (row) vs. Technique (col) Matrix | Rules Based/Kn owledgeBased | Classical ML |  |  |  | Deep Learning |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Perceptron | Logistic Regression | SVM | Graphical Models (HMM, MEMM, CRF) | Dense FF with BP and softmax | $\begin{aligned} & \text { RNN- } \\ & \text { LSTM } \end{aligned}$ | CNN |
| Morphology |  |  |  |  |  |  |  |  |
| POS |  |  |  |  |  |  |  |  |
| Chunking |  |  |  |  |  |  |  |  |
| Parsing |  |  |  |  |  |  |  |  |
| NER, MWE |  |  |  |  |  |  |  |  |
| Coref |  |  |  |  |  |  |  |  |
| WSD |  |  |  |  |  |  |  |  |
| Machine Translation |  |  |  |  |  |  |  |  |
| Semantic Role Labeling |  |  |  |  |  |  |  |  |
| Sentiment |  |  |  |  |  |  |  |  |
| Question Answering |  |  |  |  |  |  |  |  |

## Part of Speech Tagging

- Attach to each word a tag from Tag-Set
- Standard Tag-set : Penn Treebank (for English).


## Penn POS TAG Set

2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10. 
11. 
12. 
13. 
14. 
15. 
16. 
17. 
18. 
19. 
20. 
21. 

CC
CD
DT
EX
FW

IN

JJ
JJR
JJS
LS
MD
NN
NNS
NNP
NNPS
PDT
POS
PRP
PRP\$
RB
RBR

Coordinating conjunction
Cardinal number
Determiner
Existential there
Foreign word
Preposition or subordinating conjun

Adjective
Adjective, comparative
Adjective, superlative
List item marker
Modal
Noun, singular or mass
Noun, plural
Proper noun, singular
Proper noun, plural
Predeterminer
Possessive ending
Personal pronoun
Possessive pronoun
Adverb
Adverb, comparative

## Penn POS TAG Set (cntd)

| 22. | RBS |
| :--- | :--- |
| 23. | RP |
| 24. | SYM |
| 25. | TO |
| 26. | UH |
| 27. | VB |
| 28. | VBD |
| 29. | VBG |
| 30. | VBP |
| 31. | VBZ |
| 32. | WDT |
| 33. | WP |
| 34. | WP\$ |
| 35. | WRB |
| 36. |  |

Adverb, superlative
Particle
Symbol
to
Interjection
Verb, base form
Verb, past tense
Verb, gerund or present participle

Verb, past participle
Verb, non-3rd person singular present

Verb, 3rd person singular present

Wh-determiner
Wh-pronoun
Possessive wh-pronoun
Wh-adverb

# A dialogue text POS tagged from Treebank 

[ SpeakerB1/SYM ]
[ SpeakerA2/SYM ] ./.
So/UH how/WRB many/JJ ,/, um/UH ,/,
[ credit/NN cards/NNS ] do/VBP
[ you/PRP ] have/VB ?/.
./.
[ Um/UH ]
,/,
[ I/PRP ]
think/VBP
[ I/PRP ]
'm/VBP down/IN to/IN
[ one/CD ]
https://catalog.Idc.upenn.edu/debc/addenda/LDC99T42 .pos.txt

Redirafapg8ดpak

## Mathematics of POS tagging

## Noisy Channel Model



$$
\left(w_{n}, w_{n-1}, \ldots, w_{1}\right)
$$

$\left(t_{m}, t_{m-1}, \ldots, t_{1}\right)$

## Sequence $W$ is transformed into sequence $T$

$$
\begin{aligned}
& \mathrm{T}^{*}=\underset{\mathrm{T}}{\operatorname{argmax}}(\mathrm{P}(\mathrm{~T} \mid \mathrm{W})) \\
& \mathrm{W}^{*}=\underset{\mathrm{W}}{\operatorname{argmax}}(\mathrm{P}(\mathrm{~W} \mid \mathrm{T}))
\end{aligned}
$$

## Argmax computation (1/2)

Best tag sequence
$=\mathrm{T}^{*}$
$=\operatorname{argmax} \mathrm{P}(\mathrm{T} \mid \mathrm{W})$
$=\operatorname{argmax} \mathrm{P}(\mathrm{T}) \mathrm{P}(\mathrm{W} \mid \mathrm{T})$
(by Baye's Theorem)
$P(T)=P\left(t_{0}=\wedge t_{1} t_{2} \ldots t_{n+1}=.\right)$
$=P\left(t_{0}\right) P\left(t_{1} \mid t_{0}\right) P\left(t_{2} \mid t_{1} t_{0}\right) P\left(t_{3} \mid t_{2} t_{1} t_{0}\right) \ldots$ $P\left(t_{n} \mid t_{n-1} t_{n-2} \ldots t_{0}\right) P\left(t_{n+1} \mid t_{n} t_{n-1} \ldots t_{0}\right)$
$=P\left(t_{0}\right) P\left(t_{1} \mid t_{0}\right) P\left(t_{2} \mid t_{1}\right) \ldots P\left(t_{n} \mid t_{n-1}\right) P\left(t_{n+1} \mid t_{n}\right)$
$\mathrm{N}+1$
$\overline{\bar{i}=0} \prod_{0}\left(\mathbf{t}_{\mathbf{i}} \mid \mathbf{t}_{\mathrm{i}-1}\right)$
Bigram Assumption

## Argmax computation (2/2)

$P(W \mid T)=P\left(w_{0} \mid t_{0}-t_{n+1}\right) P\left(w_{1} \mid w_{0} t_{0}-t_{n+1}\right) P\left(w_{2} \mid w_{1} w_{0} t_{0}-t_{n+1}\right) \ldots$

$$
P\left(w_{n} \mid w_{0}-w_{n-1} t_{0}-t_{n+1}\right) P\left(w_{n+1} \mid w_{0}-w_{n} t_{0}-t_{n+1}\right)
$$

Assumption: A word is determined completely by its tag. This is inspired by speech recognition

$$
\begin{aligned}
& =P\left(w_{0} \mid t_{0}\right) P\left(w_{1} \mid t_{1}\right) \ldots P\left(w_{n+1} \mid t_{n+1}\right) \\
& =\prod_{i=0}^{n+1} P\left(w_{i} \mid t_{i}\right) \\
& =\prod_{i=1}^{n+1} P\left(w_{i} \mid t_{i}\right) \quad \text { (Lexical Probability Assumption) }
\end{aligned}
$$

## Generative Model



This model is called Generative model.
Here words are observed from tags as states.
This is similar to HMM.


Lawrence R. Rabiner:a tutorial on hidden Markov models and selected applications in speech recognition. Proceedings of the IEEE, 1989, pages 257-286
https://web.ece.ucsb.edu/Faculty/Rabiner/ece259/Re prints/tutorial\%20on\%20hmm\%20and\%20application s.pdf

## Definition of HMM and URN example

- An HMM is defined by
<S, V, A, B, ா>

Here:

$$
-\mathrm{S}=\{\mathrm{U} 1, \mathrm{U} 2, \mathrm{U} 3\}
$$

$$
-V=\{R, G, B\}
$$

For observation sequence:

$$
\mathrm{O}=\left[\mathrm{O}_{1} \ldots \mathrm{O}_{n}\right]
$$

and State sequence

$$
Q=\left[S_{1} \ldots S_{n}\right]
$$

## URN Example

Colored Ball choosing


Urn 3 \# of Red $=60$ \# of Green =10 $\#$ of Blue $=30$

# Viterbi Decoding to find state sequence 

- Observation : RRGGBRGR
- Find best possible state sequence


## Noting probabilities again

$A=$|  | $\mathrm{U}_{1}$ | $\mathrm{U}_{2}$ | $\mathrm{U}_{3}$ |
| :--- | :--- | :--- | :--- |
| $\mathrm{U}_{1}$ | 0.1 | 0.4 | 0.5 |
| $\mathrm{U}_{2}$ | 0.6 | 0.2 | 0.2 |
| $\mathrm{U}_{3}$ | 0.3 | 0.4 | 0.3 |


$B=$|  | $R$ | $G$ | $B$ |
| :--- | :--- | :--- | :--- |
| $U_{1}$ | 0.3 | 0.5 | 0.2 |
| $U_{2}$ | 0.1 | 0.4 | 0.5 |
| $U_{3}$ | 0.6 | 0.1 | 0.3 |

$$
\pi_{i}=P\left(q_{1}=U_{i}\right)
$$

## Diagrammatic representation (1/2)



## Diagrammatic representation (2/2)



## Observations and states

$$
\begin{array}{rlllllll}
O_{1} & O_{2} & O_{3} & O_{4} & O_{5} & O_{6} & O_{7} & O_{8}^{8} \\
\text { OBS: } R & R & G & G & B & R & G & R \\
& & & & & & & \\
\text { State: } S_{1} S_{2} & S_{3} & S_{4} & S_{5} & S_{6} & S_{7} & S_{8}
\end{array}
$$

S* = "best" possible state (urn) sequence
Goal: Maximize $\mathrm{P}\left(\mathrm{S}^{*} \mid \mathrm{O}\right)$ by choosing "best" S

## Goal

- Maximize $\mathrm{P}(\mathrm{S} \mid \mathrm{O})$ where S is the State Sequence and O is the Observation Sequence

$$
S^{*}=\arg \max _{S}(P(S \mid O))
$$

## False Start

|  | $\mathrm{O}_{1}$ | $\mathrm{O}_{2}$ | $\mathrm{O}_{3}$ | $\mathrm{O}_{4}$ | $\mathrm{O}_{5}$ | $\mathrm{O}_{6}$ | $\mathrm{O}_{7}$ | $\mathrm{O}_{8}$ |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| OBS: | R | R | G | G | B | R | G | R |
| State: | $\mathrm{S}_{1}$ | $\mathrm{~S}_{2}$ | $\mathrm{~S}_{3}$ | $\mathrm{~S}_{4}$ | $\mathrm{~S}_{5}$ | $\mathrm{~S}_{6}$ | $\mathrm{~S}_{7}$ | $\mathrm{~S}_{8}$ |
| $P(S \mid O)=$ | $P\left(S_{1-8} \mid O_{1-8}\right)$ |  |  |  |  |  |  |  |
| $P(S \mid O)=$ | $P\left(S_{1} \mid O\right) . P\left(S_{2} \mid S_{1}, O\right) . P\left(S_{3} \mid S_{1-2, O}, \ldots\right) . . P\left(S_{8} \mid S_{1-7}, O\right)$ |  |  |  |  |  |  |  |

By Markov Assumption (a state depends only on the previous state)

$$
P(S \mid O)=P\left(S_{1} \mid O\right) \cdot P\left(S_{2} \mid S_{1}, O\right) \cdot P\left(S_{3} \mid S_{2}, O\right) \ldots P\left(S_{8} \mid S_{7}, O\right)
$$

## Bayes Theorem

$$
P(A \mid B)=P(A) \cdot P(B \mid A) / P(B)
$$

$\mathrm{P}(\mathrm{A})$-: Prior
$P(B \mid A)$-: Likelihood

$$
{\arg \max _{S} P(S \mid O)=\arg \max _{S} P(S) \cdot P(O \mid S), ~}_{\text {S }}
$$

## State Transitions Probability

$$
\begin{aligned}
& P(S)=P\left(S_{1-8}\right) \\
& P(S)=P\left(S_{1}\right) \cdot P\left(S_{2} \mid S_{1}\right) \cdot P\left(S_{3} \mid S_{1-2}\right) \cdot P\left(S_{4} \mid S_{1-3}\right) \ldots P\left(S_{8} \mid S_{1-7}\right)
\end{aligned}
$$

## By Markov Assumption (k=1)

$$
P(S)=P\left(S_{1}\right) \cdot P\left(S_{2} \mid S_{1}\right) \cdot P\left(S_{3} \mid S_{2}\right) \cdot P\left(S_{4} \mid S_{3}\right) \ldots P\left(S_{8} \mid S_{7}\right)
$$

## Observation Sequence probability

$P(O \mid S)=P\left(O_{1} \mid S_{1-8}\right) \cdot P\left(O_{2} \mid O_{1}, S_{1-8}\right) \cdot P\left(O_{3} \mid O_{1-2}, S_{1-8}\right) . . . P\left(O_{8} \mid O_{1-7, S} S_{1-8}\right)$
Assumption that ball drawn depends only on the Urn chosen
$P(O \mid S)=P\left(O_{1} \mid S_{1}\right) \cdot P\left(O_{2} \mid S_{2}\right) \cdot P\left(O_{3} \mid S_{3}\right) \ldots P\left(O_{8} \mid S_{8}\right)$
$P(S \mid O)=P(S) \cdot P(O \mid S)$
$P(S \mid O)=P\left(S_{1}\right) \cdot P\left(S_{2} \mid S_{1}\right) \cdot P\left(S_{3} \mid S_{2}\right) \cdot P\left(S_{4} \mid S_{3}\right) \ldots P\left(S_{8} \mid S_{7}\right)$.
$P\left(O_{1} \mid S_{1}\right) . P\left(O_{2} \mid S_{2}\right) . P\left(O_{3} \mid S_{3}\right) \ldots P\left(O_{8} \mid S_{8}\right)$

## Grouping terms

| $\mathrm{O}_{0}$ | $\mathrm{O}_{1}$ | $\mathrm{O}_{2}$ | $\mathrm{O}_{3}$ | $\mathrm{O}_{4}$ | $\mathrm{O}_{5}$ | $\mathrm{O}_{6}$ | $\mathrm{O}_{7}$ | $\mathrm{O}_{8}$ |  |
| ---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Obs: $\varepsilon$ | R | R | G | G | B | R | G | R |  |
| State: $\mathrm{S}_{0}$ | $\mathrm{~S}_{1}$ | $\mathrm{~S}_{2}$ | $\mathrm{~S}_{3}$ | $\mathrm{~S}_{4}$ | $\mathrm{~S}_{5}$ | $\mathrm{~S}_{6}$ | $\mathrm{~S}_{7}$ | $\mathrm{~S}_{8}$ | $\mathrm{~S}_{9}$ |

$$
\begin{aligned}
& \mathrm{P}(\mathrm{~S}) \cdot \mathrm{P}\left(\mathrm{O}^{\mid S}\right) \\
&= {\left[\mathrm{P}\left(\mathrm{O}_{0} \mid \mathrm{S}_{0}\right) \cdot \mathrm{P}\left(\mathrm{~S}_{1} \mid \mathrm{S}_{0}\right)\right] . } \\
& {\left[\mathrm{P}\left(\mathrm{O}_{1} \mid \mathrm{S}_{1}\right) \cdot\right.} \\
&\left.\mathrm{P}\left(\mathrm{~S}_{2} \mid \mathrm{S}_{1}\right)\right] . \\
& {\left[\mathrm{P}\left(\mathrm{O}_{2} \mid \mathrm{S}_{2}\right) \cdot \quad \mathrm{P}\left(\mathrm{~S}_{3} \mid \mathrm{S}_{2}\right)\right] . } \\
& {\left[\mathrm{P}\left(\mathrm{O}_{3} \mid \mathrm{S}_{3}\right) \cdot \mathrm{P}\left(\mathrm{~S}_{4} \mid \mathrm{S}_{3}\right)\right] . } \\
& {\left[\mathrm{P}\left(\mathrm{O}_{4} \mathrm{~S}_{4}\right) \cdot \mathrm{P}\left(\mathrm{~S}_{5} \mathrm{~S}_{4}\right)\right] . } \\
& {\left[\mathrm{P}\left(\mathrm{O}_{5} \mid \mathrm{S}_{5}\right) \cdot \mathrm{P}\left(\mathrm{~S}_{6} \mid \mathrm{S}_{5}\right)\right] . } \\
& {\left[\mathrm{P}\left(\mathrm{O}_{6} \mid \mathrm{S}_{6}\right) \cdot \mathrm{P}\left(\mathrm{~S}_{7} \mathrm{~S}_{6}\right)\right] . } \\
& {\left[\mathrm{P}\left(\mathrm{O}_{7} \mid \mathrm{S}_{7}\right) \cdot \mathrm{P}\left(\mathrm{~S}_{8} \mid \mathrm{S}_{7}\right)\right] . } \\
& {\left[\mathrm{P}\left(\mathrm{O}_{8} \mid \mathrm{S}_{8}\right) \cdot \mathrm{P}\left(\mathrm{~S}_{9} \mid \mathrm{S}_{8}\right)\right] . }
\end{aligned}
$$

We introduce the states $\mathrm{S}_{0}$ and $\mathrm{S}_{9}$ as initial and final states respectively.
After $\mathrm{S}_{8}$ the next state is $\mathrm{S}_{9}$ with probability 1 , i.e., $P\left(S_{9} \mid S_{8}\right)=1$
$\mathrm{O}_{0}$ is $\varepsilon$-transition

## Introducing useful notation

| $\mathrm{O}_{0}$ | $\mathrm{O}_{1}$ | $\mathrm{O}_{2}$ | $\mathrm{O}_{3}$ | $\mathrm{O}_{4}$ | $\mathrm{O}_{5}$ | $\mathrm{O}_{6}$ | $\mathrm{O}_{7}$ | $\mathrm{O}_{8}$ |  |
| ---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Obs: $\varepsilon$ | R | R | G | G | B | R | G | R |  |
| State: $\mathrm{S}_{0}$ | $\mathrm{~S}_{1}$ | $\mathrm{~S}_{2}$ | $\mathrm{~S}_{3}$ | $\mathrm{~S}_{4}$ | $\mathrm{~S}_{5}$ | $\mathrm{~S}_{6}$ | $\mathrm{~S}_{7}$ | $\mathrm{~S}_{8}$ | $\mathrm{~S}_{9}$ |



Recall



Probability of a path (e.g. Top most path) $=P(\mathrm{~T}){ }^{*} P(\mathrm{~W} \mid \mathrm{T})$ $P\left({ }^{\wedge}\right) \cdot P(N N / \wedge) . P(N N S / N N) . P(V B D / N N S) . P(N N / V B D)$. $P(D T / N N) . P(N N / D T) . P(. \mid N N) . P($.
$P(\wedge / \wedge) . P(b r o w n / N N) . P(f o x e s / N N S) . P(j u m p e d / V B D)$. P(over/NN) . P(the/DT) . P(fence/NN) . P(...)

## Viterbi Decoding

## Probabilistic FSM



The question here is:
"what is the most likely state sequence given the output sequence seen"

## Developing the tree



## Tree structure contd...



The problem being addressed by this tree is $S^{*}=\arg \max P\left(S \mid a_{1}-a_{2}-a_{1}-a_{2, \mu}\right)$ $\mathrm{a} 1-\mathrm{a} 2-\mathrm{a} 1-\mathrm{a} 2$ is the output sequence and $\mu$ the model or the machine

Path found:
(working backward)


Problem statement: Find the best possible sequence

$$
S^{*}=\arg \max P(S \mid O, \mu)
$$

where, $S \rightarrow$ State Seq, $O \rightarrow$ Output Seq, $\mu \rightarrow$ Model or Machine


T is defined as $P\left(S_{i} \xrightarrow{a_{k}} S_{j}\right) \quad \forall_{i, j, k}$

## Tabular representation of the tree

| Latest symbol <br> observed | $€$ | $\mathrm{a}_{1}$ | $\mathrm{a}_{2}$ | $\mathrm{a}_{1}$ | $\mathrm{a}_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Ending state |  |  |  |  |  |

Note: Every cell records the winning probability ending in that state
Final winner
The bold faced values in each cell shows the sequence probability ending in that state. Going backward from final winner sequence which ends in state $S_{2}$ (indicated By the $2^{\text {nd }}$ tuple), we recover the sequence.

## Algorithm

## (following James Alan, Natural Language Understanding (2nd edition), Benjamin Cummins (pub.), 1995

## Given:

1. The HMM, which means:
a. Start State: $\mathrm{S}_{1}$
b. Alphabet: $A=\left\{a_{1}, a_{2}, \ldots a_{p}\right\}$
c. Set of States: $S=\left\{S_{1}, S_{2}, \ldots S_{n}\right\}$
d. Transition probability which is equal to

$$
P\left(S_{i} \xrightarrow{a_{k}} S_{j}\right) \quad \forall_{i, j, k}
$$

2. The output string $\mathrm{o}_{1} \mathrm{O}_{2} \ldots \mathrm{o}_{\mathrm{T}}$

$$
P\left(S_{j}, a_{k} \mid S_{i}\right)
$$

To find:
The most likely sequence of states $\mathrm{C}_{1} \mathrm{C}_{2} \ldots \mathrm{C}_{\mathrm{T}}$ which produces the given output sequence, i.e., $\mathrm{S}_{1} \mathrm{~S}_{2} \ldots \mathrm{~S}_{\mathrm{T}}=\quad \underset{c}{\arg \max \left[P\left(S \mid o 1, o_{2}, \ldots o r, \mu\right]\right.}$

```
\(\operatorname{SEQSCORE}(1,1)=1.0\)
BACKPTR(1,1)=0
For(i=2 to N) do
    SEQSCORE(i,1)=0.0
[expressing the fact that first state is \(\mathrm{S}_{1}\) ]
```


## 2. Iteration

```
For( \(\mathrm{t}=2\) to T ) do
    For( \(\mathrm{i}=1\) to N ) do
        \(\operatorname{SEQSCORE}(\mathrm{i}, \mathrm{t})=\operatorname{Max}_{(\mathrm{j}=1, \mathrm{~N})}\)
```

            \(\left[\operatorname{SEQSCORE}(j,(t-1)) * P\left(S j \xrightarrow{a_{k}} S i\right)\right]\)
    \(\operatorname{BACKPTR}(I, \mathrm{t})=\) index \(j\) that gives the MAX above
    
## 3. Seq. Identification

$\mathrm{S}(\mathrm{T})=\mathrm{i}$ that maximizes $\operatorname{SEQSCORE}(\mathrm{i}, \mathrm{T})$
For ifrom (T-1) to 1 do

$$
S(i)=\operatorname{BACKPTR}[S(i+1),(i+1)]
$$

## Optimizations possible:

1. BACKPTR can be $1^{*} \top$
2. SEQSCORE can be T*2

Homework:- Compare this with A*, Beam Search [Homework]
Reason for this comparison:
Both of them work for finding and recovering sequence

## Back to POS tag problem

## Viterbi for POS Tagging

- E.g.
-T: Tags
- W: Words
- Two special symbol: ‘^’ and ‘’'

Find out number of paths in the tree given word sequence.

Exponential w.r.t. number of words in the sentence of length $L$

Number of path $=$ Number of leaves in the tree.

$$
O\left(T^{L}\right)
$$

## We do not need exponential work!

- Suppose our tags are - DT, NN, VB, JJ, RB and OT

| $\wedge$ | The | black | dog | barks | . |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\wedge$ | DT | DT | DT | DT |  |
|  | NN | NN | NN | NN |  |
|  | VB | VB | VB | VB |  |
|  | JJ | JJ | JJ | JJ |  |
|  | RB | RB | RB | RB |  |
|  | OT | OT | OT | OT |  |

DT- determiner<br>NN- Noun<br>VB- Verb<br>JJ- Adjective<br>RB- Adverb<br>OT- others<br>Possible tags

So, $6^{4}$ possible path

dog: $6^{3}$
barks: $6^{4}$
Total $6^{4}$ paths

## Consider the paths that end in NN after seeing input "The black"

| $\wedge$ | The | black |  |
| :---: | :---: | :---: | :---: |
| $\wedge$ | DT | NN | $\begin{aligned} & P(\mathrm{~T}) \cdot P(\mathrm{~W} \mid \mathrm{T})=P(\mathrm{DT} \mid \wedge) \cdot P(\mathrm{NN} \mid \mathrm{DT}) \cdot P(\text { The } \mid \mathrm{DT}) . \\ & P(\mathrm{Black} \mid \mathrm{NN}) \end{aligned}$ |
| $\wedge$ | NN | NN | $P(\mathrm{~T}) \cdot P(\mathrm{~W} \mid \mathrm{T})=P(\mathrm{NN} \mid \wedge) \cdot P(\mathrm{NN} \mid \mathrm{NN}) . P($ The $\mid \mathrm{NN})$ $P$ (Black\|NN) |
| $\wedge$ | VB | NN | $\begin{aligned} & P(\mathrm{~T}) \cdot P(\mathrm{~W} \mid \mathrm{T})=P(\mathrm{VB} \mid \wedge) \cdot P(\mathrm{NN} \mid \mathrm{VB}) \cdot P(\text { The } \mid \mathrm{VB}) . \\ & P(\mathrm{Black} \mid \mathrm{NN}) \end{aligned}$ |
| $\wedge$ | JJ | NN | $\boldsymbol{P}(\mathrm{T}) \cdot \boldsymbol{P}(\mathrm{W} \mid \mathrm{T})=P(\mathrm{JJ} \mid \wedge) \cdot P(\mathrm{NN} \mid \mathrm{JJ}) \cdot P(\mathrm{The} \mid \mathrm{JJ})$. $P$ (Black\|NN) |
| $\wedge$ | RB | NN | $P(\mathrm{~T}) \cdot P(\mathrm{~W} \mid \mathrm{T})=P\left(\left.\mathrm{RB}\right\|^{\wedge}\right) \cdot P(\mathrm{NN} \mid \mathrm{RB}) . P($ The $\mid \mathrm{RB})$. $P$ (Black\|NN) |
| $\wedge$ | OT | NN | $\begin{aligned} & P(\mathrm{~T}) \cdot P(\mathrm{~W} \mid \mathrm{T})=P(\mathrm{OT} \mid \wedge) \cdot P(\mathrm{NN} \mid \mathrm{OT}) \cdot P(\mathrm{The} \mid \mathrm{OT}) . \\ & P(\text { Black } \mid \mathrm{NN}) \end{aligned}$ |

Complexity $=L * T^{2} \begin{aligned} & \text { For each tag, only path with highest probability } \\ & \text { value are retained, others are discarded. }\end{aligned}$



Probability of a path (e.g. Top most path) $=P(\mathrm{~T}){ }^{*} P(\mathrm{~W} \mid \mathrm{T})$ $P\left({ }^{\wedge}\right) \cdot P(N N / \wedge) . P(N N S / N N) . P(V B D / N N S) . P(N N / V B D)$. $P(D T / N N) . P(N N / D T) . P(. \mid N N) . P($.
$P(\wedge / \wedge) . P(b r o w n / N N) . P(f o x e s / N N S) . P(j u m p e d / V B D)$. P(over/NN) . P(the/DT) . P(fence/NN) . P(...)

## Decoding Summary

- On every word compute the partial path probability
- Out of all partial paths ending in a particular state, choose the one with highest path probability
- Advance only that leaf
- In case of tie, choose any one arbitrarily


## Assignment Discussion

## Brown Corpus

- $1,014,312$ words of running text of edited English prose printed in the United States
- 500 samples of 2000+ words each
- Facilitate automatic or semi-automatic syntactic analysis


## Universal POS Tag Set

 (https://universaldependencies.org/ u/pos/)| Open class words | Closed class words | Other |
| :---: | :---: | :---: |
| ADJ (The car is green.) | ADP | PUNCT |
| ADV (arguably wrong) | AUX | SYM |
| $\frac{\mathrm{INTJ}}{\text { etc.) }} \text { (yes, no, uhuh, }$ | CCONJ | $\underline{X}$ |
| NOUN (tree, man) | DET |  |
| PROPN | NUM |  |
| VERB | PART |  |
|  | PRON |  |
|  | SCONJ |  |

## Noun (1/2)

## Definition

- Nouns are a part of speech typically denoting a person, place, thing, animal or idea.
- The NOUN tag is intended for common nouns only. See PROPN for proper nouns and PRON for pronouns.
- Note that some verb forms such as gerunds and infinitives may share properties and usage of nouns and verbs. Depending on language and context, they may be classified as either VERB or NOUN.
Swimming_noun is a good exercise; He is swimmina verb


## Noun (2/2)

## Examples

- girl
- cat
- tree
- air
- beauty


## References

- Loos, Eugene E., et al. 2003. Glossary of linguistic terms: What is a noun?
- Wikipedia


## Annotation matter

## Tag repository and probability

-Where do tags come from?

- Tag set
- How to get probability values i.e. $P($.$) ?$
- Annotated corpora

After modeling of the problem, emphasis should be on the corpus

## Computing $\mathrm{P}($.$) values$

Let us suppose annotated corpus has the following sentence

| I | have | a | brown | bag |
| :---: | :---: | :---: | :---: | :---: |
| PRN | VB | DT | JJ | NN |

$$
P(N N \mid J J)=\frac{\text { Number_of_times_}_{-} J J_{-} \text {followed_by_} N N}{N u m b e r_{-} \text {of_times_JJ_appeared }}
$$

$$
P(B r o w n \mid J J)=\frac{\text { Number_of_times_Brown_tagged_as_JJ }_{\text {Number_of_times_JJ_appeared }}}{\text { Nut }}
$$

## Why Ratios?

- This way of computing parameter probabilities: is this correct?
-What does "correct" mean?
- Is this principled?
- We are using Maximum Likelihood Estimate (MLE)
- Assumption: underlying distribution is multinomial


## Explanation with coin tossing

- A coin is tossed 100 times, Head appears 40 times
- $P(H)=0.4$
- Why?
- Because of maximum likelihood


## $N$ tosses, K Heads, parameter $P(H)=p$

- Construct Maximum Likelihood Expression
- Take log likelihood and take derivative
- Equate to 0 and Get $p$

$$
\begin{aligned}
& L=p^{K}(1-p)^{N-K} \\
& \Rightarrow L L=\log (L)=K \log p+(N-K) \log (1-p) \\
& \Rightarrow \frac{d(L L)}{d p}=\frac{K}{p}-\frac{N-K}{1-p} \\
& \Rightarrow \frac{d(L L)}{d p}=0 \text { gives } p=\frac{K}{N}
\end{aligned}
$$

## Exercise

- Following the process for finding the probability of Head from N tosses of coin yielding K Heads, prove that the transition probabilities can be found from MLE
- Most important: get the likelihood expression
- Use chapter 2 of the book
- Pushpak Bhattacharyya: Machine translation, CRC Press, Taylor \& Francis Group, Boca Raton, USA, 2015, ISBN: 978-1-4398-9718-8


## Appendix

## Appendages to tags in Penn Tag Set

$S$ = plural
\$ = possessive

R = comparative

T = superlative

O = objective case of pronoun

D = past tense
$Z=3 r d$ singular verb

N = past participle
$G=$ present participle or gerund

## Machine Translation v/s POS

 tagging!- Similarity
- POS
- Every word in a sentence has one corresponding tag.
- MT
- Every word in a sentence has one (or more) corresponding translated word.
- Difference
- Order: Order of translated word may change.
- Fertility: One word corresponds to many. Many to one also possible.


## Complexity

- POS and HMM
- Linear time complexity
- MT and Bean search
- Exponential time complexity
- Permutation of words produces exponential searc space
- However, for related languages, MT is like POS tagging


## Properties of related languages

## 1. Order preserving

## 2. Fertility ~ 1

## 3. Morphology preserving

| Hindi | Jaaunga |
| :--- | :--- |
| Bengali | Jaabo |
| English | Will go |

Hindi \& Bengali $\uparrow$ Hindi \& English $\downarrow$

## Properties of related languages

4. Syncretism: Suffix features should be similarly loaded

| Hindi | Main jaaunga | Hum jaayenge |
| :--- | :--- | :--- |
| Bengali | Ami jaabo | Aamra jaabo |
|  |  |  |

5. Idiomaticity: Literal translation should be high

| Hindi | Aap Kaise Ho? |
| :--- | :--- |
| Bengali | Aapni Kemon Achen? |
| English | How do you do? |

Hindi \& Bengali $\uparrow$
Hindi \& English $\downarrow$

## Points to ponder wrt HMM and Viterbi

## Viterbi Algorithm

- Start with the start state.
- Keep advancing sequences that are "maximum" amongst all those ending in the same state


## Viterbi Algorithm

Tree for the sentence: "^ People laugh ."


Claim: We do not need to draw all the subtrees in the algorithm

## Viterbi phenomenon (Markov process)



LAUGH

Next step all the probabilities will be multiplied by identical probability (lexical and transition). So children of N2 will have probability less than the children of N 1 .

## What does $\mathrm{P}(\mathrm{A} \mid \mathrm{B})$ mean?

- $P(A \mid B)=P(B \mid A)$ If $P(A)=P(B)$
- $\mathrm{P}(\mathrm{A} \mid \mathrm{B})$ means??
- Causality?? B causes A??
- Sequentiality?? A follows B?


## Classic problems with respect to

 HMM1.Given the observation sequence, find the possible state sequences- Viterbi
2. Given the observation sequence, find its probability- forward/backward algorithm
3. Given the observation sequence find the HMM prameters.- Baum-Welch algorithm

## Illustration of Viterbi

- The "start" and "end" are important in a sequence.
- Subtrees get eliminated due to the Markov Assumption.

POS Tagset

- N (noun), V (verb), O(other) [simplified]
- ^ (start), . (end) [start $\&$ end states]

Illustration of Viterbi
Lexicon
people: N, V
laugh: N, V

Corpora for Training
$\qquad$ ${ }^{\wedge} \mathrm{w}_{11-} \mathrm{t}_{11} \mathrm{w}_{12-} \mathrm{t}_{12} \mathrm{w}_{13-} \mathrm{t}_{13}$ $\qquad$ $\mathrm{w}_{1 \mathrm{k} \_1-} \mathrm{t}_{1 \mathrm{k} \_1}$.
${ }^{\wedge} W_{21-} t_{21} W_{22-} t_{22} W_{23-} t_{23}$ $\qquad$ $W_{2 k \_2-} t_{2 k \_2}$.
${ }^{\wedge} W_{n 1-} t_{n 1} W_{n 2-} t_{n 2} W_{n 3-} t_{n 3}$ $\qquad$ . $W_{n k \_n-} t_{n k \_n}$.

## Inference



Partial sequence graph

|  | $\wedge$ | $N$ | $V$ | $O$ | . |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\wedge$ | 0 | 0.6 | 0.2 | 0.2 | 0 |
| $N$ | 0 | 0.1 | 0.4 | 0.3 | 0.2 |
| $V$ | 0 | 0.3 | 0.1 | 0.3 | 0.3 |
| 0 | 0 | 0.3 | 0.2 | 0.3 | 0.2 |
| . | 1 | 0 | 0 | 0 | 0 |

This transition table will change from language to language due to language divergences.

## Lexical Probability Table

|  | $\epsilon$ | people | laugh | $\ldots$ | $\ldots$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\wedge$ | 1 | 0 | 0 | $\ldots$ | 0 |
| $N$ | 0 | $1 \times 10^{-3}$ | $1 \times 10^{-5}$ | $\ldots$ | $\ldots$ |
| V | 0 | $1 \times 10^{-6}$ | $1 \times 10^{-3}$ | $\ldots$ | $\ldots$ |
| 0 | 0 | 0 | 0 | $\ldots$ | $\ldots$ |
| . | 1 | 0 | 0 | 0 | 0 |

Size of this table = \# pos tags in tagset $X$ vocabulary size
vocabulary size = \# unique words in corpus

## Inference

New Sentence:
people laugh .

$\mathrm{p}\left({ }^{\wedge} \mathrm{N} N .\left.\right|^{\wedge}\right.$ people laugh .)
$=(0.6 \times 0.1) \times\left(0.1 \times 1 \times 10^{-3}\right) \times\left(0.2 \times 1 \times 10^{-5}\right)$

## Computational Complexity

- If we have to get the probability of each sequence and then find maximum among them, we would run into exponential number of computations.
- If $|\mathrm{s}|=$ \#states (tags $+^{\wedge}+$. ) and $|\mathrm{o}|=$ length of sentence ( words $+^{\wedge}+$.) Then, \#sequences $=s^{|0|-2}$
- But, a large number of partial computations can be reused using Dynamic Programming.


## Dynamic Programming



## Computational Complexity

- Retain only those N / V / O nodes which ends in the highest sequence probability.
- Now, complexity reduces from $|s|^{|0|}$ to $|s| .|0|$
- Here, we followed the Markov assumption of order 1.

Reeledhrafapg8:paa

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## Reading List

- https://www.nltk.org/book/ch05.html
- TnT (http://www.aclweb.org/anthology-new/A/A00/A001031.pdf)
- Hindi POS Tagger built by IIT Bombay (http://www.cse.iitb.ac.in/pb/papers/ACL-2006-Hindi-POS-Tagging.pdf)

