



Program Analysis
<https://www.cse.iitb.ac.in/~karkare/cs618/>

Data Flow Analysis (contd...)

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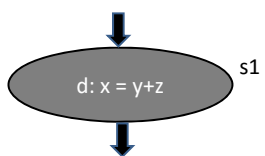



Available Expressions

- Expression e is available at a point p :
 - **Every** path from entry to p has at least one evaluation of e
 - There is no assignment to any component variable of e **after last evaluation** of e prior to p
- Expression e is *generated* by its evaluation
- Expression e is *killed* by assignment to its component variables

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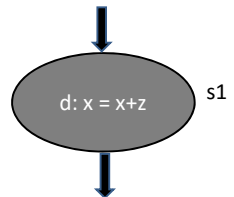
Available Expr Analysis



$gen(s1) = \{y+z\}$
 $kill(s1) = E_x$ // E_x : set of all expressions having x as a component
 $out(s1) = in(s1) - kill(s1) \cup gen(s1)$
THIS MAY NOT WORK IN GENERAL! WHY?

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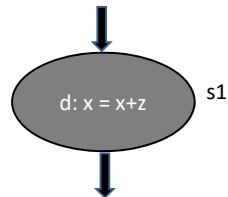
Available Expr Analysis



INCORRECT FORMULATION
 $out(s1) = in(s1) - kill(s1) \cup gen(s1)$
 $gen(s1) = \{x+z\}$
 $kill(s1) = E_x$ // E_x : set of all expressions having x as a component

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Available Expr Analysis



INCORRECT FORMULATION

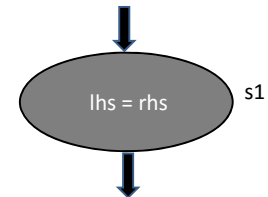
$$\text{out}(s1) = \text{in}(s1) - \text{kill}(s1) \cup \text{gen}(s1)$$

$$\text{gen}(s1) = \{x+z\}$$

$$\text{kill}(s1) = E_x \quad // \quad E_x: \text{set of all expressions having } x \text{ as a component}$$

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Available Expr Analysis



CORRECT FORMULATION

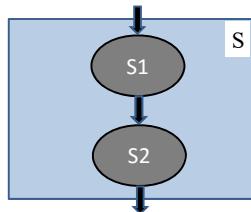
$$\text{out}(s1) = \text{in}(s1) - \text{kill}(s1) \cup \text{gen}(s1)$$

$$\text{gen}(s1) = \{ \text{rhs} \mid \text{lhs is not part of rhs} \}$$

$$\text{kill}(s1) = E_{\text{lhs}} \quad // \quad E_{\text{lhs}}: \text{set of all expressions having lhs as a component}$$

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Available Expr Analysis



$$\text{gen}(s) = \text{gen}(s2) \cup (\text{gen}(s1) - \text{kill}(s2))$$

$$\text{kill}(s) = \text{kill}(s2) \cup (\text{kill}(s1) - \text{gen}(s2))$$

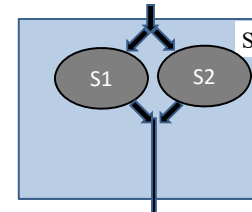
$$\text{in}(s1) = \text{in}(s)$$

$$\text{in}(s2) = \text{out}(s1)$$

$$\text{out}(s) = \text{out}(s2)$$

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Analysis of Structured Programs



$$\text{gen}(s) = \text{gen}(s1) \cap \text{gen}(s2)$$

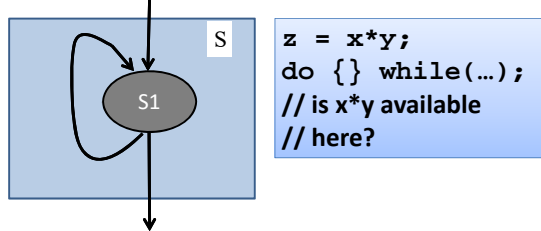
$$\text{kill}(s) = \text{kill}(s1) \cup \text{kill}(s2)$$

$$\text{in}(s1) = \text{in}(s2) = \text{in}(s)$$

$$\text{out}(s) = \text{out}(s1) \cap \text{out}(s2)$$

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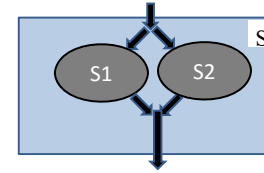
Available Expr Analysis



$gen(s) = gen(s1)$
 $kill(s) = kill(s1)$
 $in(s1) = in(s) \cap gen(s1)$
 $out(s) = out(s1)$

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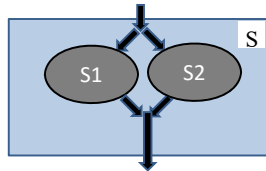
Again: Conservative Analysis



- Assumption: All paths are feasible.
 - Consider: if (true) s1; else s2
 - s2 is never executed
- $gen(s) = gen(s1) \supseteq gen(s1) \cap gen(s2)$
 $kill(s) = kill(s1) \subseteq kill(s1) \cup kill(s2)$

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Again: Conservative Analysis



- Thus: $true\ gen(s) \supseteq analysis\ gen(s)$
 $true\ kill(s) \subseteq analysis\ kill(s)$
- True is what is computed at run time
- This is **SAFE** estimate
 - prevents optimization
 - but no wrong optimization

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Available Expressions

- Expr e is available at the start of a block
 - It is available at the end of **all** predecessors

$$in(B) = \bigcap_{P \text{ is pred of } B} out(P)$$

- Expr e is available at the end of a block
 - either it is generated by the block
 - or it is available at the start of the block and not killed by the block

$$out(B) = in(B) - kill(B) \cup gen(B)$$

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Available Expressions

- Kill & gen known for each block.
- A program with N blocks has 2N equations with 2N unknowns
 - solution is possible.
 - iterative approach (on next slide)

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```

for each block B {
    out(B) = U; // U = "universal" set of all exprs
}
out(Entry) =  $\emptyset$ ; // remember reaching defs?
change = true;
while (change) {
    change = false;
    for each block B other than Entry {
        in(B) =  $\bigcap_{P \text{ is pred of } B} \text{out}(P)$ ;
        oldOut = out(B);
        out(B) = in(B) - kill(B)  $\cup$  gen(B);
        if (oldOut != out(B)) then {
            change = true;
        }
    }
}

```

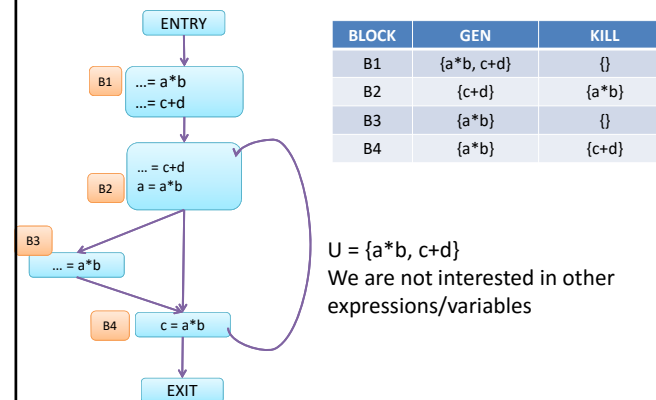
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Some Issues

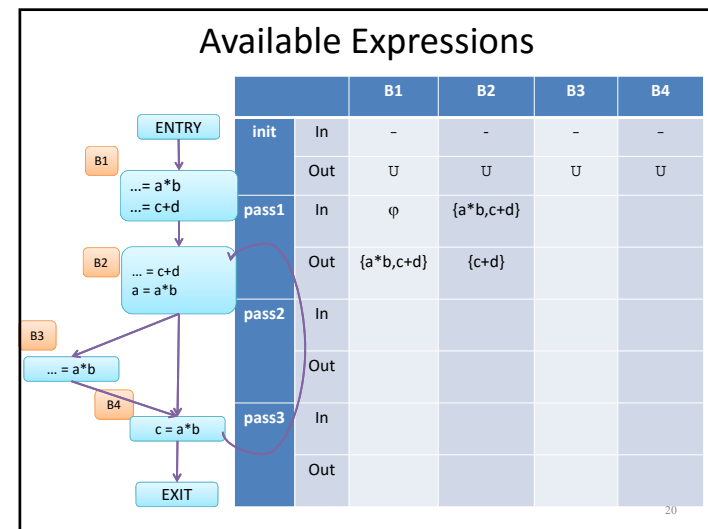
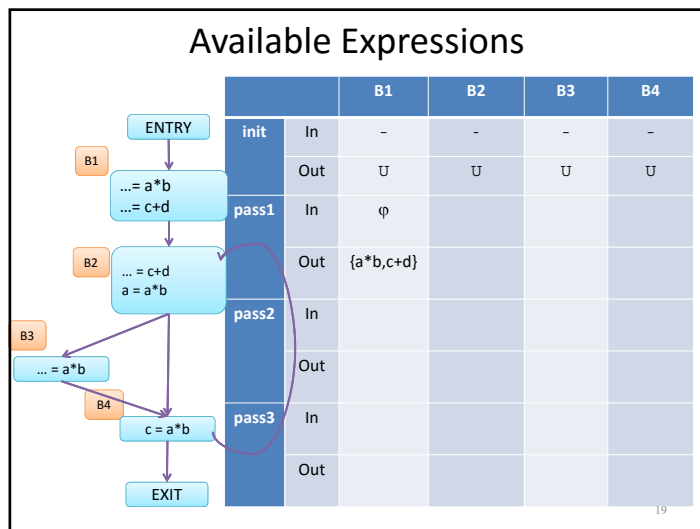
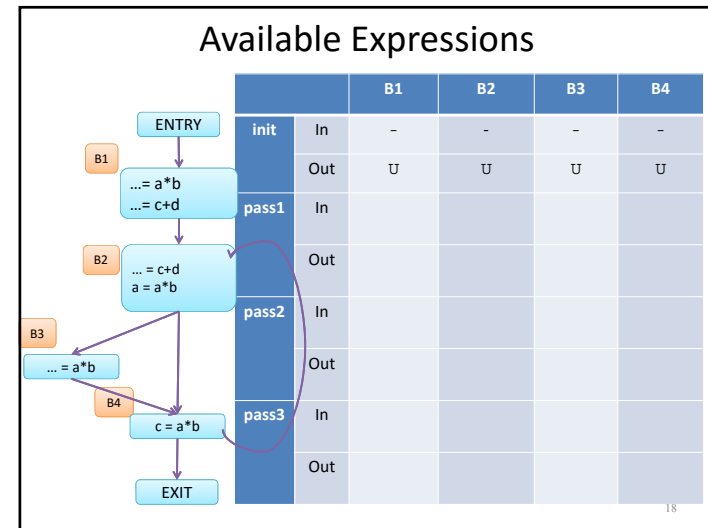
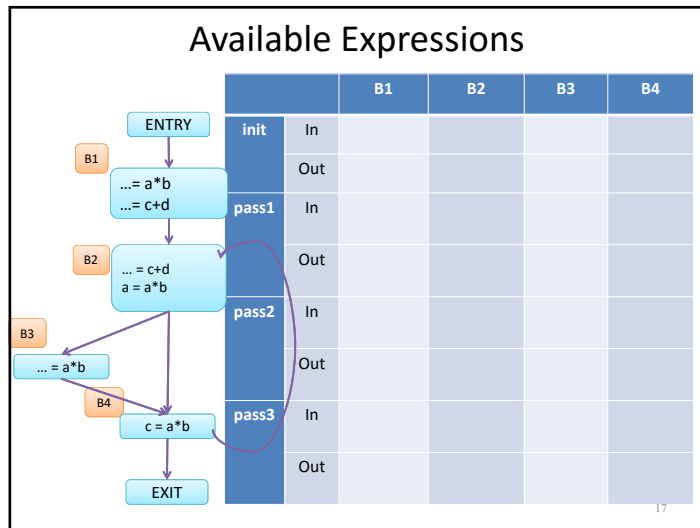
- What is the set of all expressions?
- How to compute it efficiently?
- Why Entry block is initialized differently?

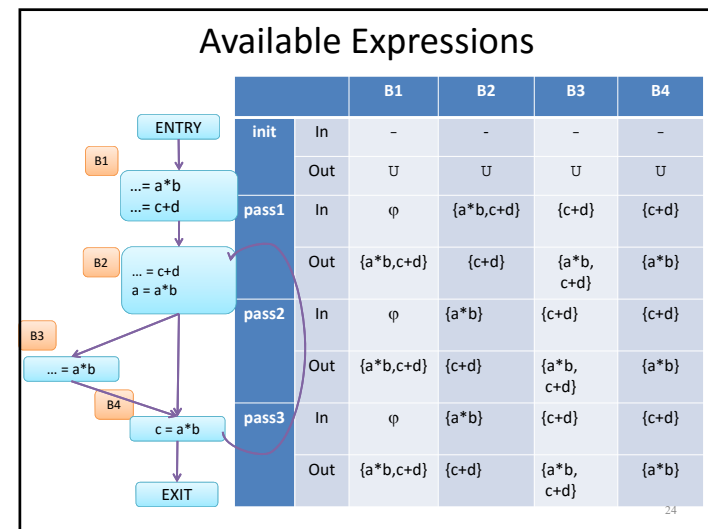
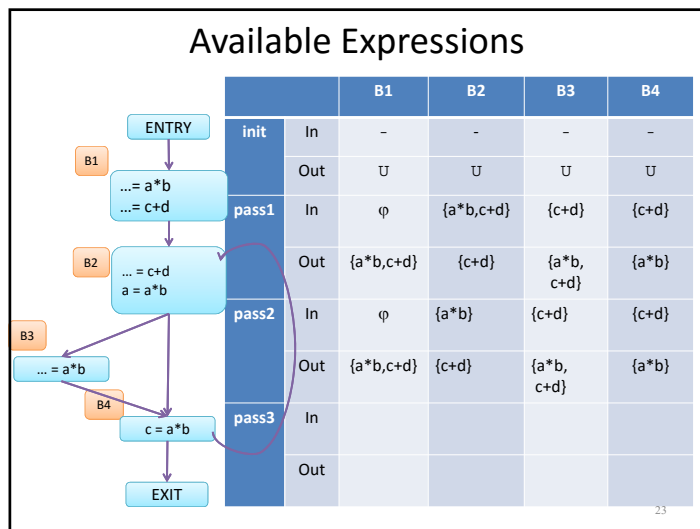
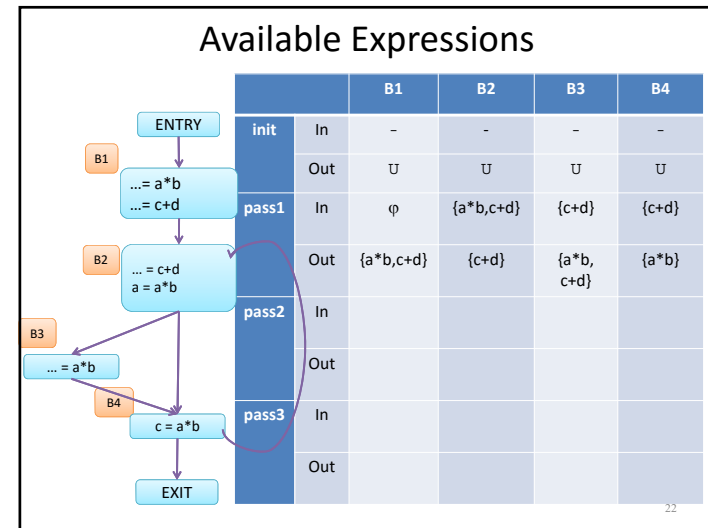
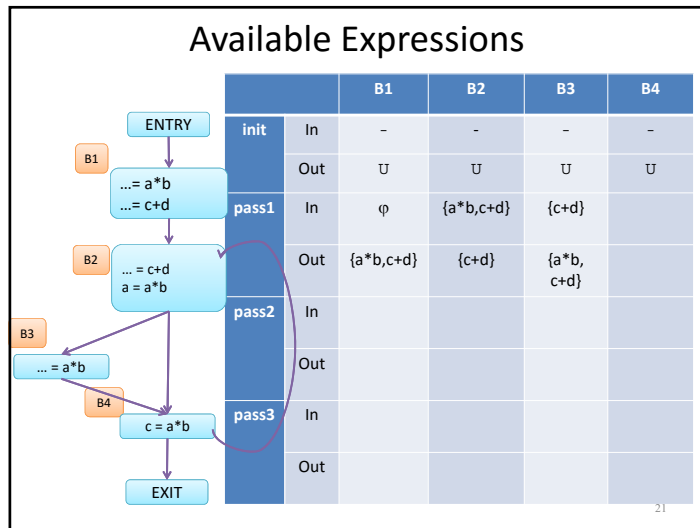
15

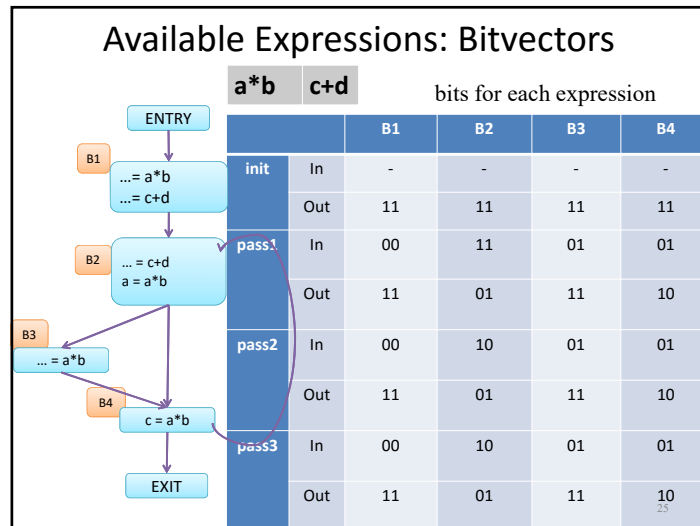
Available Expressions



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Available Expressions: Bitvectors

$$\text{in}(B) = \bigcap_{P \text{ is pred of } B} \text{out}(P)$$

$$\text{out}(B) = \text{in}(B) - \text{kill}(B) \cup \text{gen}(B)$$

- With bit vectors,

$$\text{in}(B) = \bigwedge_{P \text{ is pred of } B} \text{out}(P)$$

$$\text{out}(B) = (\text{in}(B) \wedge \neg \text{kill}(B)) \vee \text{gen}(B)$$

- Bitwise \wedge , \vee , \neg operations.

Available Expressions: Application

- Common subexpression elimination in a block B
 - Expression e available at the entry of B
 - e is also computed at a point p in B
 - Components of e are not modified from entry of B to p
- e is “upward exposed” in B
- Expressions generated in B are “downward exposed”

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Recap: Summary of Reaching Definitions

$\text{gen} = \{ d_x \mid d_x \text{ in } B \text{ defines variable } x \text{ and is not followed by another definition of } x \text{ in } B \}$

$\text{kill} = \{ d_x \mid \text{block contains some definition of } x \}$

$$\text{in}(B) = \bigcup_{P \text{ is pred of } B} \text{out}(P)$$

$$\text{out}(B) = \text{in}(B) - \text{kill}(B) \cup \text{gen}(B)$$

meet (\wedge) operator is \bigcap

Initialization:

$$\text{out}(B_{\text{entry}}) = \text{Entry Info} = \varnothing$$

$$\text{out}(B) = \varnothing$$

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Summary of Available Expressions

gen = downward exposed expressions

$kill = \{ e_x \mid \text{block contains some definition of } x \}$

$in(B) = \bigcap_{P \text{ is pred of } B} out(P)$

$out(B) = in(B) - kill(B) \cup gen(B)$

meet (\wedge) operator is \cap

Initialization:

$out(B_{entry}) = \text{Entry Info} = \varphi$

$out(B) = \cup$

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Comparing Reaching Definition and Available Expressions Analysis

- Class Discussion about
 - Similarities
 - Differences

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Summary of Available Expressions

- What if we Initialize:

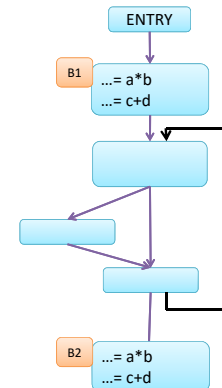
$out(B_{entry}) = \text{Entry Info} = \varphi$

$out(B) = \varphi$

- We might miss some expressions that are available
- Loose on opportunity to optimize!

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What are the expressions available at B2 when $out(B)$ initialized with
i) \cup ii) φ



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Live Variable Analysis

- A variable x is *live* at a point p if
 - There is a point p' along some path in the flow graph starting at p to the EXIT
 - Value of x could be used at p'
 - There is no definition of x between p and p' along this path
- Otherwise x is *dead* at p

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Live Variable Analysis: Gen

- $gen(B)$
- Set of variables whose values may be used in B prior to any definition
- Also called “ $use(B)$ ”
- “upward exposed” use of a variable is generated by B

Live Variable Analysis: Kill

- $kill(B)$
- Set of variables defined in B prior to any use
- Also called “ $def(B)$ ”
- “upward exposed” definition of a variable kills its liveness in B

Live Variable Analysis

- $$out(B) = \bigcup_{S \text{ is succ of } B} in(S)$$
- $$in(B) = out(B) - kill(B) \cup gen(B)$$
- Alt: $in(B) = out(B) - def(B) \cup use(B)$
- With bit vectors,

$$out(B) = \bigvee_{S \text{ is succ of } B} in(S)$$

$$in(B) = (out(B) \wedge \neg kill(B)) \vee gen(B)$$
 - Bitwise \wedge , \vee , \neg operations.

Very Busy Expressions

- Expression e is very busy at a point p
 - **Every** path from p to exit has at least one evaluation of e
 - There is no assignment to any component variable of e **before first evaluation** of e following p
- Also called ***Anticipable*** expression

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Very Busy Expression

- Practice Assignment
 - Set the data flow equations for Very Busy Expression
 - Hint: Available Expression Analysis