CS618: Program Analysis
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Liveness based Garbage Collection

Amey Karkare
karkare@cse.iitk.ac.in
karkare@cse.iitb.ac.in

Department of CSE, IIT Kanpur/Bombay

Ideal Garbage Collection

. . . garbage collection (GC) is a form of automatic memory management. The garbage collector, or just collector, attempts to reclaim garbage, or memory occupied by objects that are no longer in use by the program. . . .

From Wikipedia

`https://en.wikipedia.org/wiki/Garbage_collection_(computer_science)`

Real Garbage Collection

. . . All garbage collectors use some efficient approximation to liveness. In tracing garbage collection, the approximation is that an object can't be live unless it is reachable. . . .

From Memory Management Glossary

`www.memorymanagement.org/glossary/g.html#term-garbage-collection`

Liveness based GC

- During execution, there are significant amounts of heap allocated data that are *reachable but not live*.
  - Current GCs will retain such data.
- Our idea:
  - We do a liveness analysis of *heap data* and provide GC with its result.
  - Modify GC to mark data for retention *only if it is live*.
- Consequences:
  - Fewer cells marked. More garbage collected per collection. Fewer garbage collections.
  - Programs expected to run faster and with smaller heap.

The language analyzed

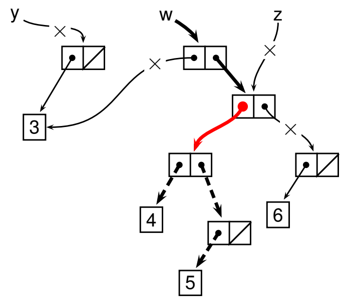
- First order eager Scheme-like functional language.
- In Administrative Normal Form (ANF).

$$
\begin{array}{lll}
p \in \mathit{Prog} & ::= & d_1 \ldots d_n\ e_{\text{main}} \\
d \in \mathit{Fdef} & ::= & (\textbf{define}\ (f\ x_1\ \ldots\ x_n)\ e) \\
e \in \mathit{Expr} & ::= & \left\{ \begin{array}{l} (\textbf{if}\ x\ e_1\ e_2) \\ (\textbf{let}\ x \leftarrow a\ \textbf{in}\ e) \\ (\textbf{return}\ x) \end{array} \right. \\
a \in \mathit{App} & ::= & \left\{ \begin{array}{ll} k & \\ (\textbf{cons}\ x_1\ x_2) & \\ (\textbf{car}\ x) & (\textbf{cdr}\ x) \\ (\textbf{null?}\ x) & (+\ x_1\ x_2) \\ (f\ x_1\ \ldots\ x_n) & \end{array} \right.
\end{array}
$$

An Example

```
(define (append l1 l2)
 (if (null? l1) l2
  (cons (car l1)
        (append (cdr l1) l2))))

(let z ←(cons (cons 4 (cons 5 nil))
         (cons 6 nil)) in
(let y ← (cons 3 nil) in
 (let w ← (append y z) in
    π:(car (cdr w)))))
```



- *Though all cells are reachable at π, a liveness-based GC will retain only the cells pointed by thick arrows.*

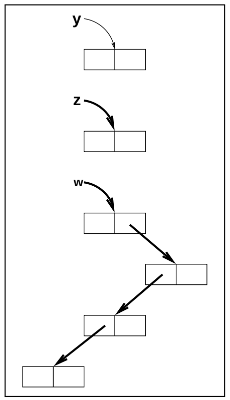
Liveness – Basic Concepts and Notations

- *Access paths:* Strings over {**0**, **1**}.
  - **0** – access **car** field
  - **1** – access **cdr** field
- Denote traversals over the heap graph
- *Liveness environment:* Maps root variables to set of access paths.

$$
\mathsf{L}_i \quad : \quad \left\{ \begin{array}{l} y \mapsto \emptyset \\ z \mapsto \{\epsilon\} \\ w \mapsto \{\epsilon, 1, 10, 100\} \end{array} \right.
$$

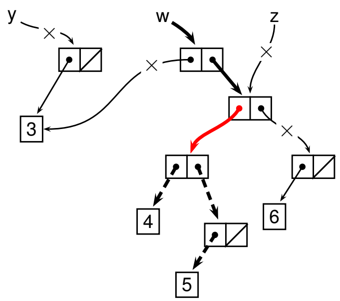
Alternate representation.

$$
\mathsf{L}_i \quad : \quad \left\{ \begin{array}{l} \emptyset\ \cup \\ \{z.\epsilon\}\ \cup \\ \{w.\epsilon, w.1, w.10, w.100\} \end{array} \right.
$$



Demand

(**car** (**cdr** w))



- Demand (notation: σ) is a description of intended use of the result of an expression.

Demand

- Demand (notation: σ) is a description of intended use of the result of an expression.
- We assume the demand on the main expression to be $(0+1)^*$, which we call σ_{all}.
- The demands on each function body, σ_f, have to be computed.

Liveness analysis – The big picture

π_{main}: (**let** z ← ... **in**
(**let** y ← ... **in**
π_9: (**let** w ← (**append** y z) **in**
π_{10}: (**let** a ← (**cdr** w) **in**
π_{11}: (**let** b ← (**car** a) **in**
π_{12}: (**return** b)))))))

(**define** (**append** l1 l2)
π_1: (**let** test ← (**null**? l1) **in**
π_2: (**if** test π_3:(**return** l2)
π_4: (**let** tl ← (**cdr** l1) **in**
π_5: (**let** rec ← (**append** tl l2) **in**
π_6: (**let** hd ← (**car** l1) **in**
π_7: (**let** ans ← (**cons** hd rec) **in**
π_8: (**return** ans))))))))

Liveness analysis – The big picture

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π_8: (**return** ans))))))))

Liveness environments:

$$
\begin{aligned}
L_1 &= \ldots \\
L_2 &= \ldots \\
\ldots \\
L_9 &= \ldots \\
L_{10} &= \ldots
\end{aligned}
$$

Liveness analysis

- **GOAL:** Compute Liveness Environment at various program points, statically.

$\mathcal{L}app(a,\sigma)$ – Liveness environment generated by an *application* a, given a demand σ.

$\mathcal{L}exp(e,\sigma)$ – Liveness environment before an *expression* e, given a demand σ.

Liveness analysis of Expressions

$\mathcal{L}exp((\text{return } x), \sigma) = \{x.\sigma\}$

$\mathcal{L}exp((\textbf{if } x \;\; e_1 \;\; e_2), \sigma) = \{x.\epsilon\} \cup \mathcal{L}exp(e_1, \sigma) \cup \mathcal{L}exp(e_2, \sigma)$

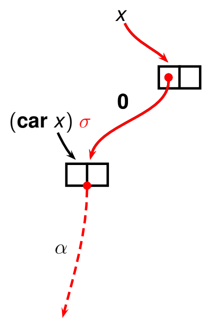
$$\mathcal{L}exp((\textbf{let } x \leftarrow s \textbf{ in } e), \sigma) = \mathsf{L} \setminus \{x.*\} \cup \mathcal{L}app(s, \mathsf{L}(x)) \quad \text{where } \mathsf{L} = \mathcal{L}exp(e, \sigma)$$

Notice the similarity with:

$$live_{in}(B) = live_{out}(B) \setminus kill(B) \cup gen(B)$$

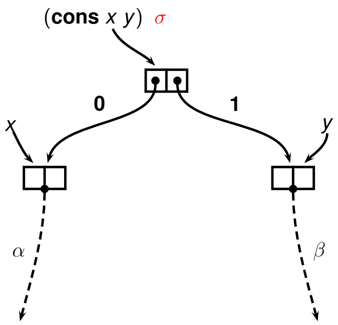
in classical dataflow analysis for imperative languages.

Liveness analysis of Primitive Applications



$\mathcal{L}app((\textbf{car } x), \sigma) = \{x.\epsilon, \; x.\mathbf{0}\sigma\}$

Liveness analysis of Primitive Applications

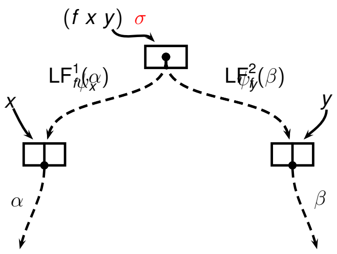


$$\mathcal{L}app((\textbf{cons } x \; y), \sigma) = \{x.\alpha \mid \mathbf{0}\alpha \in \sigma\} \cup \{y.\beta \mid \mathbf{1}\beta \in \sigma\}$$

- $\bar{\mathbf{0}}$ – Removal of a leading $\mathbf{0}$
  $\bar{\mathbf{1}}$ – Removal of a leading $\mathbf{1}$

$$\mathcal{L}app((\textbf{cons } x \; y), \sigma) = x.\bar{\mathbf{0}}\sigma \cup y.\bar{\mathbf{1}}\sigma$$

Liveness Analysis of Function Applications



$\mathcal{L}app((f \; x \; y), \sigma) = x.\overline{\psi_x}\sigma \cup \; y.\overline{\psi_y}\sigma$

- We use LF_f: context independent summary of f.
- To find $\mathsf{LF}^i_f(\ldots)$:
  - Assume a symbolic demand σ_{sym}.
  - Let e_f be the body of f.
  - Set $\mathsf{LF}^i_f(\sigma_{sym})$ to $\mathcal{L}exp(e_f, \sigma_{sym})(x_i)$.
  - How to handle recursive calls? Use LF_f with appropriate demand !!

Liveness analysis – The big picture

```
πmain: (let z ← ... in
         (let y ← ... in
 π9:  (let w ← (append y z) in
 π10: (let a ← (cdr w) in
 π11: (let b ← (car a) in
 π12: (return b)))))))
```

```
(define (append l1 l2)
 π1: (let test ← (null? l1) in
 π2: (if test  π3:(return l2)
 π4: (let tl ← (cdr l1) in
 π5: (let rec ← (append  tl  l2) in
 π6: (let hd ← (car l1) in    LF²append(1̄σ)
 π7: (let ans ← (cons  hd rec)  in    1̄σ
 π8:  (return ans))))))))   σ
```

Liveness environments:

$L_1^{l1} = \{\epsilon\} \cup \mathbf{0\bar{0}}\sigma_{append} \cup \mathbf{1}LF^1_{append}(\bar{\mathbf{1}}\sigma_{append})$

$L_1^{l2} = \sigma \cup LF^2_{append}(\bar{\mathbf{1}}\sigma_{append})$

...

$L_9^{y} = LF^1_{append}(\{\epsilon, \mathbf{1}\} \cup \mathbf{10}\sigma_{all})$

Demand summaries:

Function summaries:

$LF^1_{append}(\sigma) = \{\epsilon\} \cup \mathbf{0\bar{0}}\sigma \cup \mathbf{1}LF^1_{append}(\bar{\mathbf{1}}\sigma)$

$LF^2_{append}(\sigma) = \sigma \cup LF^2_{append}(\bar{\mathbf{1}}\sigma)$

Liveness analysis – Demand Summary

$\sigma_{main} = \sigma_{all}$ $\sigma_{append} = \sigma_1 \cup \ldots \sigma_2$

```
πmain: (let z ← ... in          σ1
         (let y ← ... in
 π9:  (let w ← (append y z) in
 π10: (let a ← (cdr w) in
 π11: (let b ← (car a) in
 π12: (return b)))))))
```

```
(define (append l1 l2)
 π1: (let test ← (null? l1) in
 π2: (if test  π3:(return l2)
 π4: (let tl ← (cdr l1) in          σ2
 π5: (let rec ←  (append  tl  l2) in
 π6: (let hd ← (car l1) in
 π7: (let ans ← (cons  hd  rec) in
 π8:  (return ans))))))))
```

Liveness environments:

$L_1^{l1} = \{\epsilon\} \cup \mathbf{0\bar{0}}\sigma_{append} \cup \mathbf{1}LF^1_{append}(\bar{\mathbf{1}}\sigma_{append})$

$L_1^{l2} = \sigma \cup LF^2_{append}(\bar{\mathbf{1}}\sigma_{append})$

...

$L_9^{y} = LF^1_{append}(\{\epsilon, \mathbf{1}\} \cup \mathbf{10}\sigma_{all})$

Demand summaries:

$\sigma_{main} = \sigma_{all}$

$\sigma_{append} = \{\epsilon, \mathbf{1}\} \cup \mathbf{10}\sigma_{all} \cup \bar{\mathbf{1}}\sigma_{append}$

Function summaries:

$LF^1_{append}(\sigma) = \{\epsilon\} \cup \mathbf{0\bar{0}}\sigma \cup \mathbf{1}LF^1_{append}(\bar{\mathbf{1}}\sigma)$

$LF^2_{append}(\sigma) = \sigma \cup LF^2_{append}(\bar{\mathbf{1}}\sigma)$

Obtaining a closed form solution for LF

- Function summaries will always have the form:

$$LF_f^i(\sigma) = I_f^i \cup D_f^i\sigma$$

- Consider the equation for LF^1_{append}

$$LF^1_{append}(\sigma) = \{\epsilon\} \cup \mathbf{0\bar{0}}\sigma \ \cup \mathbf{1}LF^1_{append}(\bar{\mathbf{1}}\sigma)$$

- Substitute the assumed form in the equation:

$$I^1_{append} \cup D^1_{append}\sigma = \{\epsilon\} \ \cup \ \mathbf{0\bar{0}}\sigma \cup \mathbf{1}(I^1_{append} \cup D^1_{append}\bar{\mathbf{1}}\sigma)$$

- Equating the terms without and with σ, we get:

$$I^1_{append} = \{\epsilon\} \ \cup \ \mathbf{1}I^1_{append}$$
$$D^1_{append} = \mathbf{0\bar{0}} \cup \mathbf{1}D^1_{append}\bar{\mathbf{1}}$$

Summary of Analysis Results

Liveness at program points:

$L_1^{l1} = \{\epsilon\} \cup \mathbf{0\bar{0}}\sigma \cup \mathbf{1}(I^1_{append} \cup D^1_{append}\bar{\mathbf{1}}\sigma_{append})$

$L_1^{l2} = \{\epsilon\} \cup I^2_{append} \cup D^2_{append}\bar{\mathbf{1}}\sigma_{append}$

$L_5^{l1} = \{\epsilon\} \cup \mathbf{0\bar{0}}\sigma_{append}$

$L_5^{tl} = I^1_{append} \cup D^1_{append}\bar{\mathbf{1}}\sigma_{append}$

$L_5^{l2} = I^2_{append} \cup D^2_{append}\bar{\mathbf{1}}\sigma_{append}$

...

Demand summaries:

$\sigma_{append} = \{\epsilon, \mathbf{1}\} \cup \bar{\mathbf{1}}\sigma_{append} \cup \mathbf{10}\sigma_{all}$

Function summaries:

$I^1_{append} = \{\epsilon\} \ \cup \ \mathbf{1}I^1_{append}$

$D^1_{append} = \mathbf{0\bar{0}} \cup \mathbf{1}D^1_{append}\bar{\mathbf{1}}$

$I^2_{append} = I^2_{append}$

$D^2_{append} = \{\epsilon\} \cup D^2_{append}\bar{\mathbf{0}}$

Solution of the equations

View the equations as grammar rules:

$$
\begin{aligned}
\mathsf{L}_1^{\mathtt{11}} &\rightarrow \epsilon \mid \mathbf{0}\bar{\mathbf{0}}\sigma \mid \mathbf{1}(\mathsf{I}^1_{\textbf{append}} \mid \mathsf{D}^1_{\textbf{append}}\bar{\mathbf{1}}\sigma_{\textbf{append}}) \\
\mathsf{I}^1_{\textbf{append}} &\rightarrow \epsilon \mid \mathbf{1}\mathsf{I}^1_{\textbf{append}} \\
\mathsf{D}^1_{\textbf{append}} &\rightarrow \mathbf{0}\bar{\mathbf{0}} \mid \mathbf{1}\mathsf{D}^1_{\textbf{append}}\bar{\mathbf{1}}
\end{aligned}
$$

The solution of $\mathsf{L}_1^{\mathtt{11}}$ is the language $\mathscr{L}(\mathsf{L}_1^{\mathtt{11}})$ generated by it.

Working of Liveness-based GC (Mark phase)

- GC invoked at a program point π
- GC traverses a path α starting from a root variable x.
- GC consults L_π^x:
  - Does $\alpha \in \mathscr{L}(\mathsf{L}_\pi^x)$?
  - If yes, then mark the current cell
- Note that α is a *forward*-only access path
  - consisting only of edges **0** and **1**, but not $\bar{\mathbf{0}}$ or $\bar{\mathbf{1}}$
  - But $\mathscr{L}(\mathsf{L}_\pi^x)$ has access paths marked with $\bar{\mathbf{0}}/\bar{\mathbf{1}}$ for **0**/**1** removal arising from the **cons** rule.

$\bar{\mathbf{0}}/\bar{\mathbf{1}}$ handling

- **0** removal from a set of access paths:

$$
\begin{aligned}
&\alpha_1\bar{\mathbf{0}}\mathbf{0}\alpha_2 \hookrightarrow \alpha_1\alpha_2 \\
&\alpha_1\bar{\mathbf{0}}\mathbf{1}\alpha_2 \hookrightarrow \text{ drop } \alpha_1\bar{\mathbf{0}}\mathbf{1}\alpha_2 \text{ from the set}
\end{aligned}
$$

- **1** removal from a set of access paths:

$$
\begin{aligned}
&\alpha_1\bar{\mathbf{1}}\mathbf{1}\alpha_2 \hookrightarrow \alpha_1\alpha_2 \\
&\alpha_1\bar{\mathbf{1}}\mathbf{0}\alpha_2 \hookrightarrow \text{ drop } \alpha_1\bar{\mathbf{1}}\mathbf{0}\alpha_2 \text{ from the set}
\end{aligned}
$$

GC decision problem

- Deciding the membership in a CFG augmented with a fixed set of unrestricted productions.

$$
\begin{aligned}
\bar{\mathbf{0}}\mathbf{0} &\rightarrow \epsilon \\
\bar{\mathbf{1}}\mathbf{1} &\rightarrow \epsilon
\end{aligned}
$$

- The problem shown to be undecidable[1].
  - Reduction from Halting problem.

[1] Prasanna, Sanyal, and Karkare. *Liveness-Based Garbage Collection for Lazy Languages*, ISMM 2016.

Practical $\bar{\mathbf{0}}/\bar{\mathbf{1}}$ simplification

- The simplification is possible to do on a finite state automaton.
- Over-approximate the CFG by an automaton (Mohri-Nederhoff transformation).
- Perform **0**/**1** removal on the automaton.

Example

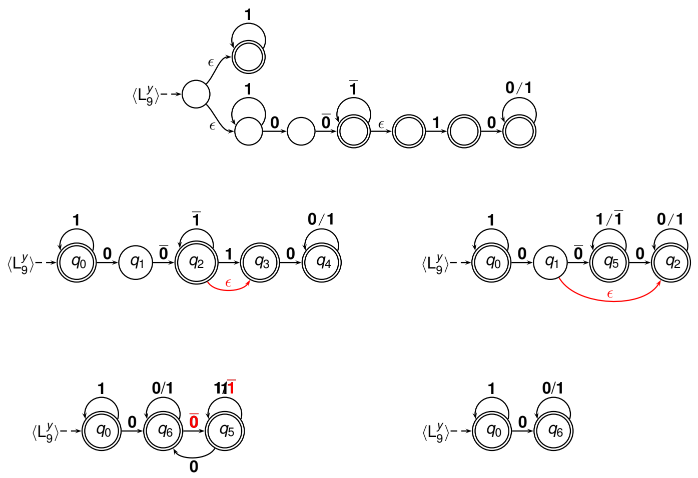
Grammar for L_9^y

$$
\begin{aligned}
L_9^y &\to I^1_{\textbf{append}} \mid D^1_{\textbf{append}}(\epsilon \mid \mathbf{1} \mid \mathbf{10}\sigma_{all}) \\
I^1_{\textbf{append}} &\to \epsilon \mid \mathbf{1} I^1_{\textbf{append}} \\
D^1_{\textbf{append}} &\to \mathbf{0}\bar{\mathbf{0}} \mid \mathbf{1} D^1_{\textbf{append}} \bar{\mathbf{1}} \\
\sigma_{all} &\to \epsilon \mid \mathbf{0}\sigma_{all} \mid \mathbf{1}\sigma_{all}
\end{aligned}
$$

After Mohri-Nederhoff transformation

$$
\begin{aligned}
L_9^y &\to I^1_{\textbf{append}} \mid D^1_{\textbf{append}}(\epsilon \mid \mathbf{1} \mid \mathbf{10}\sigma_{all}) \\
I^1_{\textbf{append}} &\to \epsilon \mid \mathbf{1} I^1_{\textbf{append}} \\
D^1_{\textbf{append}} &\to \mathbf{0}\bar{\mathbf{0}}\widehat{D}^1_{\textbf{append}} \mid \mathbf{1} D^1_{\textbf{append}} \\
\widehat{D}^1_{\textbf{append}} &\to \bar{\mathbf{1}}\widehat{D}^1_{\textbf{append}} \mid \epsilon \\
\sigma_{all} &\to \epsilon \mid \mathbf{0}\sigma_{all} \mid \mathbf{1}\sigma_{all}
\end{aligned}
$$

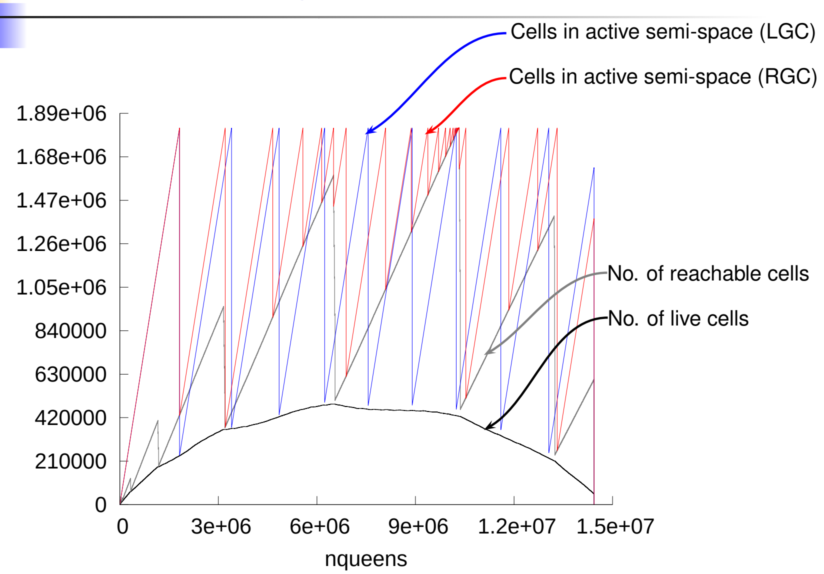
Automaton for L_9^y



Experimental Setup

- Built a prototype consisting of:
  - An ANF-scheme interpreter
  - Liveness analyzer
  - A single-generation copying collector.
- The collector optionally uses liveness
  - Marks a link during GC only if it is live.
- Benchmark programs are mostly from the no-fib suite.

GC behavior as a graph



Results as Tables

Analysis Performance:

Program	sudoku	lcss	gc_bench	knightstour	treejoin	nqueens	lambda
Time (msec)	120.95	2.19	0.32	3.05	2.61	0.71	20.51
DFA size	4251	726	258	922	737	241	732
Precision(%)	**87.5**	**98.8**	**99.9**	**94.3**	**99.6**	**98.8**	**83.8**

Garbage collection performance

Program	# Collected cells per GC		#GCs		MinHeap (#cells)		GC time (sec)	
	RGC	LGC	RGC	LGC	RGC	LGC	RGC	LGC
sudoku	**490**	**1306**	22	9	1704	589	.028	.122
lcss	**46522**	**51101**	8	7	52301	1701	.045	.144
gc_bench	**129179**	**131067**	9	9	131071	6	.086	.075
nperm	**47586**	**174478**	14	4	202597	37507	1.406	.9
fibheap	**249502**	**251525**	1	1	254520	13558	.006	.014
knightstour	**2593**	**314564**	1161	10	508225	307092	464.902	14.124
treejoin	**288666**	**519943**	2	1	525488	7150	.356	.217

Lazy evaluation

- An evaluation strategy in which evaluation of an expression is postponed until its value is needed
  - Binding of a variable to an expression **does not force evaluation** of the expression
- Every expression is evaluated at most once

Laziness: Example

```
(define (length l)
  (if (null? l)
    return 0
    return (+ 1 (length (cdr l)))))
```

```
(define (main)
  (let a ← ( a BIG closure ) in
    (let b ← (+ a 1) in
      (let c ← (cons b nil) in
        (let w ← (length c) in
          (return w))))))
```

Handling lazy semantics: Challenges

- Laziness complicates liveness analysis itself.
  - Data is made live by evaluation of closures
  - In lazy languages, the place in the program where this evaluation takes place cannot be statically determined
- Liveness-based garbage collector significantly more complicated than that for an eager language.
  - Need to track liveness of closures
  - But a closure can escape the scope in which it was created
  - Solution: carry the liveness information in the closure itself
  - For precision: need to update the liveness information as execution progresses

Handling possible non-evaluation

- Liveness no longer remains independent of demand σ
  - If (**car** x) is not evaluated at all, it does not generate any liveness for x
- Require a new terminal **2** with following semantics

$$\mathbf{2}\sigma \hookrightarrow \begin{cases} \emptyset & \text{if } \sigma = \emptyset \\ \{\epsilon\} & \text{otherwise} \end{cases}$$

$$\mathcal{L}app((\textbf{car}\ \mathrm{x}), \sigma) = \mathrm{x}.\{\mathbf{2}, \mathbf{0}\}\sigma$$

Scope for future work

- Reducing GC-time.
  - Reducing re-visits to heap nodes.
  - Basing the implementation on full Scheme, not ANF-Scheme
- Increasing the scope of the method.
  - Lazy languages. (Under review)
  - Higher order functions.
    - Specialize all higher order functions (Firstification)
    - Analysis on the firstified program
    - For partial applications, carry information about the *base* function
- Using the notion of *demand* for other analysis.
  - Program Slicing
  - Strictness Analysis
    - All path problem, requires doing intersection of demands
    - $\Rightarrow$ intersection of CFGs $\Rightarrow$ under-approximation

Conclusions

- Proposed a liveness-based GC scheme.
- Not covered in this talk:
  - The soundness of liveness analysis.
  - Details of undecidability proof.
  - Details of handling lazy languages.
- A prototype implementation to demonstrate:
  - the precision of the analysis.
  - reduced heap requirement.
  - reduced GC time for a majority of programs.
- Unfinished agenda:
  - Improving GC time for a larger fraction of programs.
  - Extending scope of the method.