CS618: Program Analysis
2016-17 Ist Semester

Typed Arithmetic Expressions

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Reference Book

Types and Programming Languages by Benjamin C. Pierce

Recap: Untyped Arithmetic Expression Language

t :=	– *terms*
`true`	– *constant true*
`false`	– *constant false*
`if` t `then` t `else` t	– *conditional*
0	– *constant zero*
`succ` t	– *successor*
`pred` t	– *predecessor*
`iszero` t	– *zero test*

Recap: The Set of Values

v :=	– *values*
`true`	– *value true*
`false`	– *value false*
0	– *value zero*
`succ` v	– *successor value*

Let's add Types to the Language

T :=	– *Types*
Bool	– *Booleans*
Nat	– *Natural Numbers*

The Typing Relation

- A set of rules assigning types to terms
- $\vdash t : T$ denotes "term t has type T"

$$0 : \text{Nat}$$

$$\frac{t_1 : \text{Nat}}{\texttt{succ}\ t_1 : \text{Nat}}$$

$$\frac{t_1 : \text{Nat}}{\texttt{pred}\ t_1 : \text{Nat}}$$

$$\frac{t_1 : \text{Nat}}{\texttt{iszero}\ t_1 : \text{Bool}}$$

The Typing Relation (contd. . .)

- A set of rules assigning types to terms
- $\vdash t : T$ denotes "term t has type T"

$$\texttt{true} : \text{Bool}$$

$$\texttt{false} : \text{Bool}$$

$$\frac{t_1 : \text{Bool} \qquad t_2 : T \qquad t_3 : T}{\texttt{if}\ t_1\ \texttt{then}\ t_2\ \texttt{else}\ t_3 : T}$$

The Typing Relation: Definition

- The *typing relation* for arithmetic expressions is the smallest binary relation between terms and types satisfying all instances of the rules defined earlier.
- A term t is *typable* (or *well typed*) if there is some T sych that $t : T$.

Inversion of the Typing Relation

- If $\vdash 0 : R$, then $R = \text{Nat}$.
- If $\vdash \texttt{succ}\ t_1 : R$, then $R = \text{Nat}$ and $\vdash t_1 : \text{Nat}$.
- If $\vdash \texttt{pred}\ t_1 : R$, then $R = \text{Nat}$ and $\vdash t_1 : \text{Nat}$.
- If $\vdash \texttt{iszero}\ t_1 : R$, then $R = \text{Bool}$ and $\vdash t_1 : \text{Nat}$.
- If $\vdash \texttt{true} : R$, then $R = \text{Bool}$.
- If $\vdash \texttt{false} : R$, then $R = \text{Bool}$.
- If $\Gamma \vdash \texttt{if}\ t_1\ \texttt{then}\ t_2\ \texttt{else}\ t_3 : R$, then
  - $\Gamma \vdash t_1 : \text{Bool}$
  - $\Gamma \vdash t_2 : R$
  - $\Gamma \vdash t_3 : R$

Uniqueness of Types

- Every term t has at most one type.
- If t is typeable, then its type is unique.
- Moreover, there is just one derivation of this typing built from the inference rules.

Safety = Preservation + Progress

- The type system is *safe* (also called *sound*)
- Well-typed programs do not “go wrong.”
  - Do not reach a “stuck state.”
- **Progress:** A well-typed term is not stuck.
  - If $\vdash t : T$, then t is either a value or there exists some t' such that $t \to t'$.
- **Preservation:** If a well-typed term takes a step of evaluation, then the resulting term is also well-typed.
  - If $\vdash t : T$ and $t \to t'$, then $\vdash t' : T$.