

# Game theoretic Verification of Timed systems

## Research proposal

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Game theory has been a valuable tool for modeling the interaction of several agents with individual and possibly conflicting objectives. Games can be categorized based on the number of players, winning conditions, collaboration between players, the way the game progresses, amount of information available to players, zero-sum games and so on. Given the kind of game, there are several interesting and relevant questions to be answered ranging from *whether a given game is determined, existence of winning strategies, equilibrium* and so on.

In computer science, games are used for verification and synthesis of reactive systems. Though the focus is usually on *zero-sum two player* games, people have now started looking at non-zero-sum games useful in the verification of multi-component systems. Research is centered around the notions of *Nash equilibrium* and *subgame perfect equilibrium* for games with Borel winning conditions and simpler games having winning objectives given as *reachability, Muller, Rabin, Street and Parity* conditions on directed graphs. The decision questions of the existence of *Nash equilibria, subgame perfect equilibria* as well as the complexity of these have also been studied [1], [2].

Connections of logics with games has been another area of research in computer science. Logics such as *Game Logic,  $\mu$ -calculus* and *Alternating Temporal Logic* have been used to specify properties of open systems. Relationships between these logics and model checking [6] these logics over concurrent/Kripke/synchronous/asynchronous game structures have been looked at. This has been extended to a timed variant of ATL and its corresponding model checking over timed concurrent game structures [5].

I would like to work on the following problems:

1. **A timed extension of a suitable fragment of Parikh's *Game Logic***  
Its comparison with timed ATL(TATL) [8], its model checking question with respect to timed automata, and its subsequent complexity. It is known [8] that model checking a fragment of TATL on timed automaton is EXPTIME-complete.
2. **The notions of *Nash equilibria, subgame perfect equilibria, mixed strategies* in the context of timed games (game extension of timed automata as well as timed extension of game structures).**

The notion of equilibria in the context of timed games has not been explored. The question of existence and computability of equilibria (Nash equilibria for stochastic zero-sum games) [2], [3] as well as subgame perfect equilibria for infinite multiplayer zero-sum games with reachability, safety, Buchi, co-Buchi and parity objectives [4] has been studied. It would be

interesting to pursue this question in a timed setting. Another interesting long standing open question in this area (for untimed games) is whether we can improve the  $NP \cap co - NP$  algorithm for the existence of a winning strategy in parity games to a  $P$  algorithm.

3. **Coalitional games on timed game structures (concurrent/turn based/synchronous/asynchronous).**

These are games where a set of players form a *group* with a common objective and the focus is on existence of a “stable” coalition resulting in an equilibrium.

## References

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