### NISC: Non-Interactive Secure Computation

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Dating for cryptographers

Dating for cryptographers

+----+ | Alice | +----+

status: looking <u>encrypted preferences</u>

Prefs Age range: Sex: Interests include:



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Feature: Alice can post her preferences and go offline

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NIZK

Our Problem: Obtain the same result against malicious players 0 With blackbox PRG (and as little overhead as possible) no good!

### NISC/OT

(NISC over OT)

- Functionality (single instance version): Alice and Bob give x and y respectively. Alice gets f(x,y).
- Structure of protocol:
  - Alice and Bob invoke <u>several instances of OT in parallel</u> with Alice as receiver
  - Alice then carries out a <u>local computation</u>, and outputs f(x,y) (or "abort")
- Security: UC security (against active corruption) in the OT-hybrid model

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- NISC/OT can be converted to NISC/CRS using [PVW'08]

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- Honest-but-curious players:
  - NISC/OT using Yao's garbled circuit [Y'86]
  - NISC from <u>fully homomorphic encryption</u> [RAD'78,G'09,GHV'10,...]: Low communication (but currently less practical); uses more than PRG+OT
- Malicious players:
  - Use a <u>NIZK</u> to prove correctness of messages sent [CCKM'00,HK'07]: Expensive, and non-blackbox use of PRG (used for encryption in Yao)
  - IPS'08 (full version)]: using "MPC-in-the-head." Non-blackbox use of PRG

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  - Today: NISC/OT using PRG as a black-box
    - Wide Open: Statistically secure NISC/OT (even constant round MPC) possible for general functions?
      - Open for honest-majority and honest-but-curious settings too

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  - Also, few PRG calls: polylog( $\kappa$ ) per gate of the function's (large) circuit (previously  $\Omega(\kappa)$  even for interactive constant-round SFE [LP'07])
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- Also, Reusable NISC in CRS model (using PRG + OT protocol): One reusable "public-key" that Alice publishes and uses in many executions.
  - Issue: public-key must be refreshed each time Alice interacts with the environment (possibly after receiving messages from many Bobs)
  - We show how to allow t such interactions before refreshing, with public-key much shorter than t times the original

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  - Input Value Disjunction (IVD) predicate:
     P<sub>S</sub>(x)=1 iff x<sub>i</sub>=b<sub>i</sub> for some (i,b<sub>i</sub>) ∈ S
  - Wire Value Disjunction (WVD) predicate:
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- Maybe good enough in practice: leaks at most one bit (or less, if Alice aggregates many executions before taking any action) of information about Alice's input

NISC for NC<sup>0</sup> with IVD-abort

<u>Step 1</u>: NISC/OT for NC<sup>0</sup>
 functions (with IVD-abort)

Using Yao's garbled circuit and Oblivious MAC



- Step 1: NISC/OT for NC<sup>0</sup> functions (with IVD-abort)
- Step 2: NISC/H for NC<sup>0</sup> function H. Use H to compile Yao's garbled circuit. (Three variants.)

#### Using Yao's garbled circuit and Oblivious MAC



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- Step 3: Plug-in NISC/OT for NC<sup>0</sup> into NISC/NC<sup>0</sup> schemes


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- Step 4: Handle IVD/WVD-aborts







### Using Yao's garbled circuit and Oblivious MAC



0

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NISC

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NISC











#### Honest-Using Yao's garbled circuit and Oblivious MAC Majority MPC Lean NISC/NC<sup>0</sup> NISC/NC<sup>0</sup> NISC/NC<sup>0</sup> MPC NISC for NISC for with inp.dep-abort using cut&choose with WVD-abort in the NC<sup>0</sup> with cert-OT with head IVD-abort IVD-abort NISC with NISC with NISC with For NC<sup>0</sup> functions, unconditionally IVD-abort WVD-abort inp.dep-abort secure NISC/OT [Kilian'88, IPS'08], Input-Private but not very efficient encoding circuits First build NISC/OT with IVD-abort for "certified-OT" NISC







NISC



Bob sends a garbled circuit to Alice

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Seach wire w has a secret mask r<sub>w</sub>, and two encryption keys K<sub>w</sub>(0) and K<sub>w</sub>(1). r<sub>w</sub> = 0 for Alice's input wires and the output wires.

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Gate	a	b'	c'=F <sub>G</sub> (a,b)'
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1	0	1	0
1	1	0	1
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  - Store [c',K<sub>w</sub>(c')]<sub>G,a',b'</sub> : (c',K<sub>w</sub>(c')) encrypted using K<sub>u</sub>(a') & K<sub>v</sub>(b')

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- For each gate G, and each pair of masked values (a',b') of inputs to the gate, let c' be the masked output Malicious Bob can pick arbitrary c'!
  - Store [c',K<sub>w</sub>(c')]<sub>G,a',b'</sub> : (c',K<sub>w</sub>(c')) encrypted using K<sub>u</sub>(a') & K<sub>v</sub>(b')
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- Correctness: Verify(msg,tag;key,rcpt) = 1
- Onforgeability: can't find (msg,tag) and (msg',tag') s.t. msg' ≠ msg, and for rcpt=OM<sub>key</sub>(msg,tag), Verify(msg',tag';key,rcpt) = 1



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- Input-dependent abort security: since abort can depend on the inputs in a fairly complicated way







- creates the garbled circuit, using purported PRG values given by Bob
- applies OM to those PRG values: using "NC<sup>0</sup> MAC" [IKOS'08]

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NISC/NC<sup>0</sup>

using cut&choose

NISC with NISC with IVD-abort WVD-abort Input-Private encoding circuits NISC

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Using Yao's garbled circuit and Oblivious MAC

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NISC

























